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A Multi-objective Slack Based Measure of Efficiency Model for Weight Derivation in the Analytic Hierarchy Process

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Abstract: The DEAHP which is a Data Envelopment Analysis (DEA) model for weight derivation in the Analytic Hierarchy Process (AHP), has been found to have some fundamental drawbacks. The principal drawback of DEAHP is that it is only powerful in weight generation for completely consistent pairwise comparison matrices but it may generate illogical and counterintuitive weights for matrices with acceptable consistency ratio. In this paper, a multi objective non-radial DEA model is proposed for weight derivation and aggregation in AHP. It will be shown that the proposed model generates true weights for completely consistent pairwise comparison matrices in addition to producing logical weights for matrices with satisfactory consistency ratio. The proposed model is applied in a hierarchical structure in order to verify its performance in weight generation and aggregation. Finally, the local weights vector for a large scale pairwise comparison matrix is obtained.

Key words: Data envelopment analysis, DEAHP, weight derivation, slacks based measure, MOLP

INTRODUCTION

Analytic Hierarchy Process (AHP) is a powerful tool which is used to deal with complex decision making problems. Each problem can be broken down into several levels where each level represents a set of criteria or sub-criteria or alternatives. Constituting the hierarchical structure, the pairwise comparison matrices are made for each level in order to generate the final priorities for the alternatives (Saaty, 1980). Because of the simple nature of AHP, it has been widely used in numerous decision making problems such as supplier selection (Ghodsypour and O'Brien, 1998), banking (Ray and Mukherjee, 1988; Sueyoshi and Kirihara, 1988), marketing (Kwak *et al*., 2005), maintenance (Bertolini *et al*., 2004), facility location (Partovi, 2006), manufacturing (Yurdakul, 2004; Bhattacharya *et al*., 2005), logistics (Korpela *et al*., 2002; Chan *et al*., 2005), etc.

Acquiring the local weights of pairwise comparison matrices is an important part of AHP. Several methods have been proposed for this objective. The most usual method is Eigen Vector Method (EVM) (Saaty, 2000). Furthermore, various methods were proposed in this area, such as weighted least-squares method (Chu *et al*., 1979), logarithmic least-squares method (Crawford, 1987), least-squares method (Saaty and Vargas, 1984), geometric least-squares method (Islei and Lockett, 1988), goal programming method (Bryson, 1995), fuzzy programming method (Mikhailov, 2000), robust estimation method (Lipovetsky and Conklin, 2002), etc.

Ramanathan (2006) developed a DEA model for weight derivation in AHP, namely DEAHP and he proved that DEAHP produces accurate weights for completely pairwise comparison matrices. However, the principal drawback of DEAHP is to produce counterintuitive weights for pairwise comparison matrices with satisfactory consistency ratio. Wang *et al*. (2008) developed a new DEA model comprising assurance region (DEA/AR) to prevail the DEAHP's drawbacks. Mirhedayatian and Farzipoor Saen (2011) proposed a revised DEAHP approach in which super-efficiency DEA model was used and they claimed that their approach overcomes DEAHP's drawbacks and also it prevents rank reversal.

In this study, a non-radial DEA model, namely Slacks Based Measure of Efficiency for Weight Derivation (SBMWD) is explained to cover the DEAHP's drawbacks. Then, it is extended to multi-objective case in order to decrease the number of models which have to be solved for local weights derivation. We call this multi objective

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model as MOSBMWD (Multi-Objective Slacks Based Measure of efficiency for Weight Derivation) and it is revealed that all information of pairwise comparison matrix is considered in weight derivation process. Furthermore, it is shown that MOSBMWD generates true weights for completely consistent pairwise comparison matrices. Afterwards, it is applied for weight derivation and aggregation in a hierarchical structure and it is disclosed that the results are logical and compatible with the information of pairwise comparison matrices. Ultimately, the local weights vector of a large scale pairwise comparison matrix is acquired using the MOSBMWD model.

DEAHP

Let:

$$P = (a_{rj})_{n\times n} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{r1} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{r2} & \cdots & a_{n2} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{1j} & a_{2j} & \cdots & a_{rj} & \cdots & a_{nj} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{rn} & \cdots & a_{nn} \end{bmatrix} \qquad (1)$$

be a pairwise comparison matrix where r and j are the column and row indicators of matrix P, respectively, $a_{rj} = 1/a_{jr}$ for $r \neq j$ and $W = (w_1,..., w_n)^T$ represents its weights vector. In the DEAHP, each row of the matrix is viewed as a Decision Making Unit (DMU) and the columns are regarded as outputs. A dummy input which is equal to 1 is added in order to use DEA models. Ramanathan (2006) applied following input oriented CCR (Charnes-Cooper-Rhodes) model (Charnes *et al*., 1978) to obtain the local weights of pairwise comparison matrix P:

$$\begin{aligned} &\text{Maximize} && w_o = \sum_{r=1}^{n} a_{ro}\, u_r \\ &\text{Subject to} && v_1 = 1 \\ & && \sum_{r=1}^{n} a_{rj} u_r - v_1 \le 0, \qquad j = 1,...,n \\ & && u_r, v_1 \ge 0, \qquad r = 1,...,n \end{aligned} \qquad (2)$$

where w_o represents the CCR efficiency score of DMU_o which is considered as its local weight and a_{rj} is the rth output of DMU_j. Model (2) is solved for all DMUs to acquire local weights vector $W^* = (w_1,.., w_n)^T$ of the pairwise comparison matrix P. It was claimed (Ramanathan, 2006) that the DEAHP generates accurate weights for completely consistent comparison matrices ($a_{rj} = a_{rk}$, a_{kj} for r, j, k$\in$\{1,..., n\}).

Determining the local weights of criteria and alternatives with respect to each criterion solving Model (2), the final weights of alternatives are obtained considering the alternatives as DMUs and their local weights with respect to each criterion as outputs. Two models are used in DEAHP for weight aggregation. In the first model the final weights of alternatives are determined ignoring the local weights of criteria. In this case the model is as follows:

$$\begin{aligned} &\text{Maximize} && \sum_{r=1}^{M} w_{ro}\, u_r \\ &\text{Subject to} && v_1 = 1 \\ & && \sum_{r=1}^{M} w_{rj} u_r - v_1 \le 0, \qquad j = 1,...,N \\ & && v_1, u_r \ge 0, \qquad r = 1,...,M \end{aligned} \qquad (3)$$

where, w_{rj} represents the local weight of jth (j = 1,…, N) alternative with respect to rth criterion (r = 1, …, M).

In the second proposed model for weight aggregation, the local weights of criteria are considered in Model 4:

$$\begin{aligned} &\text{Maximize} && u_1 (\sum_{r=1}^{M} w_{ro} d_r) \\ &\text{Subject to} && u_1 (\sum_{r=1}^{M} w_{rj} d_r) \le 1, \qquad j = 1,...,N \\ & && u_1 \ge 0 \end{aligned} \qquad (4)$$

An additional constraint $u_r = d_r u_1$ (r = 1,…,M, d_1 = 1) is incorporated into the Model (4) to take into account the importance of criteria. However, it is reasonable to use Model (4) for weight aggregation, because the importance of criteria impacts the final ranking of alternatives. The principal drawback of DEAHP is its inability in generating logical weights for pairwise comparison matrices which are not completely consistent. For instance, consider the following pairwise comparison matrix:

$$A = \begin{bmatrix} 1 & 4 & 2 & 1/3 & 1/3 \\ 1/4 & 1 & 1/3 & 1/4 & 1/4 \\ 1/2 & 3 & 1 & 1/6 & 1/2 \\ 3 & 4 & 6 & 1 & 2 \\ 3 & 4 & 2 & 1/2 & 1 \end{bmatrix}$$

Which has acceptable consistency ratio (CR = 0.067<0.1). For matrix A, the normalized local weights vector derived using EVM and DEAHP are, respectively, as W^*_{EVM} (A) = (0.154, 0.057, 0.104, 0.426, 0.259) and W^*_{DEAHP}(A) = (0.250, 0.063, 0.188, 0.250, 0.250), so the ranking of alternatives/criteria obtained via the two methods are, respectively, as A4>A5>A1>A3>A2 and A1~A4~A5~A3>A2 (where > means "is preferred to" and~stands for "as important as").

