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Estimation of P (Y<X) in the Rayleigh Distribution in the Presence of k Outliers

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Abstract: In present study, we considered the problem of estimating R = P (Y<X), where Y had Rayleigh distribution with parameter λ and X had Rayleigh distribution with presence of k outliers with parameters θ and β, such that X and Y are independent. The moment, maximum likelihood and mixture estimators of R are derived and had been shown that the moment estimator of R is asymptotically unbiased estimator. At the end, we concluded that mixture estimators are better than the maximum likelihood and moment estimators.

Key words: Rayleigh distribution, moment estimator, maximum likelihood estimator, mixture estimator, outliers, Newton-Raphson method, Monte-Carlo simulation

INTRODUCTION

In reliability context inferences about R = P (Y<X), where X and Y independent distribution, are a subject of interest. For example in mechanical reliability of a system if X is the strength of a component which is subject to stress Y, then R is a measure of system performance. The system fails, if at any time the applied stress is greater than its strength. Stress-strength reliability has been discussed by Kapur and Lamberson (1977). Some other aspects of inference about R are given in Al-Hussaini *et al*. (1997) and Sathe and Dixit (2001) have been estimate of R = P (Y<X) in the negative binomial distribution and recently, Nasiri and Pazira (2010b) have done estimation of R = P (Y<X) with presence of outliers in exponential case.

Dixit *et al*. (1996) assumed that random variables (X_1, X_2,..., X_n) represent the distance of an infected sampled plant from a plant in a plot of plants inoculated with a virus. Some of the observations are derived from the airborne dispersal of the spores and are distributed according to the exponential distribution. The other observations out of n random variables (say k) are present because aphids which are know to be carriers of Barley Yellow Mosaic Dwarf Virus (BYMDV) have passed the virus into the plants when the aphids feed on the sap. Dixit and Nasiri (2001) considered estimation of parameters of the exponential distribution in the presence of outliers generated from uniform distribution (Khamis *et al*., 2006; Elahi *et al*., 2009; Muiru *et al*., 2010; Shittu and Shangodoyin, 2008; Xie *et al*., 2007).

In present study, we obtain the moment, maximum likelihood and mixture estimators of R in Rayleigh distribution with presence of k outliers generated from the same distribution. Rayleigh distribution is a special case of the Weibull distribution. It has been used to study the scattering of radiation, wind speeds or to make certain transformation. The probability density function of the Rayleigh distribution with parameter of θ is given by:

$$f(x,\theta)=\frac{2x}{\theta}e^{-\frac{x^2}{\theta}}, \qquad x,\theta>0$$

The Maximum Likelihood Estimator (MLE) of θ is:

$$\frac{\sum_{i=1}^{n}x_i^2}{n}$$

Thus, we assume that the random variables (X_1, X_2,..., X_n) are such that k of them are distributed with p.d.f f_1 (x, θ, β):

$$f_1(x,\theta,\beta)=\frac{2x\beta}{\theta}e^{-\frac{\beta x^2}{\theta}}, x>0, \theta>0, 0<\beta\le 1 \tag{1}$$

and the remaining (n-k) random variables are distributed with p.d.f f_2 (x, θ):

$$f_2(x,\theta)=\frac{2x}{\theta}e^{-\frac{x^2}{\theta}}, x>0, \theta>0 \tag{2}$$

METHOD OF MOMENT

Let Y_1, Y_2,..., Y_m be a random sample for Y with P.d.f.:

$$g(y,\lambda)=\frac{2y}{\lambda}e^{-y^2/\lambda}, y>0, \lambda>0 \tag{3}$$

and X_1, X_2,..., X_n be a random sample for X with P.d.f.:

$$f(x,\theta,\beta)=\frac{k}{n}\cdot\frac{2x\beta}{\theta}e^{-\beta\frac{x^2}{\theta}}+\frac{n-k}{n}\cdot\frac{2x}{\theta}e^{-\frac{x^2}{\theta}}, x>0,\theta>0, 0<\beta\le 1 \tag{4}$$

with presence of k (Know) outliers (Dixit, 1989; Dixit and Nasiri, 2001; Nasiri and Pazira, 2010a; Abu-Shawiesh *et al*., 2009; Goegebeur *et al*., 2005; Guo and Zhang, 2011; Ismail, 2008). The parameter R we want to estimate is:

