

# Journal of Applied Sciences

ISSN 1812-5654





Journal of Applied Sciences 11 (2): 300-307, 2011 ISSN 1812-5654 / DOI: 10.3923/jas.2011.300.307 © 2011 Asian Network for Scientific Information

# Risk of Static Security Assessment of a Power System using Non-sequential Monte Carlo Simulation

M. Marsadek, A. Mohamed and Z.M. Norpiah Department of Electrical, Electronic and Systems Engineering, Faculty of Engineering and Built Environment, University Kebangsaan Malaysia, Bangi, 43600 Selangor, Malaysia

Abstract: Power system security assessment based on the concept of risk is required in the current power environment. In risk based static security assessment, the likelihood and severity of security violation are the two main factors that determine the security level of a power system. The objectives of this study was to perform a feasibility study of non-sequential Monte Carlo in probability estimation of contingency and provides an indepth interpretation of risk index value by classifying the risk into low, medium and high risk operating point. The probability estimation of contingency that causes security violation is determined using non-sequential Monte Carlo simulation. In this approach, the continuous severity function is used due to its ability to capture both near violating and violating impact of a contingency. A risk classification technique is also developed so as to provide a qualitative interpretation of the risk index value by classifying the risk as low, medium and high degree of risk. Implementation of the feasibility study of the proposed approach to determine the probabilistic risk in risk based static security assessment using the non-sequential Monte Carlo simulation and to classify the risk has been demonstrated on the IEEE RTS-96 test system.

Key words: Line overload, low voltage, non-sequential monte carlo, risk, severity

# INTRODUCTION

Present power system's interconnections are more complex and its operation has also become complicated. This condition makes it more difficult to monitor and handle power system since the actual power system operating conditions are difficult to predict. In addition, the unprecedented changes in the world's technology have also changed customers' expectation towards availability in the electricity supplies. For example, momentary events that have gone unnoticed a few years ago are now of utmost importance and cannot be neglected. A few seconds of interruption may cause millions of profit loss due to the increased level of dependency on electricity supply in our daily activities. The increase in today's world density population has also forced power systems to operate under increasingly stressed condition and close to their limits. As a consequence, power systems become more heavily loaded and vulnerable to disturbances, hence, putting the security of power systems at risk.

Power system security refers to the degree of risk in its ability to survive imminent contingencies without interruption of customer service (Kundur *et al.*, 2004). It

relates to robustness of the system to imminent contingencies and hence, depends on the system operating condition as well as the contingent probability of disturbances (Kundur et al., 2004). The security of a power system can be violated when it is subjected to contingencies, such as outages of lines and load variation. In the conventional power system security assessment, deterministic security limit is usually referred to as a worst case scenario. This somehow restricts the feasible secure operating condition and hence, limits the economic potential and technical ability of power systems to supply load (Morison, 2002). Furthermore, deterministic approach does not provide information on the condition of current operating point and the extent of security violation (Wan et al., 2000), but only provides information on whether the current operating condition is secure or insecure (Mohammadi and Gharehpetian, 2008). In the current power system environment, security assessment with respect to deterministic security boundary region is no longer relevant (McCalley et al., 1999, 2004; Kirschen and Jayaweera, 2007; Wan et al., 2000; Santo et al., 2004).

Risk Based Security Assessment (RBSA) is a relatively new approach that takes into consideration the uncertainty introduced by an actual power system

operating condition as well as the severity of security violation should a contingency occur. The risk index developed through RBSA can quantitatively capture the probability of occurrence of each possible contingency that may cause security violation and the impact of the event. In general, the study of RBSA can be categorized as risk based static security assessment and risk based dynamic security assessment. Risk Based Static Security Assessment (RBSSA) considers risk of equipment overload and voltage limit violation whereas risk based dynamic security assessment (RBDSA) considers risk of voltage instability and early swing transient instability. This study focuses on RBSSA that considers both low bus voltage (LV) and line overload violation (LO) as security limits.

A considerable amount of research has been done in determining the risk of line overload in power systems in which the first work began in 1994 (McCalley *et al.*, 1999). In reference (McCalley *et al.*, 1999), a predefined list of transmission line outages in the order of N-1 is considered when calculating the risk index value. A comparison was made between risk based and deterministic security assessments of power systems based on single criterion contingency (Kirschen and Jayaweera, 2007). For simplicity, the values of probability of line outage are assumed in McCalley *et al.* (1999) and Kirschen and Jayaweera (2007).