To validate the results derived from EVM and DEAHP, the ranking of alternatives/criteria is directly obtained from the pairwise comparison matrix A. From the fourth row of pairwise comparison matrix A, it is clear that alternative 4 (A4) is the most important alternative, so it can be removed from matrix A. From this reduced matrix, it is evident that A5 is superior to other existing alternatives and it can be positioned in the second place. Removing A5 leads to constitute a new reduced matrix, in which A1 is more important than A2 and A3, because all of its row elements are greater or equal to one, so it can be placed in the third position in the ranking. Finally, the rest of alternatives, A2 and A3 can be easily ranked as A3>A2 in terms of their direct comparison $a_{32} = 3$. Hence, the final ranking of alternatives/criteria is concluded as A4>A5>A1>A3>A2. This implies that results of DEAHP are counterintuitive. The main reason for the inaccuracy of the results of DEAHP for non-complete consistent pairwise comparison matrices is the use of radial CCR model for weight derivation. To explain this issue, consider the envelopment form of the CCR model (Charnes *et al*., 1978) used in the DEAHP:

$$\begin{aligned} &\text{Minimize} \quad \theta \\ &\text{subject to} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta x_{io} \quad , \ i = 1,\ldots,m \\ &\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro} \quad , \ r = 1,\ldots,s \\ &\qquad \lambda_j \ge 0, \quad j = 1,\ldots,n \end{aligned} \tag{5}$$

where, x_{io} and y_{ro} are the ith input and rth output of DMU_o, respectively and θ is a ratio which should be multiplied by the inputs of DMU_o to project it on the efficient frontier. Since the outputs of alternatives/criteria determine their efficiency score in DEAHP (see objective function of Model (2)); the output-oriented CCR model is represented as below to demonstrate the reason of DEAHP's incapability in weight generation:

$$\begin{aligned} &\text{Maximize} \quad \mu \\ &\text{subject to} \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io} \quad , \ i = 1,\ldots,m \\ &\qquad \sum_{j=1}^{n} \lambda_j y_{rj} \ge \mu y_{ro} \quad , \ r = 1,\ldots,s \\ &\qquad \lambda_j \ge 0, \quad j = 1,\ldots,n \end{aligned} \tag{6}$$

where, μ is a ratio which should be multiplied by the outputs of DMU_o to project it on the efficient frontier. Both Model 5 and 6 have the same optimum solution λ_j^* and their objective function can be converted via following equation (Charnes *et al*., 1978):

$$\mu^* = 1/\theta^*$$

The efficiency scores of alternatives/criteria θ_j^* obtained by Model 2 and their projected outputs obtained by Model 6 are represented in Table 1. For example, the Model 6 is used to derive efficiency score of DMU_1 which is presented as follows:

$$\begin{aligned} &\text{Maximize} \quad \mu_1 \\ &\text{subject to} \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 \le 1, \\ &\qquad \lambda_1 + \frac{1}{4}\lambda_2 + \frac{1}{2}\lambda_3 + 3\lambda_4 + 3\lambda_5 \ge \mu_1, \\ &\qquad 4\lambda_1 + \lambda_2 + 3\lambda_3 + 4\lambda_4 + 4\lambda_5 \ge 4\mu_1, \\ &\qquad 2\lambda_1 + \frac{1}{3}\lambda_2 + \lambda_3 + 6\lambda_4 + 2\lambda_5 \ge 2\mu_1, \\ &\qquad \frac{1}{3}\lambda_1 + \frac{1}{4}\lambda_2 + \frac{1}{6}\lambda_3 + \lambda_4 + \frac{1}{2}\lambda_5 \ge \frac{1}{3}\mu_1, \\ &\qquad \frac{1}{3}\lambda_1 + \frac{1}{4}\lambda_2 + \frac{1}{2}\lambda_3 + 2\lambda_4 + \lambda_5 \ge \frac{1}{3}\mu_1, \\ &\qquad \lambda_1,\ \lambda_2,\ \lambda_3,\ \lambda_4,\ \lambda_5 \ \ge 0 \end{aligned}$$

The reference set for each DMU (alternative) is also represented in Table 1. According to Charnes *et al*. (1978), the reference set for efficient DMUs only consists of the same efficient DMUs (i.e., the reference set for efficient DMU_k only involves DMU_k). It can be realized from Table 1 that three DMUs, A2, A3 and A5 are viewed as inefficient, because A1 and A4 are identified as their reference set. The inefficient DMUs may be projected on efficient frontier by multiplying their outputs by their corresponding ratio μ^*_j (i.e., $y^*_{rj} = \mu^*_j\, y_j$). However, all of these inefficient DMUs are not completely projected on the efficient frontier. For example, some outputs of alternative 2 (A2) do not coincide with their reference set outputs (i.e., outputs of A4). For instance consider the first output of A2 (y_{12}) which is not projected on the efficient frontier after it is multiplied by its corresponding ratio $\mu^*_2 = 4$, that is:

$$y_{12}^* = \mu_2^* y_{12} \ne y_{14} \, (4 \times \frac{1}{4} = 1 \ne 3)$$

Such inaccurate projection causes more DMUs are recognized as efficient. Hence, it is logical to modify the DEAHP model so that the projection of inefficient DMUs can be entirely done. For this purpose, the non-radial DEA models are suggested to be used in weight generation process. This is explained in the next section.

SBM MODEL FOR WEIGHT DERIVATION

As it explained that, the radial CCR model which is used in DEAHP decreases its discrimination power. However, Zhou *et al.* (2007) claims that non-radial DEA models are stronger than radial models in ranking DMUs. In this section, a non-radial DEA model called Slack Based Measure (SBM) is explained to generate the local weights for pairwise comparison matrix. Since the input is constant and the outputs of each DMU determine its efficiency score in DEAHP, it is suggested to use output-oriented SBM models for weight generation. The output-oriented SBM model which was developed by Tone (2001) is as follows:

$$
\begin{aligned}
\text{Minimize} \quad & \rho_o = \frac{1}{1 + 1/s\sum_{r=1}^{s} s_r^+ / y_{ro}} \\
\text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \quad i = 1,\ldots,m, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1,\ldots,s, \\
& \lambda_j \ge 0, \quad j = 1,\ldots,n, \\
& s_r^+ \ge 0, \quad r = 1,\ldots,s.
\end{aligned}
\tag{7}
$$

where, s^+_r is output slack and ρ_o is the SBM efficiency score of the evaluated DMU (DMU_o). A DMU is efficient if and only if its efficiency score is equal to 1 ($\rho^* = 1$). Tone (2001) claimed that the efficiency score of SBM model is always less than CCR model (i.e., $\rho^*_{SBM} < \theta^*_{CCR}$), so it is expectable to have less number of efficient DMUs after solving Model 7 compared with radial CCR model and this in turn, is one of the main reasons to use SBM for weight derivation.