$$
\begin{aligned}
R &= P(Y<X) \\
&= \int_0^\infty \int_0^x g(y,\lambda)\, f(x,\theta,\beta)\, dy\, dx \\
&= \int_0^\infty \left(\int_0^x \frac{2y}{\lambda} e^{\frac{-y^2}{\lambda}} dy\right)\left[\frac{k}{n}\frac{2x\beta}{\theta} e^{-\frac{\beta x^2}{\theta}} + \frac{n-k}{n}\frac{2x}{\theta} e^{-\frac{x^2}{\theta}}\right] dx \\
&= \int_0^\infty (1-e^{\frac{-x^2}{\lambda}})\frac{k}{n}\frac{2x\beta}{\theta} e^{-\frac{\beta x^2}{\theta}} dx + \int_0^\infty (1-e^{-x^2/\lambda})\frac{n-k}{n}\frac{2x}{\theta} e^{-\frac{x^2}{\theta}} dx
\end{aligned}
$$

$$
= \frac{k}{n}\frac{\theta}{\lambda\beta+\theta} + \frac{n-k}{n}\frac{\theta}{\lambda+\theta} \tag{5}
$$

For k = 0, R was proposed by Surles and Padgett (1998). Since P (Y<X) + P (Y>X) = 1, we consider P (Y>X):

$$
R = P(Y>X) = \frac{b\lambda\beta}{\lambda\beta+\theta} + \frac{\bar{b}\lambda}{\lambda+\theta} \tag{6}
$$

Where:

$$
b = \frac{k}{n}, \bar{b} = \frac{n-k}{n} \text{ and } b + \bar{b} = 1
$$

The moment estimator of R can be shown to be:

$$
\hat{R} = \frac{b\hat{\lambda}\hat{\beta}}{\hat{\lambda}\hat{\beta}+\hat{\theta}} + \frac{\bar{b}\hat{\lambda}}{\hat{\lambda}+\hat{\theta}} \tag{7}
$$

where, $\hat{\lambda} = \frac{1}{m}\sum_{i=1}^{m} y_i^2$, $\hat{\theta}$ and $\hat{\beta}$ can be obtained as;

From Eq. 4 we get:

$$
E(X) = \frac{b\theta}{\beta} + \bar{b}\theta \tag{8}
$$

$$
E(X^2) = 2b\left(\frac{\theta}{\beta}\right)^2 + 2\bar{b}\theta^2 \tag{9}
$$

$$
\text{Consider, } m_i' = \frac{1}{n}\sum_{j=1}^{n} X_j^i \text{ and let } D = \frac{m'_2}{m'_1} \text{ and } H = \frac{m_2'}{m_1'^2}
$$

$$
D = \frac{\frac{2b}{\beta^2} + 2\bar{b}}{\frac{b}{\beta} + \bar{b}}.\theta \tag{10}
$$

and:

$$
H = \frac{\frac{2b}{\beta^2} + 2\bar{b}}{\left(\frac{b}{\beta} + \bar{b}\right)^2} \tag{11}
$$

From Eq. 11, we have:

$$
\left(\bar{b}^2 H - 2\bar{b}\right)\beta^2 + 2b\bar{b}H\beta + \left(b^2 H - 2b\right) = 0 \tag{12}
$$

$$
\xi_1\beta^2 + \xi_2\beta + \xi_3 = 0 \tag{13}
$$

Where:

$$
\begin{aligned}
\xi_1 &= \bar{b}^2 H - 2\bar{b} \\
\xi_2 &= 2b\bar{b}H \\
\xi_3 &= b^2 H - 2b
\end{aligned}
$$

If $\Delta = \xi_2^2 - 4\,\xi_1\xi_3$ is non-negative then the roots are real. Therefore:

$$
\hat{\beta} = \frac{-\xi_2 + \sqrt{\xi_2^2 - 4\xi_1\xi_3}}{2\xi_1} \tag{14}
$$

and from Eq. 10:

$$
\hat{\theta} = \frac{b\hat{\beta} + \bar{b}\hat{\beta}^2}{2b + 2\bar{b}\hat{\beta}^2}.D \tag{15}
$$

Thus, we can obtain the moment estimator of β and θ by using Eq. 14 and 15, respectively.