A more comprehensive study on RBSSA of power systems can be seen in McCalley et al. (2004), Ni et al. (2003a, b). In McCalley et al. (2004), the risk index contour plotted was obtained with a limited set of N-1 contingency. Online RBSSA was developed by Ni et al. (2003a, b) to provide rapid online quantification of a security level with an existing or forecasted operating condition considering generator, transformer and transmission line outages. In McCalley et al. (2004), Ni et al. (2003a, b), the contingencies are assumed to be Poisson distributed and hence Poisson probability distribution function (pdf) is used to calculate the probability of contingency occurrence with a given failure

A condition-based risk index for line overload and low bus voltage based on the credibility theory employed to model fuzziness of component outages with a given probability of failure is developed (Feng *et al.*, 2008). Another probabilistic technique that is applied in risk assessment to determine the probability of voltage collapse is by using the Monte Carlo simulation (Arya *et al.*, 2006). In the same reference, a comparison is made between the Monte Carlo simulation and the radial basis function neural network for RBDSA of power systems. A risk index is used to quantify the degree of

risk of the current operating point but a qualitative classification as to whether the risk is deemed to be low, medium or high has never been done. This study presents a new risk classification technique to provide insight information on the security level of current operating point in RBSSA of power systems. Through risk classification, the operating limit of power systems can be clearly seen. This study also explores the feasibility of using the non-sequential Monte Carlo simulation to determine the probabilistic risk in RBSSA. The results obtained from the Monte Carlo simulation are then compared with the calculated Poisson probabilistic risk.

### RISK BASED SECURITY ASSESSMENT

**Overview:** Risk based technique has given a paradigm shift towards security assessment. Risk and reliability have the same implications in which an operating system whose risk level is high is said to be unreliable and vice versa. There are two important attributes in risk assessment, namely likelihood and impact.

Risk is defined as the product of event likelihood and its severity. It can be written as:

RISK (E) = Prob (E)
$$\times$$
Sev (E) (1)

where, E is event.

In RBSA, the probability of a contingency that can cause security violation is termed as event likelihood. RBSSA includes the assessment of risk of Low Voltage (LV) and Line Overload (LO). In a given operating condition the risk of Low Voltage (LV) is equal to the sum of individual's contingency risk and it is given as:

$$RISK(LV) = \sum_{i=1}^{N} RISK_{LV}(E_i)$$

$$= \sum_{i=1}^{N} (PROB(E_i) \times SEV_{LV}(E_i))$$
(2)

where, N is the number of contingency.

Similarly, the risk of Line Overload (LO) is equal to the sum of individual's line contingency risk and it can be written as:

$$RISK(LO) = \sum_{i=1}^{N} RISK_{LO}(E_i)$$
$$= \sum_{i=1}^{N} (PROB(E_i) \times SEV_{LO}(E_i))$$
(3)

**Probability estimation:** The probability distribution function of transmission line outage is assumed as follows (Chen *et al.*, 2006):

$$PROB(F_k) = 1 - e^{-\lambda_k}$$
 (4)

where,  $\lambda_k$  is the failure rate of line k.

Using joint probability distribution and assuming all events are independent, the probability of N-1 contingency in a power system is derived as follows:

$$\begin{split} & \text{PROB}(\overline{E}_{i}) = \text{PROB}(\overline{F}_{1} \cap \overline{F}_{2} \cap ..... \cap \overline{F}_{N-1} \cap \overline{F}_{N}) \\ & = \text{PROB}(\overline{F}_{i}) \prod_{j \neq i}^{N} \text{PROB}(\overline{F}_{j}) \end{split} \tag{5}$$

Assuming a transmission line outage is an event that is collectively exhaustive (Yates and Goodman, 2005); hence the following relationship is valid:

$$PROB(\overline{F}_k) = 1 - PROB(F_k)$$
 (6)

Substituting Eq. 6 into 5:

$$PROB(E_{i}) = PROB(F_{i}) \prod_{j \neq i}^{N} (1 - PROB(F_{j}))$$

$$= (1 - e^{-\lambda_{i}}) * e^{-\sum_{j \neq i} \lambda_{j}}$$
(7)

Equation 7 is consistent with the poisson distribution function used to determine the probability of contingency in references (McCalley *et al.*, 2004; Ni *et al.*, 2003a, b).