As it was discussed in the previous section, CCR model which was used in DEAHP prevents prefect projection of inefficient DMUs to efficient frontier. However, SBM model covers this drawback, because each DMU is projected individually through its corresponding slacks to efficient frontier. The slack values of alternatives/criteria of matrix A obtained by Model (7) are represented in Table 2. The used Model (7) to determine the slacks of alternative 1 (A1) is as below:

$$
\begin{aligned}
\text{Minimize} \quad & \rho_1 = \frac{1}{1 + (1/5) \times \left(\frac{s_1^+}{1} + \frac{s_2^+}{4} + \frac{s_3^+}{2} + \frac{s_4^+}{1/3} + \frac{s_5^+}{1/3}\right)} \\
\text{subject to} \quad & \lambda_1 + \lambda_1 + \lambda_1 + \lambda_1 + \lambda_1 \le 1, \\
& \lambda_1 + 1/4\lambda_2 + 1/2\lambda_3 + 3\lambda_4 + 3\lambda_5 - s_1^+ = 1, \\
& 4\lambda_1 + \lambda_2 + 3\lambda_3 + 4\lambda_4 + 4\lambda_5 - s_2^+ = 4, \\
& 2\lambda_1 + 1/3\lambda_2 + \lambda_3 + 6\lambda_4 + 2\lambda_5 - s_3^+ = 2, \\
& 1/3\lambda_1 + 1/4\lambda_2 + 1/6\lambda_3 + \lambda_4 + 1/2\lambda_5 - s_4^+ = 1/3, \\
& 1/3\lambda_1 + 1/4\lambda_2 + 1/2\lambda_3 + 2\lambda_4 + \lambda_5 - s_5^+ = 1/3, \\
& s_1^+, s_2^+, s_3^+, s_4^+, s_5^+ \ge 0, \\
& \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5 \ge 0
\end{aligned}
$$

It is clear from Table 2 that four alternatives A1, A2, A3 and A5 are inefficient (i.e., have efficiency score less than 1) when the SBM Model 7 is used for weight generation. The reference set for these inefficient DMUs is A4 (i.e., $\lambda^*_4 = 1$, $\lambda^*_1 = \lambda^*_2 = \lambda^*_3 = \lambda^*_5 = 0$ for all inefficient DMUs). As Table 1 addresses, the DEAHP has fundamental weakness in projecting inefficient DMUs to efficient frontier. However, all of these DMUs are perfectly located on efficient frontier using Model 7. For example the first output of A3 (y_{13}) can be projected to its corresponding reference set output (y_{14}) through slack s^+_1, i.e., $y^*_{13} = y_{13}+s^+_1 = 1/2+5/2 = 3$ (see the bold values in the third row of Table 2). This reveals that non-radial models are suggested to be used in weight derivation process because of their discrimination power and capability in prefect projection of inefficient DMUs.

Table 1: Efficiency scores of alternatives and their projected outputs for matrix A using DEAHP

Alternatives	Matrix A					θ^*_j	$\mu^*_j = 1/\theta^*_j$	Reference set	Projected outputs of DMU_j ($y^*_{rj} = \mu_j\, y_{rj}$)				
A1	1	4	2	1/3	1/3	1.000	1.000	A1	1	4	2	1/3	1/3
A2	1/4	1	1/3	1/4	1/4	0.250	4.000	A4	1	4	4/3	1	1
A3	1/2	3	1	1/6	1/2	0.750	1.333	A1,A4	2/3	4	4/3	2/9	2/3
A4	3	4	6	1	2	1.000	1.000	A4	3	4	6	1	2
A5	3	4	2	1/2	1	1.000	1.000	A4	3	4	2	1/2	1

Table 2: Slack values and reference set of alternatives of matrix A using SBM model

									Slacks values (S^+_r)								
Alternatives	Matrix A					Reference set	ρ^*_j	S_1^+	S_2^+	S_3^+	S_4^+	S_5^+	Projected outputs ($y^*_{rj} = y_{rj}+S^+_r$)				
A1	1	4	2	1/3	1/3	A4	0.313	2	0	4	2/3	5/3	3	4	6	1	2
A2	1/4	1	1/3	1/4	1/4	A4	0.109	11/4	3	17/3	3/4	7/4	3	4	6	1	2
A3	1/2	3	1	1/6	½	A4	0.214	5/2	1	5	5/6	3/2	3	4	6	1	2
A4	3	4	6	1	2	A4	1.000	0	0	0	0	0	3	4	6	1	2
A5	3	4	2	1/2	1	A4	0.556	0	0	4	1/2	1	3	4	6	1	2

Model 7 is the envelopment form of SBM model and its multiplier form is as below:

$$
\begin{aligned}
\text{Maximize} \quad & 1+\sum_{r=1}^{s} y_{ro}u_r - \sum_{i=1}^{m} x_{io}v_i \\
\text{subject to} \quad & \sum_{r=1}^{s} y_{rj}u_r - \sum_{i=1}^{m} x_{ij}v_i \le 0, \qquad j=1,\ldots,n, \\
& v_i \ge \frac{1}{m}\left[\frac{1}{x_{io}}\right], \qquad i=1,\ldots,m, \\
& u_r \ge \frac{1+\sum_{r=1}^{s} y_{ro}u_r - \sum_{i=1}^{m} x_{io}v_i}{s}\left[\frac{1}{y_{ro}}\right], \qquad r=1,\ldots,s
\end{aligned} \tag{8}
$$

In DEAHP, each alternative/criterion is regarded as a DMU and the elements of each column are viewed as outputs. Therefore, Model 8 can be expressed as follows:

$$
\begin{aligned}
\text{Maximize} \quad & w_o = 1+\sum_{r=1}^{n} a_{ro}u_r - v_1 \\
\text{subject to} \quad & \sum_{r=1}^{n} a_{rj}u_r - v_1 \le 0, \qquad j=1,\ldots,n, \\
& v_1 = 1, \\
& u_r \ge \frac{1+\sum_{r=1}^{n} a_{ro}u_r - v_1}{n}\left[\frac{1}{a_{ro}}\right], \qquad r=1,\ldots,n
\end{aligned} \tag{9}
$$

where, w_o represents the local weight of the alternative/criterion under evaluation. The above model is solved for all alternatives/criteria to obtain the local weights vector $W^* = (w^*_1,\ldots,w^*_n)^T$. Model 9 is called SBMWD.

MULTI-OBJECTIVE NON-RADIAL DEA MODEL FOR WEIGHT DERIVATION

The SBMWD Model 9 has to be solved n times (n is the number of alternatives/criteria) in order to obtain the local weights vector. Here, we want to propose a multi-objective non-radial DEA model by which the local weights vector is obtained by solving only one model.

Now-a-days, multi-objectives DEA models are preferred to be used for evaluation of systems which have multi-dimensional performance evaluation criteria, because single-objective models are failed in the assessment process of such systems. Many studies have been done for combining DEA models with Multi-objective Linear Programming (MOLP). One of the first works in this area was performed by Golany (1988) in which a multi-objective DEA model was proposed to choose the desired outputs levels given the input levels and this caused improving its discrimination power. Yu *et al*. (2005) utilized Fuzzy Multiple Objective Programming (FMOP) method to obtain the common weight for all DMUs and this leads to enhance the discrimination power of the classical DEA. Lozano and Villa (2009) developed two different multi-objective DEA approaches for target setting. The first one was an interactive method which allows the decision makers (DMs) to express their preference. The other was a lexicographic method in which a sequence of models considering the weights was solved in order to improve selected inputs and outputs. AHP was used in both approaches in order to allow DMs to articulate their preference. Wong *et al*. (2009) developed a combined model using DEA and MOLP in which DEA problem was transformed into an MOLP formulation and they demonstrated that it is not required to incorporate DMs prior judgments. Further, the DM can select the most desired solution along the efficient frontier according to his/her judgment.