Here, we shall show that $\hat{\theta}$ and $\hat{\beta}$ are asymptotically unbiased estimators. Let:

$$
W_1 = \frac{1}{n}\sum_{i=1}^{n} X_i
$$

and:

$$
W_2 = \frac{1}{n}\sum_{i=1}^{n} X_i^2
$$

then:

$$
D = \frac{W_2}{W_1}
$$

Here, we can write $\hat{\theta}$ as a function of W_1, W_2:

$$
\hat{\theta} = h(w_1, w_2) \tag{16}
$$

Let:

$$E(W_1) = \mu = \frac{b\theta}{\beta} + \bar{b}\theta$$

$$E(W_2) = v = 2b\left(\frac{\theta}{\beta}\right)^2 + 2\bar{b}\theta^2$$

Expand the function h (w_1, w_2) around (μ, v) by Taylor series:

$$\hat{\theta} = h(w_1, w_2) = h(\mu, v) + (w_1 - \mu)\frac{\partial h}{\partial w_1}\Big|_{w_1=\mu, w_2=v} + (w_2 - v)\frac{\partial h}{\partial w_2}\Big|_{w_1=\mu, w_2=v} + \ldots \quad (17)$$

Then:

$$E(\hat{\theta}) = h(\mu, v) = \frac{b\beta + \bar{b}\beta^2}{2b + 2\bar{b}\beta^2} \cdot \frac{v}{\mu}$$

$$= \frac{b\beta + \bar{b}\beta^2}{2b + 2\bar{b}\beta^2} \cdot \frac{2b\left(\frac{\theta}{\beta}\right)^2 + 2\bar{b}\theta^2}{b\left(\frac{\theta}{\beta}\right) + \bar{b}\theta}$$

$$= \frac{b\beta + \bar{b}\beta^2}{2b + 2\bar{b}\beta^2} \cdot \frac{2b + 2\bar{b}\beta^2}{b\beta + \bar{b}\beta^2} \cdot \theta$$

$$= \theta$$

and similarly:

$$E(\hat{\beta}) = \frac{b\theta}{b\left(\frac{\theta}{\beta}\right) + \bar{b}\theta - \bar{b}\theta} = \beta$$

Note that if we consider:

$$W_3 = \frac{1}{m}\sum_{i=1}^{m} Y_i^2$$

and $\hat{R} = h(w_1, w_2, w_3)$, It is easy to show that $E(\hat{R}) = R$.

METHOD OF MAXIMUM LIKELIHOOD

The maximum likelihood estimator of R can be shown to be:

$$\hat{R} = \frac{b\hat{\lambda}\hat{\beta}}{\hat{\lambda}\hat{\beta} + \hat{\theta}} + \frac{\bar{b}\hat{\lambda}}{\hat{\lambda} + \hat{\theta}} \quad (18)$$

Where:

$$\hat{\lambda} = \frac{1}{m}\sum_{i=1}^{m} Y_i^2$$

and to obtain the maximum likelihood estimator of θ and β, we consider the joint distribution of $\underline{X}$ with presence of k outliers:

$$L(\underline{x}, \theta, \beta) = \frac{2^n \beta^k \prod_{i=1}^{n} x_i}{C(n,k)\theta^n} e^{-\frac{\sum_{i=1}^{n} x_i^2}{\theta}} \sum_{\underline{A}} e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2} \quad (19)$$

where,

$$C(n,k) = \binom{n}{k}$$

and:

$$\sum_{\underline{A}} = \sum_{A_1=1}^{n-k+1} \sum_{A_2=A_1+1}^{n-k+2} \sum_{A_k=A_{k-1}+1}^{n}$$

For the more details see Dixit and Nasiri (2001) and Nasiri and Pazira (2010a).

If L (θ, β) = In [L ($\underline{x}$, θ, β)], then from Eq. 19:

$$L(\theta, \beta) = n\ln 2 + \sum_{i=1}^{n} \ln x_i + k\ln\beta - n\ln\theta - \ln C(n,k) - \frac{\sum_{i=1}^{n} x_i^2}{\theta} + \ln\sum_{\underline{A}} e^{\frac{1-\beta}{\theta}\sum_{i=1}^{k} x_{A_i}^2} \quad (20)$$