**Severity function modeling:** Severity functions are adopted to uniformly quantify the severity of network performance for low voltage and line overload. In general, there are three types of severity functions, namely; discrete severity function, continuous severity function and percentage of violation severity function (McCalley *et al.*, 2000). In this study, the continuous severity function is chosen due to its ability to capture the near violating and violating impact of security violation (Marsadek *et al.*, 2009). The continuous severity function for low voltage of each bus is shown in Fig. 1.

For each bus, its severity function evaluates to 1 at the deterministic limits of 0.95 p.u and increases linearly as voltage magnitude falls below the specified limit. Therefore, the severity of low voltage for each contingency can be calculated as follows:

$$SEV_{LV}(k) = \sum_{i=1}^{N} SEV_{LV}(i_k)$$
(8)

$$SEV_{LV}(i_k) = \begin{cases} -20(|V|-1) \\ 0 \end{cases}$$
 (9)

Where:

i = Bus number

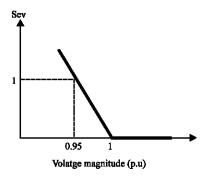


Fig. 1: Continuous severity function for low voltage

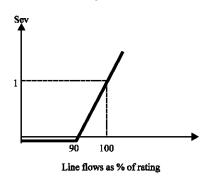


Fig. 2: Continuous severity function for line overload

k = Contingency number

|V| = Bus voltage magnitude

N = Total bus

The continuous severity function for line overload is shown in Fig. 2. For each circuit, its severity function evaluates to 1 at its deterministic limits of 100% and increase linearly as line flow exceeds the limit. Severity of line outage for each contingency can be calculated as follows:

$$SEV_{LO}(k) = \sum_{s=1}^{M} SEV_{LO}(S_k)$$
 (10)

$$SEV_{LV}(S_k) = \begin{cases} 10(P_k - 0.9), \text{for } P_k > 90\% \\ 0, \text{for } P_k \le 90\% \end{cases}$$
 (11)

Where:

S = Circuit number

k = Contingency number

Pk = Percent of line rating

M = Total circuit number

# FRAMEWORK OF MONTE CARLO BASED RBSSA

In RBSSA, risk evaluation is performed in order to assess the overall risk of power systems when subjected to contingencies. There are two main stages in risk evaluation process which involves selection of power system state and calculation of state probability and determination of risk index value. The process of computing the risk index value is a straightforward process. There are two commonly used methods for system state selection in a power system namely state enumeration and Monte Carlo simulation (Li, 2005). In state enumeration, system state is selected based on the enumerated contingency list. However, it is not computationally efficient to enumerate all possible contingencies when the power system size is large. In some extent, state enumeration process is very similar to deterministic process (Rei and Schilling, 2008). On the other hand, computational burden of Monte Carlo simulation does not depend on system size or complexity. Monte Carlo simulation uses random event generator that works by sampling a system state and it can be categorized into sequential and non-sequential sampling. Non-sequential sampling is adopted in this study since, it does not require chronological time dependant event. The concept of non-sequential Monte Carlo simulation is based on the fact that a system state is a combination of all component states and each component state can be determined by sampling the probability of the component appearing in that state (Li, 2005). As a stopping criterion in Monte Carlo simulation, the coefficient of variance,  $\beta$ , is often used.

Under the scope of this study, only uncertainty in transmission line outage is considered. Probability estimation using non-sequential Monte Carlo simulation implemented is performed by considering the following steps:

**Step 1:** Initialize counter w = 0

**Step 2:** Generate random column vector,  $R = [r_1, r_2, ... r_{N-1}, r_N]$  consisting N elements, where, N is the number of transmission lines

**Step 3:** Transform the evenly distributed random vector R generated in step 2 into exponential random vector  $Z = \lfloor z_1, z_2, ... z_{N-1}, z_N \rfloor$  using the following relationship (Rubinstein and Kroese, 2008):

$$z_{i} = \frac{1}{\lambda_{i}} \ln \left( r_{i} \right)$$

**Step 4:** Determine vector  $Q = \lfloor q_1, q_2, \dots q_{N-1}, q_N \rfloor$  that contains status of transmission lines by comparing each element in Z with the respective probability of failure, i.e.,:

$$\begin{aligned} \boldsymbol{q}_i = \begin{cases} 0 \text{ (success)} &, & \boldsymbol{z}_i > & PROB(\boldsymbol{F}_i) \\ 1 \text{ (failure)} &, & 0 \leq \boldsymbol{z}_i \leq & PROB(\boldsymbol{F}_i) \end{cases} \end{aligned}$$