An MOLP problem which is used to optimize a vector of linear functions subject to linear constraints can be generally expressed as follows:

$$
\begin{aligned}
\text{Maximize} \quad & F(x) = [f_1(x),\ldots,f_i(x),\ldots,f_n(x)] \\
\text{s.t.} \quad & x \in X
\end{aligned} \tag{10}
$$

The above MOLP model consists of n (≥ 2) different objective functions $f_i(x)$, ($i = 1,\ldots,n$) which should be maximized simultaneously. The linear set of constraints $Ax = b$ (where A is $m\times n$ matrix), create the feasible region $X = \{x: Ax = b, x\ge 0, b\in R^m\}$. The objective functions $f_i(x)$ ($i = 1,\ldots,n$) portray the objective space $Z = F(x)$ which is the image of feasible region X.

It should be noted that the Model 10 does not have unique solution when the judgments of DMs are not incorporated into the model. Hence, it is not expected to obtain the best solution for all objective functions when MOLP model is solved but instead, it is desired to generate solutions which are non-dominated. A solution is called non-dominated if it is not possible to find a better point in feasible region to improve at least one of the objective functions without worsening the others. In other words, $x^*\in X$ is non-dominated solution if there does not exist another $x\in X$ such that $f_j(x) > f_j(x^*)$ for at least one objective function j and $f_i(x) \ge f_i(x^*)$ for all $i \ne j$, $i = 1,2,\ldots n$. For more details, see Hwang and Masud (1979).

Model (9) has n different objectives with respect to the efficiency scores of each alternative/criterion w_j, $j = 1,\ldots n$. Such a problem can be represented in a general form of MOLP as follows:

$$
\begin{aligned}
&\text{Maximize} \quad W(u) = [w_1(u),\ldots,w_j(u),\ldots,w_n(u)] \\
&\text{s.t.} \\
&\qquad u \in U
\end{aligned}
\tag{11}
$$

where, U represents the feasible region and $w_j(u)$ ($j = 1,2,\ldots n$) represents objective functions (i.e., local weights of alternatives/criteria). It should be noted that in MOLP problem, it is not possible to optimize all objective functions simultaneously; instead, the following min-ordering approach is suggested to be used to obtain a specific non-dominated point:

$$
\begin{aligned}
&\text{Maximize} \quad \min_{1\le j\le n}\{w_j(u)\} \\
&\text{s.t.} \\
&\qquad u \in U
\end{aligned}
\tag{12}
$$

The above formulation can be expressed as a single objective function model by auxiliary variable t:

$$
\begin{aligned}
&\text{Maximize} \quad t \\
&\text{s.t.} \\
&\qquad t \le w_j(u) \quad j = 1,2,\ldots,n \\
&\qquad u \in U
\end{aligned}
\tag{13}
$$

Regarding formulation (9), the multi-objective SBMWD model can be written as follows:

$$
\begin{aligned}
&\text{Maximize} \quad w_1 = 1 + \sum_{r=1}^{n} a_{r1} u_{r_1} - v_1 \\
&\vdots \\
&\text{Maximize} \quad w_j = 1 + \sum_{r=1}^{n} a_{rj} u_{r_j} - v_1 \\
&\vdots \\
&\text{Maximize} \quad w_n = 1 + \sum_{r=1}^{n} a_{rn} u_{r_n} - v_1 \\
&\text{s.t.} \quad \sum_{r=1}^{n} a_{rj} u_{r_j} - v_1 \le 0, \qquad j = 1,\ldots,n, \\
&\qquad v_1 = 1 \;, \\
&\qquad u_{r_1} \ge \frac{1 + \sum_{r=1}^{n} a_{r1} u_{r_1} - v_1}{n} \left[\frac{1}{a_{r1}}\right], \qquad r = 1,\ldots,n, \\
&\qquad \vdots \\
&\qquad u_{r_j} \ge \frac{1 + \sum_{r=1}^{n} a_{rj} u_{r_j} - v_1}{n} \left[\frac{1}{a_{rj}}\right], \qquad r = 1,\ldots,n, \\
&\qquad \vdots \\
&\qquad u_{r_n} \ge \frac{1 + \sum_{r=1}^{n} a_{rn} u_{r_n} - v_1}{n} \left[\frac{1}{a_{rn}}\right], \qquad r = 1,\ldots,n
\end{aligned}
\tag{14}
$$

where, w_j is local weight of jth alternative/criterion ($j = 1,\ldots,n$) and u_{rj} represents the corresponding multipliers of jth alternative/criterion ($r = 1,\ldots,n$, $j = 1,\ldots n$).

The set of objective functions in Model (14) can be reduced to a single objective function using the min-ordering formulation (13). But the problem is to determine a common set of multipliers for u_{rc}, $j = 1,\ldots,n$ in order to convert the Model (14) to an integrated multi-objective model. To overcome this problem, a common set of multipliers u_{rc} can be defined as below:

$$
u_{r_c} \ge \frac{1 + \sum_{r=1}^{n} a_{r\min} u_{r_c} - v_1}{n} \left[\frac{1}{a_{r\min}}\right], \qquad r = 1,\ldots,n
\tag{15}
$$

where, $a_{r\min}$ represents the minimum of rows in a pairwise comparison matrix:

$$
a_{r\min} = \min_{j\in\{1,\ldots,n\}} \{a_{rj}\}
\tag{16}
$$

Hence, the integrated multi-objective model for weight derivation can be presented as below:

$$
\begin{aligned}
&\text{Maximize} \quad t \\
&\text{s.t.} \quad 1 + \sum_{r=1}^{n} a_{r1} u_{r_c} - v_1 \ge t, \\
&\qquad \vdots \\
&\qquad 1 + \sum_{r=1}^{n} a_{rj} u_{r_c} - v_1 \ge t, \\
&\qquad \vdots \\
&\qquad 1 + \sum_{r=1}^{n} a_{rn} u_{r_c} - v_1 \ge t, \\
&\qquad \sum_{r=1}^{n} a_{rj} u_{r_c} - v_1 \le 0, \qquad j = 1,\ldots,n, \\
&\qquad v_1 = 1 \;, \\
&\qquad u_{r_c} \ge \frac{1 + \sum_{r=1}^{n} a_{r\min} u_{r_c} - v_1}{n} \left[\frac{1}{a_{r\min}}\right], \qquad r = 1,\ldots,n
\end{aligned}
\tag{17}
$$

The Model (17) is called MOSBMWD. Solving the Model (17), the optimal common set of multipliers u^*_{rc} will be obtained. Then, the local weights of alternatives/criteria will be calculated substituting u^*_{rc} in the objective function corresponding to each alternative i.e.:

$$
w^*_j = 1 + \sum_{r=1}^{n} a_{rj} u^*_{r_c} - v_1, \; j = 1,\ldots,n
$$

Therefore, it is possible to generate the local weights vector of a pairwise comparison matrix by solving the Model (17).

Theorem 1: The Model (17) will be feasible despite incorporating the common set of multipliers:

$$u_{r_c} \geq \frac{1+\sum_{r=1}^{n} a_{r\min} u_{r_c} - v_1}{n}\left[\frac{1}{a_{r\min}}\right], \qquad r=1,...,n$$

into the model.

Proof: To prove the Theorem 1, consider Model (9). Assume that a new DMU with minimum outputs (a_{rmin}) is added to the current set of DMUs. In this case, two new constraints:

$$\sum_{r=1}^{n} a_{r\min} u_{r_c} - v_1 \leq 0$$

and:

$$u_{r_c} \geq \frac{1+\sum_{r=1}^{n} a_{r\min} u_{r_c} - v_1}{n}\left[\frac{1}{a_{r\min}}\right], \quad r=1,...,n$$

will be added to the model. The new DMU is inefficient because it has minimum outputs, so the final ranking of other DMUs is not changed when it is added to the current set of DMUs. Furthermore, since the SBM model is always feasible, it retains its feasibility despite adding the new DMU with minimum outputs.