The solve for our MLEs of θ and β we take the derivative of the log likelihood (L (θ, β)) with respect to each parameter set the partial derivatives equal to zero and solve for $\hat{\theta}$ and $\hat{\beta}$:

$$\frac{\partial L(\theta, \beta)}{\partial \theta} = -\frac{n}{\theta} + \frac{\sum_{i=1}^{n} x_i^2}{\theta^2} - (1-\beta)\frac{\sum_{\underline{A}}\left(\sum_{i=1}^{k} x_{A_i}^2\right) e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}}{\theta^2 \sum_{\underline{A}} e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}} \overset{\text{set}}{=} 0 \quad (21)$$

$$\frac{\partial L(\theta, \beta)}{\partial \beta} = \frac{k}{\beta} - \frac{\sum_{\underline{A}}\left(\sum_{i=1}^{k} x_{A_i}^2\right) e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}}{\theta \sum_{\underline{A}} e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}} \overset{\text{set}}{=} 0 \quad (22)$$

There is no closed-from solution to this system of equations, so we will solve for $\hat{\theta}$ and $\hat{\beta}$ iteratively, using the Newton-Raphson method, a tangent method for root finding. In our case we will estimate α = (θ, β) iteratively:

$$\hat{\alpha}_{i+1} = \hat{\alpha}_i - G^{-1}g \quad (23)$$

where, g is the vector of normal equations for which we want g = [g_1, g_2] with:

$$g_1 = \sum_{i=1}^{n} x_i^2 - n\theta - \frac{(1-\beta)\sum_{\underline{A}}\left(\sum_{i=1}^{k} x_{A_i}^2\right) e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}}{\sum_{\underline{A}} e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}} \quad (24)$$

$$g_2 = \frac{k}{\beta} - \frac{\sum_{\underline{A}}\left(\sum_{i=1}^{k} x_{A_i}^2\right) e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}}{\theta \sum_{\underline{A}} e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}} \qquad (25)$$

and G is the matrix of second derivatives:

$$G = \begin{bmatrix} \frac{dg_1}{d\theta} & \frac{dg_1}{d\beta} \\ \frac{dg_2}{d\theta} & \frac{dg_2}{d\beta} \end{bmatrix} \qquad (26)$$

Where:

$$\frac{dg_1}{d\theta} = -n + \left(\frac{1-\beta}{\theta}\right)^2 \left(\frac{\gamma_3}{\gamma_1} + \frac{\gamma_2^2}{\gamma_1^2}\right) \qquad (27)$$

$$\frac{dg_1}{d\beta} = \frac{dg_2}{d\theta} = \left(\frac{1-\beta}{\theta}\right)\left(\frac{\gamma_3}{\gamma_1} - \frac{\gamma_2^2}{\gamma_1^2} + \frac{\theta\gamma_2}{(1-\beta)\gamma_1}\right) \qquad (28)$$

$$\frac{dg_2}{d\beta} = -\frac{k}{\beta^2} + \frac{1}{\theta^2}\left(\frac{\gamma_3}{\gamma_1} - \frac{\gamma_2^2}{\gamma_1^2}\right) \qquad (29)$$

With:

$$\gamma_1 = \sum_{\underline{A}} e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}$$

$$\gamma_2 = \sum_{\underline{A}} \left(\sum_{i=1}^{k} x_{A_i}^2\right) e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}$$

$$\gamma_3 = \sum_{\underline{A}} \left(\sum_{i=1}^{k} x_{A_i}^2\right)^2 e^{\frac{(1-\beta)}{\theta}\sum_{i=1}^{k} x_{A_i}^2}$$

The Newton-Raphson algorithm converges, as our estimates of θ and β change by less than a tolerated amount with each successive iteration, to $\hat{\theta}$ and $\hat{\beta}$. For k = 0, $\hat{\theta}$ was proposed by Surles and Padgett (1998).

Note: Here, the number of k outlying observations is known, k = 1, 2,..., n. But if k is unknown, then k can be selected by evaluating the likelihood for different value of k choosing the one that maximizes the likelihood.