**Step 5:** Update counter w = w+1

**Step 6:** Check if there is only one nonzero element in vector Q, if yes proceed with Step 7, otherwise, repeat Step 2

**Step 7:** Obtain the value of P<sup>i</sup><sub>w</sub>using the following rule:

If 
$$q_i = 1$$
, then  $P_w^i = 1$  for  $i = 1, 2, ...$ 

**Step 8:** Estimate the probability of line outage by using:

$$p_{i} = \frac{\sum_{i=1}^{w} P_{w}^{i}}{W}, \text{ For } i = 1, 2, \dots, N$$
 (12)

**Step 9:** Obtain coefficient of variation,  $\beta$ , which is given by Arya *et al.* (2006):

$$\beta_i = \frac{\sqrt{V(p_i)}}{p_i} \tag{13}$$

Where:

$$V(p_i) = \frac{V_i(X)}{w}$$

$$V_{i}(X) = \frac{\sum_{t=1}^{w} (P_{t}^{i} - P_{i})^{2}}{(w - 1)}$$
 For  $i = 1, 2, ...., N$ 

**Step 10:** If  $\beta \le \xi$ , terminate the Monte Carlo simulation, otherwise repeat from Step 2 onwards

The value of  $\xi$  selected is 0.04.

**Step 11:** Compute the relative error which is given by:

$$Error(\%) = \frac{|prob_{MC} - prob_{Poisson}|}{prob_{Poisson}} \times 100$$
 (14)

where,  $\operatorname{prob}_{MC}$  is the probability calculated from monte carlo simulation,  $\operatorname{prob}_{Poisson}$  is the probability calculated using Poisson pdf.

#### RISK CLASSIFICATION

The value of risk index quantifies the degree of risk of the current operating condition. However, further interpretation on whether the risk index value is deemed to be high, medium or low has yet to be made. This study explores on how risk classification can be made in RBSSA and the proposed risk classification is shown in Fig. 3.

Figure 3 shows the plot of risk index values with respect to the operating points. Points P<sub>0</sub> and P<sub>3</sub> indicate the range of possible loading conditions in the acceptable region. This acceptable region refers to the feasible

operating condition before a power system becomes insecure. From Fig. 3, the operating point,  $P_0$  refers to the load at base case condition whilst the operating point,  $P_3$  refers to the maximum permissible load before the operating point becomes unacceptable. Unacceptable in this context means the operating point becomes insecure even when all transmission lines are in service. The acceptable operating point region is then divided into three equally spaced risks. Power system risk is classified as low, medium and high if the risk index values are between  $RI_0$  and  $RI_1$ ,  $RI_1$  and  $RI_2$  and  $RI_2$  and  $RI_3$ , and  $RI_3$ , and  $RI_3$ , and  $RI_3$  and  $RI_4$  and  $RI_4$  and  $RI_5$  and  $RI_5$ 

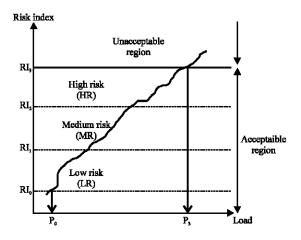


Fig. 3: Risk classification

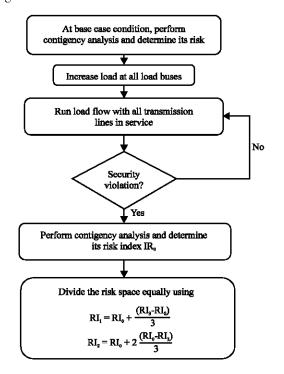


Fig. 4: Flowchart for risk classification

respectively. The implementation of the proposed risk classification is described by referring to the flowchart shown in Fig. 4.

### RESULTS AND DISCUSSION

The proposed method for RBSSA is performed on the IEEE RTS-96 (Billinton *et al.*, 1999) in which the system model consists of 24 buses, 35 transmission lines including two parallel lines and 5 transformers as shown in Fig. 5. The total real and reactive power load at base case condition is 2850 MW + j580 MVar. For contingency analysis, only transmission line outages are considered in the study. A two-state single repairable Markov model is assumed for all the transmission lines (Billinton and Allan, 1994).