The efficiency scores of alternatives/criteria obtained from matrix A using SBMWD model are less than the DEAHP scores, i.e., $\rho^*_{jSBMWD} \leq \theta^*_{jDEAHP}$ for all alternatives/criteria (see the values of θ^*_j and ρ^*_j in Table 1 and 2, respectively). It can be easily concluded that this is true for Model (17) (i.e., $\rho^*_{jMOSBMWD} \leq \theta^*_{jDEAHP}$) because MOSBMWD is a multi-objective case of SBMWD model. The reason for the bigger efficiency score of DEAHP compared with MOSBMWD stems from the values of lower bounds for multipliers u_r, v_1 in Model 2 and 17. Since the lower bound of u_r, v_1 in Model 2 is equal to zero, the outputs which their corresponding multiplier become zero in optimum solution (i.e., the outputs with $u^*_r = 0$) are disregarded in efficiency score computation. In other words, for each alternative/criterion, the efficiency score consists of the outputs which have non-zero multipliers, i.e.:

$$w_o = \sum_{r'} a_{r'o}\, u^*_{r'}, \qquad \forall r', u^*_{r'} \neq 0$$

However, all of the multipliers u_r, v_1 have positive non-zero lower bounds in Model 17. It is obvious from Model 17 that:

$$u_{r_c} \geq \frac{1+\sum_{r=1}^{n} a_{r\min} u_{r_c} - v_1}{n}\left[\frac{1}{a_{r\min}}\right], \quad r=1,...,n, \quad v_1 = 1$$

and this causes all the multipliers have the values more than zero in the optimum solution. This implies that all of outputs are taken into account in efficiency score estimation when Model (17) is used for weight derivation. At this juncture, we wish to calculate the local weights vector of matrix A using Model (17). The results are shown in Table 3. To derive local weights vector of matrix A, the MOSBMWD model based upon Model (17) is represented as below:

$$\begin{aligned}
\text{Max}\quad & t\\
\text{s.t.}\quad & 1+u_{1_c}+4u_{2_c}+2u_{3_c}+\frac{1}{3}u_{4_c}+\frac{1}{3}u_{5_c}-v_1 \geq t,\\
& 1+\frac{1}{4}u_{1_c}+u_{2_c}+\frac{1}{3}u_{3_c}+\frac{1}{4}u_{4_c}+\frac{1}{4}u_{5_c}-v_1 \geq t,\\
& 1+\frac{1}{2}u_{1_c}+3u_{2_c}+u_{3_c}+\frac{1}{6}u_{4_c}+\frac{1}{2}u_{5_c}-v_1 \geq t,\\
& 1+3u_{1_c}+4u_{2_c}+6u_{3_c}+u_{4_c}+2u_{5_c}-v_1 \geq t,\\
& 1+3u_{1_c}+4u_{2_c}+2u_{3_c}+\frac{1}{2}u_{4_c}+u_{5_c}-v_1 \geq t,\\
& u_{1_c}+4u_{2_c}+2u_{3_c}+\frac{1}{3}u_{4_c}+\frac{1}{3}u_{5_c}-v_1 \leq 0;\\
& \frac{1}{4}u_{1_c}+u_{2_c}+\frac{1}{3}u_{3_c}+\frac{1}{4}u_{4_c}+\frac{1}{4}u_{5_c}-v_1 \leq 0;\\
& \frac{1}{2}u_{1_c}+3u_{2_c}+u_{3_c}+\frac{1}{6}u_{4_c}+\frac{1}{2}u_{5_c}-v_1 \leq 0;\\
& 3u_{1_c}+4u_{2_c}+6u_{3_c}+u_{4_c}+2u_{5_c}-v_1 \leq 0;\\
& 3u_{1_c}+4u_{2_c}+2u_{3_c}+\frac{1}{2}u_{4_c}+u_{5_c}-v_1 \leq 0;\\
& v_1 = 1\quad,\\
& u_{1_c} \geq \frac{1+\frac{1}{4}u_{1_c}+u_{2_c}+\frac{1}{3}u_{3_c}+\frac{1}{6}u_{4_c}+\frac{1}{4}u_{5_c}-v_1}{5}\left[\frac{1}{1/4}\right],\\
& u_{2_c} \geq \frac{1+\frac{1}{4}u_{1_c}+u_{2_c}+\frac{1}{3}u_{3_c}+\frac{1}{6}u_{4_c}+\frac{1}{4}u_{5_c}-v_1}{5}\left[\frac{1}{1}\right],\\
& u_{3_c} \geq \frac{1+\frac{1}{4}u_{1_c}+u_{2_c}+\frac{1}{3}u_{3_c}+\frac{1}{6}u_{4_c}+\frac{1}{4}u_{5_c}-v_1}{5}\left[\frac{1}{1/3}\right],\\
& u_{4_c} \geq \frac{1+\frac{1}{4}u_{1_c}+u_{2_c}+\frac{1}{3}u_{3_c}+\frac{1}{6}u_{4_c}+\frac{1}{4}u_{5_c}-v_1}{5}\left[\frac{1}{1/6}\right],\\
& u_{5_c} \geq \frac{1+\frac{1}{4}u_{1_c}+u_{2_c}+\frac{1}{3}u_{3_c}+\frac{1}{6}u_{4_c}+\frac{1}{4}u_{5_c}-v_1}{5}\left[\frac{1}{1/4}\right]
\end{aligned}$$

As it can be seen from Table 3, the local weights vector of alternatives is obtained by running Model 17 only one time, while in the DEAHP; it has to solve n models separately in order to get local weights of alternatives. This, in turn, reveals the advantage of using Model (17) in reducing the number of models that have to be run. Solving Model (17) for matrix A, the optimum value of multipliers u_{rc} is determined. For instance, given the optimum values of multipliers u^*_{rc} and substituting it in the objective function of alternative 1 i.e.:

Table 3: Local weights and ranking obtained for matrix A using the MOSBMWD model

		Common set of weights					Objective function values		
		Matrix A					$w_j^* = 1+\sum_{r=1}^{n} a_{rj} u_{r_c}^* - v_1$		
Alternatives	t^*	$\mu^*_{1c} = 0.083$	$\mu^*_{2c} = 0.021$	$\mu^*_{3c} = 0.063$	$\mu^*_{4c} = 0.125$	$\mu^*_{5c} = 0.083$	Non-normalized	Normalized	Rank
A1	0.115	1	4	2	1/3	1/3	0.361	0.156	3
A2	0.115	¼	1	1/3	1/4	1/4	0.115	0.050	5
A3	0.115	½	3	1	1/6	1/2	0.229	0.099	4
A4	0.115	3	4	6	1	2	1.000	0.433	1
A5	0.115	3	4	2	½	1	0.604	0.262	2