MIXTURE OF METHOD OF MOMENT AND MAXIMUM LIKELIHOOD

Read (1981) proposed the methods which avoid the difficulty of complicated equations. According to Read (1981), replacement of some but not all, of the equations in the system of likelihood may make it more manageable. From Eq. 15, we have:

$$\hat{\theta} = \frac{b\hat{\beta} + \bar{b}\hat{\beta}^2}{2b + 2\bar{b}\hat{\beta}^2} \cdot D \qquad (30)$$

and from Eq. 22:

$$\hat{\beta} = \frac{k\hat{\theta}\sum_{\underline{A}} e^{\frac{(1-\hat{\beta})}{\hat{\theta}}\sum_{i=1}^{k} x_{A_i}^2}}{\sum_{\underline{A}}\left(\sum_{i=1}^{k} x_{A_i}^2\right) e^{\frac{(1-\hat{\beta})}{\hat{\theta}}\sum_{i=1}^{k} x_{A_i}^2}} \qquad (31)$$

SIMULATION STUDY

In present study, we have addressed the problem of estimating P (Y<X) for the Rayleigh distribution with presence of k outliers. The moment, maximum likelihood and mixture estimators of R are derived and has been shown that the moment estimator of R is asymptotically unbiased estimator. All the results are base on 1000 replications and are given in Table 1 and 2 for k = 1 and 2, respectively. In this case as expected when m = n and m, n increase then the average biases and the MSEs decrease. For fixed m as n increase the MSEs decrease and also for fixed n as m increase the MSEs decrease.

Table 1: Biases and Mean Squared Errors (MSE's) of the point estimates from Rayleigh distribution, when k = 1, β = 2, θ = 5, λ = 3

		Bias			MSE		
(n,m)	R	MLE	Mom	Mix	MLE	Mom	Mix
(15,15)	0.3864	0.0143	-0.0325	-0.0427	0.0133	0.0144	0.0143
(20,20)	0.3835	0.0085	-0.0220	-0.0295	0.0089	0.0089	0.0063
(25,25)	0.3818	0.0070	-0.0136	-0.0131	0.0058	0.0061	0.0051
(15,20)	0.3864	0.0131	-0.0299	-0.0405	0.0133	0.0140	0.0128
(20,15)	0.3835	0.0112	-0.0277	-0.0390	0.0125	0.0128	0.0107
(15,25)	0.3864	0.0097	-0.0276	-0.0378	0.0095	0.0099	0.0103
(25,15)	0.3818	0.0085	-0.0226	-0.0373	0.0083	0.0099	0.0097
(20,25)	0.3835	0.0075	-0.0215	-0.0273	0.0067	0.0087	0.0058
(25,20)	0.3818	0.0074	-0.0164	-0.0188	0.0063	0.0084	0.0055

Table 2: Biases and Mean Squared Errors (MSE's) of the point estimates from Rayleigh distribution, when k = 2, β = 2, θ = 5, λ = 3

		Bias			MSE		
(n,m)	R	MLE	Mom	Mix	MLE	Mom	Mix
(15,15)	0.3977	0.0142	-0.0106	-0.0199	0.0386	0.0495	0.0295
(20,20)	0.3920	0.0120	-0.0039	-0.0098	0.0196	0.0199	0.0124
(25,25)	0.3886	0.0104	-0.0017	-0.0041	0.0013	0.0020	0.0034
(15,20)	0.3977	0.0141	-0.0080	-0.0185	0.0373	0.0341	0.0214
(20,15)	0.3920	0.0140	-0.0078	-0.0115	0.0293	0.0313	0.0182
(15,25)	0.3977	0.0123	0.00770	-0.0114	0.0232	0.0262	0.0179
(25,15)	0.3886	0.0122	-0.0074	-0.0101	0.0220	0.0245	0.0133
(20,25)	0.3920	0.0109	-0.0035	-0.0066	0.0185	0.0185	0.0080
(25,20)	0.3886	0.0107	-0.0034	-0.0063	0.0096	0.0117	0.0078

CONCLUSION

From Table 1 and 2, we conjecture that the moment estimator of R is asymptotically unbiased. On the other hand, the moment and mixture estimators are underestimation but the maximum likelihood estimator is overestimation, this is true for k = 1 and 2. The MSEs of any three estimators are tending to zero and when m = n and m, n increase then the MSEs decrease and for fixed m as n increase the MSEs decrease and also for fixed n as m increases the MSEs decrease, this is true for k = 1 and 2.

Table 1 and 2 showed that the mixture estimator has the smallest estimated MSEs as compared with the moment and maximum likelihood estimators. We strongly feel mixture estimator is better and easy to calculate than the maximum likelihood and moment estimations. From the previous observations, we suggest to use mixture method for estimating R = P (Y<X) in Rayleigh case with presence of k outliers because it is easy to calculate than the rest.

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