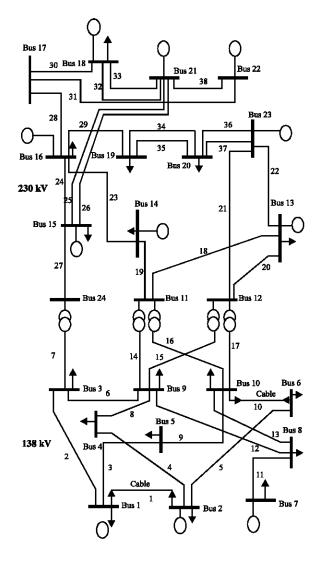


Fig. 5: IEEE RTS-96

Table 1: Probability estimation					
Line outage			Probability of line outage		
Line	From bus	To bus	Poisson (×10 <sup>-6</sup> )	Monte carlo (×10 <sup>-6</sup> )	 Error (%
L1	Bus 1	Bus 2	4.58	4.71	2.76
L2	Bus 1	Bus 3	11.20	11.80	4.6
L3	Bus 1	Bus 5	6.60	6.55	0.65
L4	Bus 2	Bus 4	8.05	7.86	2.39
L5	Bus 2	Bus 6	10.40	10.50	1.41
L6	Bus 9	Bus 3	7.80	7.49	4.06
L7	Bus 4	Bus 3	0.34	0.24	44.83
L8	Bus 9	Bus 4	7.31	7.71	5.14
L9	Bus 10	Bus 5	6.83	7.07	3.38
L10	Bus 10	Bus 6	6.60	6.59	0.11
L11	Bus 7	Bus 8	5.90	5.74	2.80
L12	Bus 9	Bus 8	9.32	9.51	1.91
L13	Bus 10	Bus 8	9.32	9.39	0.69
L14	Bus 11	Bus 9	0.34	0.26	31.67
L15	Bus 12	Bus 9	0.34	0.32	7.28
L16	Bus 11	Bus 10	0.34	0.39	12.22
L17	Bus 12	Bus 10	0.34	0.33	3.45
L18	Bus 13	Bus 11	8.30	8.62	3.79
L19	Bus 14	Bus 11	8.05	8.04	0.14
L20	Bus 13	Bus 12	8.30	8.29	0.03
L21	Bus 23	Bus 12	11.50	11.80	2.2
L22	Bus 13	Bus 23	10.70	10.60	0.52
L23	Bus 16	Bus 14	7.80	7.79	0.13
L24	Bus 16	Bus 15	6.60	6.55	0.65
L25	Bus 21	Bus 15	8.55	8.64	0.99
L26	Bus 15	Bus 24	8.55	8.46	1.08
L27	Bus 17	Bus 16	7.07	7.09	0.35
L28	Bus 19	Bus 16	6.83	6.89	0.91
L29	Bus 18	Bus 17	6.36	6.51	2.21
L30	Bus 22	Bus 17	12.10	12.20	0.71
L31	Bus 18	Bus 21	7.07	7.04	0.49

**Probability estimation:** To generate uniform random vector as stated in step 2 of the Monte Carlo simulation, the Excel program which has a built-in random number generator is used. Table 1 shows the comparison of probability estimation generated by using the Monte Carlo simulation and Poisson for N-1 contingency of transmission line outage.

7.98

2.23

2.74

L32 Bus 19

Bus 23

Bus 20

7.80

From Table 1, it is shown that the probability estimation of contingency determined using non-sequential monte carlo provide small percentage different when compared with the poisson distribution function. In most cases, the percentage different was found to be within 0.01 to 5% except for line outages of lines L7, L14, L15 and L16. As can be seen from both Poisson and Monte Carlo simulation result these 4 line outages have the lowest probability of outage, therefore small discrepancy will cause in a large relative error.