Table 4: Optimum values of multipliers obtained solving MOSBMWD and DEAHP

Matrix A	Optimum values of multipliers									
	MOSBMWD					DEAHP				
Alternatives	μ^*_{1c}	μ^*_{2c}	μ^*_{3c}	μ^*_{4c}	μ^*_{5c}	μ^*_1	μ^*_2	μ^*_3	μ^*_4	μ^*_5
A1	0.083	0.021	0.063	0.125	0.083	0.000	0.250	0.000	0.000	0.000
A2	0.083	0.021	0.063	0.125	0.083	0.000	0.000	0.000	1.000	0.000
A3	0.083	0.021	0.063	0.125	0.083	0.000	0.250	0.000	0.000	0.000
A4	0.083	0.021	0.063	0.125	0.083	0.000	0.000	0.000	0.000	0.500
A5	0.083	0.021	0.063	0.125	0.083	0.333	0.000	0.000	0.000	0.000

$$w_1^* = 1+\sum_{r=1}^{n} a_{r1} u_{r_c}^* - v_1$$

the local weights of A1 is computed as follows:

$$1+(1*0.083+4*0.021+2*0.063+\frac{1}{3}*0.125+\frac{1}{3}*0.083)-1=0.361$$

Similarly, the local weights of A2, A3, A4 and A5 using Model 17 are equal to 0.115, 0.229, 1.000 and 0.604, respectively. The local weights are normalized to compare them with the results of EVM and the DEAHP (see Table 1 in order to compare the results). The local weights vector obtained using EVM for matrix A is W^*_{EVM} = (0.154, 0.057, 0.104, 0.426, 0.259). As it is obvious from Table 3, the ranking of alternative obtained using the MOSBMWD model is consistent with the EVM ranking and there is no significant difference between the local weights values generated by both methods (see the normalized objective function values in Table 3). Furthermore, the optimal values of multipliers u_{rc} are all higher than zero (u^*_{1c} = 0.083, u^*_{2c} = 0.021, u^*_{3c} = 0.063, u^*_{4c} = 0.125, u^*_{5c} = 0.083) and as it was discussed earlier, this leads to consider all outputs in efficiency score calculation when the Model 17 is used for weight derivation.

The multipliers obtained from solving DEAHP and Model (17) are illustrated in Table 4. It is clear that all values are more than zero when MOSBMWD is solved; besides, the values of multipliers are the same as all alternatives in MOSBMWD because common set of multiplier u_{rc} is used in Model (17). Nevertheless, many of multipliers in DEAHP are zero which causes ignoring them in weight calculation. For example, only the corresponding multiplier of the second output of A1 is higher than zero, u^*_2 = 0.250 but the others are zero, $u^*_1 = u^*_3 = u^*_4 = u^*_5 = 0$ which means only the second output determines the efficiency score of A1 and the others are disregarded in weight calculation.

Now, we want to derive local weights vector for a completely consistent pairwise comparison matrix by Model (17). Consider the following pairwise comparison matrix F:

$$F=\begin{bmatrix} 1 & 2 & 4 & 8 \\ 1/2 & 1 & 2 & 4 \\ 1/4 & 1/2 & 1 & 2 \\ 1/8 & 1/4 & 1/2 & 1 \end{bmatrix}$$

The above matrix is completely consistent (i.e., CR = 0) and its local weighs vectors generated by EVM and MOSBMWD are given in Table 5.

It is easy to derive the local weights of matrix F intuitively. It can be realized that the outputs of A1 are twice, four times and eight times as the outputs of A2, A3 and A4, respectively. Therefore, the local weight vector is:

$$W^* = (1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8})$$

which can be normalized as $W^*_{normalized}$ = (0.533, 0.267, 0.133, 0067). As it is clear from Table 5, the normalized local weights vector obtained from solving Model (17) is the same as the local weights vector directly derived from pairwise comparison matrix F. This reveals that

Table 5: Local weights for completely consistent matrix F via EVM and DEAHP

					EVM		MOSBMWD	
Alternatives	Matrix F				Normalized local weights	Rank	Normalized local weights	Rank
A1	1	2	4	8	0.533	1	0.533	1
A2	1/2	1	2	4	0.267	2	0.267	2
A3	1/4	1/2	1	2	0.133	3	0.133	3
A4	1/8	1/4	1/2	1	0.067	4	0.067	4

MOSBMWD generates correct weights for completely consistent pairwise comparison matrices.

Now, we want to aggregate the local weights generated by Model (17) to obtain the global weights of alternatives. Let, $w_1, w_2, \ldots, w_M$ be the local weights of criteria and $w_{r1}, w_{r2}, \ldots, w_{rN}$ be the local weights of N alternatives with respect to rth criterion (r = 1,…,M) which are represented in the following decision making matrix:

Table 6: Aggregation of local weights via SAW method

Normalized final weights	Criteria: w_M	…	w_2	w_1	Alternatives
$\dfrac{\sum_{r=1}^{M} w_{r1} w_r}{\max_{j\in\{1,\ldots,N\}} \{\sum_{r=1}^{M} w_{rj} w_r\}}$	w_{M1}	…	w_{21}	w_{11}	A_1
⋮	⋮	…	⋮	⋮	⋮
$\dfrac{\sum_{r=1}^{M} w_{rj} w_r}{\max_{j\in\{1,\ldots,N\}} \{\sum_{r=1}^{M} w_{rj} w_r\}}$	w_{Mj}	…	w_{2j}	w_{1j}	A_j
⋮	⋮		⋮	⋮	⋮
$\dfrac{\sum_{r=1}^{M} w_{rN} w_r}{\max_{j\in\{1,\ldots,N\}} \{\sum_{r=1}^{M} w_{rj} w_r\}}$	w_{MN}	…	w_{2N}	w_{1N}	A_N

The last column of Table 6 depicts the final aggregated weights of alternatives using Simple Additive Weighting (SAW) method. We can use the following formula instead of using Model (17) for weight aggregation:

$$w_{A_j} = \frac{\sum_{r=1}^{M} w_{rj} w_r}{\max_{j\in\{1,\ldots,N\}} \{\sum_{r=1}^{M} w_{rj} w_r\}}, \qquad j=1,\ldots,N \tag{18}$$

where, W_{aj} represents the final weight of alternative j (A_j, j = 1,…,N). Model (17) for weight aggregation can be reformulated as follows:

$$\begin{aligned}
&\text{Maximize} \quad t\\
&\text{s.t.} \quad 1 + u_{1_c}(\sum_{r=1}^{M} w_{rj} d_r) - v_1 \geq t, \qquad j=1,\ldots,N,\\
&\qquad u_{1_c}(\sum_{r=1}^{M} w_{rj} d_r) - v_1 \leq 0, \qquad j=1,\ldots,N,\\
&\qquad v_1 = 1 \quad ,\\
&\qquad u_{1_c} \geq \frac{1 + u_{1_c}(\sum_{r=1}^{M} w_{r\min} d_r) - v_1}{N} \left[\frac{1}{(\sum_{r=1}^{M} w_{r\min} d_r)}\right]
\end{aligned} \tag{19}$$

where, d_r represents the proportion of the local weight of rth criterion (r = 1,…,M) divided by the local weight of the first criterion (i.e., $d_r = \frac{w_r}{w_1}$) and $w_{r\min}$ is the minimum of rows according to data of Table 6 (i.e., $w_{r\min} = \min_{j\in\{1,\ldots,N\}} \{w_{rj}\}$).

Regarding to the constraints:

$$u_{1_c}(\sum_{r=1}^{M} w_{rj} d_r) - v_1 \leq 0, \quad j=1,\ldots,N, \qquad v_1 = 1$$

we have:

$$u_{1_c} \leq \frac{1}{\sum_{r=1}^{M} w_{rj} d_r}, \quad j=1,\ldots,N \tag{20}$$

The above constraint can be restated as below:

$$u_{1_c} \leq \min_{j\in\{1,\ldots,N\}} \left\{\frac{1}{\sum_{r=1}^{M} w_{rj} d_r}\right\} = \frac{1}{\max_{j\in\{1,\ldots,N\}} \{\sum_{r=1}^{M} w_{rj} d_r\}} \tag{21}$$

Besides, regarding the third and the fourth constraints of Model (19), we have:

$$\begin{aligned}
&u_{1_c} \geq \frac{u_{1_c}(\sum_{r=1}^{M} w_{r\min} d_r)}{N(\sum_{r=1}^{M} w_{r\min} d_r)},\\
&u_{1_c} \geq 0
\end{aligned} \tag{22}$$

Therefore, with respect to the constraints (21) and (22), we have:

$$0 \leq u_{1_c} \leq \frac{1}{\max_{j\in\{1,\ldots,N\}} \{\sum_{r=1}^{M} w_{rj} d_r\}} \tag{23}$$

Considering the objective function and the first set of constraint of Model (19):

$$1+u_{1_c}(\sum_{r=1}^{M} w_{rj}d_r)-v_1 \geq t, \ \ j=1,...,N$$

the value of u_{1c} in constraint (23), should be equal to its upper bound in the optimum solution in order to maximize t. In other words:

$$u_{1_c}^* = \frac{1}{\max\limits_{j\in\{1,...,N\}}\{\sum_{r=1}^{M} w_{rj}d_r\}} \tag{24}$$

Furthermore, the set of constraints:

$$1+u_{1_c}(\sum_{r=1}^{M} w_{rj}d_r)-v_1 \geq t, \ \ j=1,...,N$$

corresponds to the objective function of alternative j which can be expressed as below:

$$w_{A_j} = 1+u_{1_c}(\sum_{r=1}^{M} w_{rj}d_r)-v_1 \tag{25}$$

Considering the third constraint of Model (19) (i.e., $v_1 = 1$) and substituting Eq. 24 in Eq. 25, the final weight of alternative j is obtained as below:

$$w_{A_j}^* = \frac{\sum_{r=1}^{M} w_{rj}d_r}{\max\limits_{j\in\{1,...,N\}}\{\sum_{r=1}^{M} w_{rj}d_r\}} = \frac{\sum_{r=1}^{M} w_{rj}(\frac{w_r}{w_1})}{\max\limits_{j\in\{1,...,N\}}\{\sum_{r=1}^{M} w_{rj}(\frac{w_r}{w_1})\}} = \frac{\sum_{r=1}^{M} w_{rj}w_r}{\max\limits_{j\in\{1,...,N\}}\{\sum_{r=1}^{M} w_{rj}w_r\}} \tag{26}$$

Hence, the final weight obtained from the MOSBMWD model is the same as the final weight obtained from the SAW method.

NUMERICAL EXAMPLE

In this section, Model (17) is applied to obtain the final weights of alternatives in a hierarchical structure. The objective of this example is to select the best strategy and its hierarchy is depicted in Fig. 1. The data set of this example is partially taken from Yuksel and Dagdeviren (2007). There are four criteria to select the best strategy including technically qualified workforce (C_1), energy costs (C_2), new foreign markets (C_3) and economic and political uncertainty (C_4). The alternatives (strategies) include working with strong suppliers (A_1), making joint investments with European suppliers (A_2), investing in former east-block countries (A_3), subcontracting (A_4).

The pairwise comparison matrices for criteria and alternatives with respect to each criterion and their corresponding local weights vectors obtained from EVM, DEAHP and Model (17) are presented in Table 7. Due to the rule suggested by Saaty (2000), all the selected pairwise comparison matrices for this example have acceptable consistency ratio (i.e., CR<0.1). The local weights vectors obtained using different methods are normalized to ease the comparison (see the bold values in Table 7). It is evident from Table 7 that the priority vectors for all matrices obtained from Model (17) are compatible with the result of EVM, while the outcomes of DEAHP, neither in ranking nor in local weights values are consistent with EVM results. For example the normalized priority vectors of pairwise comparison matrix A, obtained from EVM and MOSBMWD are, respectively as W^*_{EVM} (A) = (0.448, 0.283, 0.164, 0.106) and $W^*_{MOSBMWD}$ (A) = (0.450, 0.287, 0.162, 0.100) which rank the decision criteria as $C_1>C_2>C_3>C_4$. However, the local

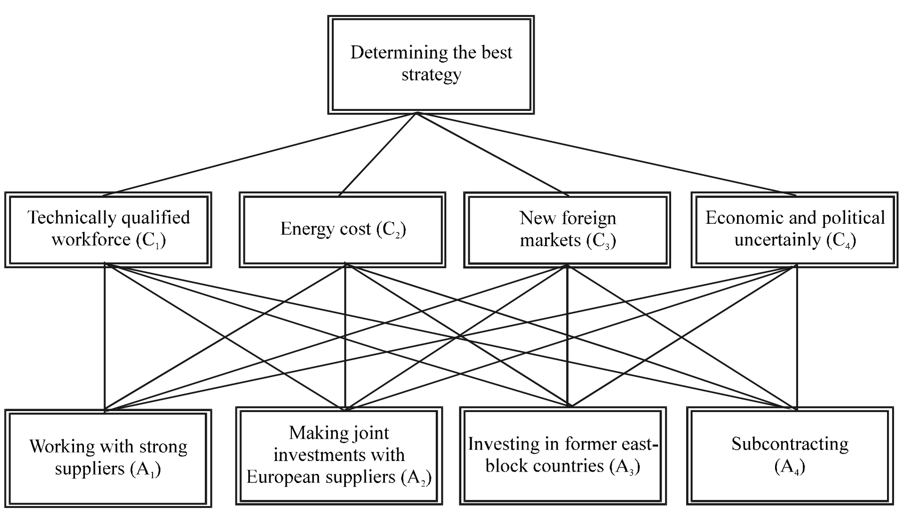

Fig. 1: Hierarchy of selecting the best strategy problem

Table 7: Pairwise comparison matrices for criteria and alternatives and their local weights vectors

					EVM			DEAHP			MOSBMWD		
Criteria	C1	C2	C3	C4	Non-normalized	Normalized	Rank	Non-normalized	Normalized	Rank	Non-normalized	Normalized	Rank
A: Pairwise comparisons of four criteria with respect to decision goal													
C1	1	2	3	3	0.793	0.448	1	1.000	0.333	1	1.000	0.450	1
C2	1/2	1	2	3	0.501	0.283	2	1.000	0.333	1	0.639	0.287	2
C3	1/3	1/2	1	2	0.290	0.164	3	0.667	0.222	2	0.361	0.162	3
C4	1/3	1/3	1/2	1	0.188	0.106	4	0.333	0.111	3	0.222	0.100	4
CR = 0.03													
					EVM			DEAHP			MOSBMWD		
Alternatives	A1	A2	A3	A4	Non-normalized	Normalized	Rank	Non-normalized	Normalized	Rank	Non-normalized	Normalized	Rank
B: Pairwise comparisons of four alternatives with respect to criterion 1													
A1	1	1/4	3	1/4	0.191	0.118	3	0.333	0.136	2	0.238	0.119	3
A2	4	1	9	2	0.829	0.511	1	1.000	0.409	1	1.000	0.500	1
A3	1/3	1/9	1	1/5	0.084	0.052	4	0.111	0.045	3	0.100	0.050	4
A4	4	1/2	5	1	0.518	0.319	2	1.000	0.409	1	0.663	0.331	2
CR = 0.03													
C: Pairwise comparisons of four alternatives with respect to criterion 2													
A1	1	1/2	1/5	1/4	0.134	0.079	4	0.200	0.077	3	0.167	0.078	4
A2	2	1	1/3	1/3	0.234	0.137	3	0.400	0.154	2	0.292	0.136	3
A3	5	3	1	2	0.800	0.470	1	1.000	0.385	1	1.000	0.466	1
A4	4	3	1/2	1	0.536	0.315	2	1.000	0.385	1	0.688	0.320	2
CR = 0.02													
D: Pairwise comparisons of four alternatives with respect to criterion 3													
A1	1	1/5	1/6	3	0.151	0.094	3	0.375	0.149	2	0.183	0.092	3
A2	5	1	1/2	8	0.560	0.351	2	1.000	0.397	1	0.719	0.362	2
A3	6	2	1	7	0.811	0.508	1	1.000	0.397	1	1.000	0.504	1
A4	1/3	1/8	1/7	1	0.075	0.047	4	0.143	0.057	3	0.083	0.042	4
CR = 0.05													
E: Pairwise comparisons of four alternatives with respect to criterion 4													
A1	1	1/5	1/8	3	0.130	0.083	3	0.333	0.136	2	0.152	0.082	3
A2	5	1	1/2	9	0.529	0.339	2	1.000	0.407	1	0.638	0.343	2
A3	8	2	1	8	0.836	0.536	1	1.000	0.407	1	1.000	0.538	1
A4	1/3	1/9	1/8	1	0.066	0.042	4	0.125	0.051	3	0.069	0.037	4
CR = 0.05													