**Severity function value:** The continuous severity functions values shown in Fig. 1 and 2 are calculated based on the test system post-contingency performance. To obtain these values, line outage simulations are carried out using the Power System Analysis Toolbox (PSAT) (Milano, 2005). From the outage simulation results, a

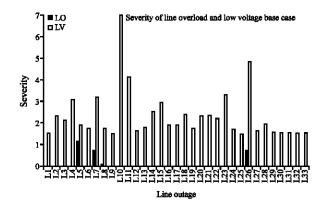


Fig. 6: Severity function value for line overload and low voltage at base case

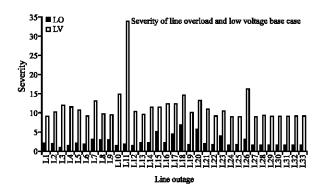


Fig. 7: Severity function value for line overload and low voltage at maximum permissible load

database consisting of bus voltages and line flows is formed and the severity functions are calculated using Eq. 9 and 11. The severity function value provides information on whether any particular contingency causes near violation or violation in static security. If the low voltage severity function value of a particular contingency is non zero, it denotes that at least one or more bus is having voltage magnitude of less than 0.95 pu. However, if the line overload severity function value of a particular contingency is non zero, it means that the line flow exceeds its 90% rating. The severity function values for line overload and low voltage at base case and maximum permissible loads are shown in Fig. 6 and 7, respectively.

At base case condition, line outage, L5 which is the transmission line connecting bus 2 and bus 6 has the highest line overload severity function value. This implies that L5 is the most severe contingency that causes line overload at base case condition. On the other hand, the outage of line connecting bus 10 and bus 6 (L10) is the most severe contingency in low voltage violation. From

Fig. 6, it is also noted that at base case condition, the severity of low voltage is more prominent compared to line overload. All the line outages at base case condition cause near violation or violation in low voltage while only 4 line outages (L5, L7, L8 and L26) cause near violation or violation in line overload with their severity values much less than low voltage severity value.

The maximum permissible load for line overload and low voltage is 30 and 40% increase in load from base case respectively. At maximum permissible load, outage in L11 cause severe low voltage violation since its low voltage severity function is the highest. As can be seen from Fig. 7, outage in L5 is no longer the most severe contingency for line overload given that line outage of L18 has the maximum line overload severity function value. In general, at maximum permissible load all line outages cause near violation or violation in static security. In addition to this, severity function values for both line overload and low voltage at maximum allowable load increase tremendously when compared to base case condition.

**Risk index classification:** The risk index values at various load conditions for low voltage and line overload are shown in Fig. 8 and 9, respectively. In this study, the risk index is calculated at every 5% increase in load from base case until it reaches its maximum permissible load.

In general, the risk index curve for low voltage shown in Fig. 8 depicts an increasing pattern of risk index with respect to total load demand. The risk classification result shows that the operating point is categorized as low risk when load is increased not more than 20% from base case. The operating point is in medium risk when load is increased from 20 to 30% of base case load. When load is increased at greater than 35% of base case load, the operating point is classified as high risk because the load margin between the current operating and maximum permissible load becomes very small. The power system is considered to remain in the acceptable region if the load does not exceed 40% increase from base case load.

A similar interpretation as low voltage risk can be made to risk of line overload in the test system. From Fig. 9, it is noted that the power system should not be operated at loads greater than 15% increase from base case load in order to remain in the low risk region. At 20% increase in load from base case, the power system is said to operate in the medium risk region. When, load is increased to 25% from base case, the operating point is classified as high risk because the load margin between the current operating and maximum permissible load becomes very small. The maximum permissible load for line overload is 30% increase from base case load, which

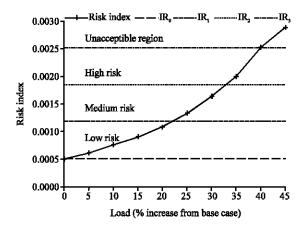


Fig. 8: Risk index for low voltage

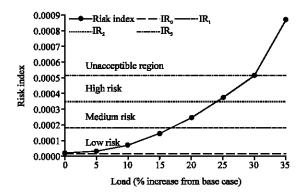


Fig. 9: Risk index for line overload

is 10% less than the maximum allowable load for low voltage. Thus, the power system is said to be at greater risk towards line overload compared to low voltage when load is increased from base case to its maximum allowable load.

### CONCLUSION

The application of Monte Carlo simulation for estimating the probability of contingency in risk based static security assessment has been presented and shown to provide a new approach for calculating likelihood in the occurrence of contingency. The selection of appropriate coefficient of variance is very important when running a Monte Carlo simulation. Large value of coefficient of variation may lead to poor error, whereas small value of coefficient may result in unnecessary computational burden. For the study made on the IEEE RTS-96 test system, a coefficient of variation of 4% is sufficient to result in a reasonable computation error.