Table 8: Final weights of alternatives

	C1	C2	C3	C4	Final weights	Rank
Final weights of alternatives obtained from EVM						
local weights of criteria	0.793	0.501	0.290	0.188		
A1	0.191	0.134	0.151	0.130	0.277	4
A2	0.829	0.234	0.560	0.529	1.000	1
A3	0.084	0.800	0.811	0.836	0.830	2
A4	0.518	0.536	0.075	0.066	0.689	3
Final weights of alternatives obtained from DEAHP						
local weights of criteria	1.000	1.000	0.667	0.333		
A1	0.333	0.200	0.375	0.333	0.373	4
A2	1.000	0.400	1.000	1.000	1.000	1
A3	0.111	1.000	1.000	1.000	0.880	3
A4	1.000	1.000	0.143	0.125	0.890	2
Final weights of alternatives obtained from MOSBMWD						
local weights of criteria	1.000	0.639	0.361	0.222		
A1	0.238	0.167	0.183	0.152	0.279	4
A2	1.000	0.292	0.719	0.638	1.000	1
A3	0.100	1.000	1.000	1.000	0.833	2
A4	0.663	0.688	0.083	0.069	0.723	3

weights vector and the ranking obtained from DEAHP are, respectively as, $W^{*}_{DEAHP}(A) = (0.333, 0.333, 0.222, 0.111)$ and $C_1 \sim C_2 > C_3 > C_4$ which contradict the direct information of matrix *A*, because the first criterion, C_1, is obviously more important than others in accordance with the first row of matrix A.

Since the final weights of alternatives in a hierarchy are obtained using these local weights vectors, such miscalculation causes deficiency of DEAHP in aggregating the local weights in order to get the final ranking of alternatives. The local weights obtained from Model (17) are aggregated using Eq. 18. The results are illustrated in Table 8. The final weights of alternatives obtained from EVM and DEAHP are also presented in Table 8.

It is clear from Table 8 that the final weights of alternatives using EVM and MOSBMWD are, respectively as, $W^{final*}_{EVM} = (0.277, 1.000, 0.830, 0.689)$ and $W^{final*}_{MOSBMWD} = (0.279, 1.000, 0.833, 0.723)$ which rank the alternatives as $A_2 \succ A_3 \succ A_4 \succ A_1$, while the final weights vector obtained from DEAHP is as $W^{final*}_{DEAHP} = (0.373, 1.000, 0.880, 0.890)$ which ranks the alternatives as $A_2 \succ A_4 \succ A_3 \succ A_1$. The reason for this difference between the results of DEAHP and EVM is miscalculation of local weights vectors in previous step. As it can be seen from Table 7, in DEAHP, C_1 is evaluated as important as C_2 but it was examined earlier that the former outperforms the latter. Furthermore, with respect to C_2, DEAHP evaluates A_3 to be as significant as A_4, while it can be easily realized from the third row of matrix C that A_3 is considered to be the most important alternative. Such overestimation leads DEAHP to be inefficient in weight derivation and aggregation process. However, considering the results of

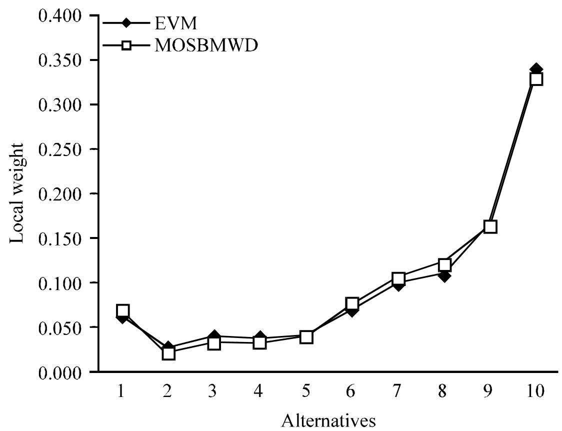

Fig. 2: Local weights obtained from EVM and MOSBMWD for matrix N

Table 8, it is revealed that MOSBMWD performs successfully in the local and final weight generation.

To examine the advantage of running Model (17), a large scale pairwise comparison matrix is represented. Consider pairwise comparison matrix N:

$$N = \begin{bmatrix} 1 & 5 & 3 & 3 & 3 & 1/2 & 1/3 & 1/5 & 1/6 & 1/9 \\ 1/5 & 1 & 1/3 & 1/3 & 1/3 & 1/2 & 1/2 & 1/3 & 1/3 & 1/6 \\ 1/3 & 3 & 1 & 1 & 1/2 & 1/2 & 1/3 & 1/2 & 1/3 & 1/5 \\ 1/3 & 3 & 1 & 1 & 1 & 1/4 & 1/3 & 1/4 & 1/2 & 1/7 \\ 1/3 & 3 & 2 & 1 & 1 & 1/2 & 1/2 & 1/3 & 1/4 & 1/9 \\ 2 & 2 & 2 & 4 & 2 & 1 & 1/3 & 1/2 & 1/2 & 1/6 \\ 3 & 2 & 3 & 3 & 2 & 3 & 1 & 1 & 1/3 & 1/4 \\ 5 & 3 & 2 & 4 & 3 & 2 & 1 & 1 & 1/3 & 1/5 \\ 6 & 3 & 3 & 2 & 4 & 2 & 3 & 3 & 1 & 1/3 \\ 9 & 6 & 5 & 7 & 9 & 6 & 4 & 5 & 3 & 1 \end{bmatrix}$$

This is a pairwise comparison matrix with satisfactory consistency ratio which is CR = 0.095. The local weights obtained from the EVM and Model (17) are depicted in Fig. 2. As Fig. 2 shows, the results obtained from Model (17) are similar to EVM results. This reveals we can derive logical and intuitive local weights by running Model (17) only one time, while in DEAHP, we need to run the model n times. Therefore, using the Model (17) generates logical weights for large scale pairwise comparison matrices.

CONCLUSIONS

In this study, the main drawbacks of DEAHP were examined and it was discussed that the main reason for deficiency of DEAHP is the use of radial CCR model for weight derivation. Then a multi-objective non-radial DEA model, called MOSBMWD, was proposed and through numerical examples it has been revealed that MOSBMWD is able to modify the DEAHP deficiencies. It was shown that MOSBMWD calculates correct weights for completely consistent pairwise comparison matrices. Furthermore, we have revealed that all the outputs related to each DMU were taken into account in efficiency score calculations when MOSBMWD is used for weight derivation.

In the previously proposed models such as DEAHP, revised DEAHP, DEA/AR, etc, it is required to run the model for all the DMUs, while in MOSBMWD model only one run is needed for weight derivation. Therefore, it is expected to use MOSBMWD model instead of previous models, for future studies.

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