The proposed risk classification technique has the ability to qualitatively interpret the numerical values of the

risk index. However, the risk index classification depends on the type of security violation considered which may be either low voltage or line overload violation. From the risk classification results, the maximum permissible load in low voltage is different than the maximum allowable load in line overload. It is shown that the power system may operate at a high line overload risk but still in the medium low voltage risk region.

## REFERENCES

- Arya, L.D., L.S. Titare and D.P. Kothari, 2006. Determination of probabilistic risk of voltage collapse using Radial Basis Function (RBF) network. Electric Power Syst. Res., 76: 426-434.
- Billinton, R. and R.N. Allan, 1994. Reliability Evaluation of Engineering Systems: Concepts and Techniques. Plenum Press, New York.
- Billinton, R., S. Kumar, N. Chowdhury, K. Chu and K. Debnath *et al.*, 1999. A reliability test system for educational purposes-basic data. IEEE Trans. Power Syst., 3: 1238-1244.
- Chen, W.H., Q.Y. Jiang and Y.J. Cao, 2006. Low Voltage Risk Assessment Using Neural Network Ensemble. Springer-Verlag, Germany.
- Feng, Y., W. Wu, B. Zhang and W. Li, 2008. Power system operation risk assessment using credibility theory. IEEE Trans. Power Syst., 23: 1309-1318.
- Kirschen, D.S. and D. Jayaweera, 2007. Comparison of risk-based and deterministic security assessments. IET Generation Transmission Distribution, 4: 527-533.
- Kundur, P., J. Paserba, V. Ajjarapu, G. Andersson and A. Bose et al., 2004. Definition and classification of power system stability. IEEE Trans. Power Syst., 19: 1387-1401.
- Li, W., 2005. Risk Assessment of Power Systems: Models, Methods and Applications. John Wiley and Sons, New York.
- Marsadek, M., A. Mohamed and M.Z. Norpiah, 2009. Risk based static security assessment of power systems considering the effects of severity functions. Proceedings of the 3rd International Power Engineering and Optimization Conference, Shah Alam, Malaysia, June 3-4.

- McCalley, J.D., V. Vittal and N. Abi-Samra, 1999. An overview of risk based security assessment. Proceedings of Power Engineering Society Summer Meeting, July 18-22, Edmonton, Alta, pp. 173-178.
- McCalley, J.D., V. Vittal, M. Ni, S. Greene and A. Phadke, 2000. On-Line Risk-Based Security Assessment. Electric Power Research Institute, USA.
- McCalley, J., S. Asgarpoor, L. Bertling, R. Billinion and H. Chao *et al.*, 2004. Probabilistic security assessment for power system operations. IEEE Soc. General Meet., 1: 212-220.
- Milano, F., 2005. An open source power system analysis toolbox. IEEE Trans. Power Syst., 20: 1199-1206.
- Mohammadi, M. and G.B. Gharehpetian, 2008. Power system on-line static security assessment by using multi-class support vector machines. J. Applied Sci., 8: 2226-2233.
- Morison, K., 2002. Power system security in the new market environment: Future directions. IEEE-Power Eng. Soc. Summer Meet., 3: 1416-1417.
- Ni, M., J.D. McCalley, V. Vittal and T. Tayyib, 2003a. Online risk-based security assessment. IEEE Trans. Power Syst., 18: 258-258.
- Ni, M., J.D. McCalley, V. Vittal, S. Greene, C.W.T. Ganugula and T. Tayyib, 2003b. Software implementation of online risk-based security assessment. IEEE Trans. Power Syst., 18: 1164-1172.
- Rei, A.M. and M.T. Schilling, 2008. Assessment of the Brazilian power system using enumeration and monte carlo. IEEE Trans.Power Syst., 23: 1480-1487.
- Rubinstein, R.Y. and D.P. Kroese, 2008. Simulation and the Monte Carlo Method. 2nd Edn., John Wiley and Sons, New York, pp. 345.
- Santo, M.D., A. Vaccaro, Domenico and E. Zimeo, 2004. A distributed architecture for online power systems security analysis. IEEE Trans. Ind. Elect., 51: 1238-1248.
- Wan, H., J.D. McCalley and V. Vittal, 2000. Risk based voltage security assessment. IEEE Trans. Power Syst., 15: 1247-1247.
- Yates, R.Y. and D.J. Goodman, 2005. Probability and Stochastic Process. John Wiley and Sons, New York.