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# Mathematical Modelling in Family Takaful

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**Abstract:** Clients at least knew if they paid a certain amount of money every month to an insurance company, they got back their money plus more if accidents or deaths happened. However, they have no idea about the "inner workings" or the "fine prints" of the insurance transactions. Compared to the other forms of insurance, Family Takaful Insurance without doubt should be the first choice of future clients. This research paper emphasized on mathematical modeling of Family Takaful and compared the mathematical part of the analysis using LM1 and LM2. According to the goal of this research, researchers have to calculate the life table premium from Takaful Companies which used mudharabah model and wakala model. To achieve this purpose, researchers have to derive a suitable formula for such life tables and compared between these two models would be on Surrender value, Maturity value and Death coverage. The results had shown that clients would be better off if they had chosen a takaful company that sold the mudharabah model and not the wakala model.

**Key words:** Premium payment, life premium tables, maturity value, surrender value, death coverage

## INTRODUCTION

Islamic jurists resolved that the system of insurance which falls within the confines of Islamic framework, should be founded on the concept of al-Takaful. An Islamic insurance transacting is a policy of mutual co-operation, solidarity and brotherhood against unpredicted risk or catastrophes, in which the parties involved are expected to contribute genuinely. The nature of the principles of Takaful is fundamentally different from the principles of conventional insurance.

The widely differing attitudes of Muslim scholars on the Islamic validity of insurance can be grouped under three broad categories which are called 'riba', 'gharar' and 'maisir' where their details are as follows.

**Riba:** Riba simply means a prohibited gain under Syariah. Riba would cover all gains from loans and debts, anything over and above the principal amounts. In the context of Islamic finance, riba can simply be construed as the prohibition of interest in the conventional sense, a concept prohibiting the making of money from money whether excessive or otherwise (Usmani, 2001). The guiding principle of Islamic finance is the prohibition of interest-based lending but capital providers and investors are not prevented from earning returns on their capital or investment appropriate to the duration of their commitment and exposure to risks (Ozsoy, 1994). It is the exposure to risks over specific time periods that justify the earning of returns which could be in the form of profits, rentals, fees or commission.

The money which has been collected through premiums is invested by the insurance company in interest-bearing deposits, un-Islamic business or dealings. Thus clearly the participants have to pay more for the premium in their businesses. Almost, all the Muslim jurists accept that riba which covers simple and compound interests and productive and non-productive loans, as interpreted by Muslehuddin (2004), is absolutely prohibited in Islam. The words of the Holy Quran (II:275):

- Allah permits trading and forbids riba

Even in case of productive loans, guaranteed return on capital is unjust, viewed against the uncertainties surrounding entrepreneurial profit. The example of riba has shown as profit in Table 1 given by RMa, RMb and RMc.

**Example 1:** Table 1 shows the total profit per year from interest rate x%.

Table 1: Total profit of interest

| Year | Age | Payment layout per year | Total profit (x % per year) |
|---|---|---|---|
| n1 | a 1 | RMy | y (1+x) = RMa |
| n2 | a 2 | RM2y | 2y (1+x) = RMb |
| n3 | a 3 | RM3y | 3y (1+x) = RMc |

x is the interest rate per year, y is payment layout per year, RMa is total profit in n1 year, RMb is the total profit in n2 year and RMc is the total profit in n3 year

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**Example 2:** A person A borrows RM1000.00 to a person B at 10 % per year. How much amount the person A will pay to the person B after the end of the year?

By using the total profit in Table 1, the researchers finds as below:

$$1000\ (1+0.1) = RM1100$$

The person A has to pay RM1100 to the person B which RM100 is called 'Riba'.

**Gharar:** Gharar refers to uncertainty. Gharar could be described as the lack of transparency in a given transaction. One of the underlying objectives of Islamic finance is to prevent dealings involving uncertainty, hazard, chance or uncontrollable risk leading to speculation (Obaidullah, 2005).

There is difference in juristic opinion on the tolerable level of gharar. A distinction is drawn between gharar yasir (minor uncertainty) and gharar Jahish (excessive uncertainty). Contemporary Muslim thinkers assert that what the Sunnah prohibits is gharar Jahish which is not presented in the contract of Insurance (Rashid, 1993).

**Example 3:** It shows that Table 2a has the monthly premium, maturity value and the interest rate. Table 2b shows the reality client proposal in life insurance.

The example of gharar has shown in Table 2b which client has earned RM 29 802 after invested 20 years at 5.738 interest rate per year. However, the Table 2b hasn't showed the value if the client surrendered earlier before 20 years. This is called 'Gharar'.

**Maisir:** Maisir has also been described as having 2 parties involved in a combative game played for the sole purpose of winning at the expense of one's opponent. The gain accruing from such a game is unlawful, as is the act of playing it, for it diverts one's attention from productive occupation and virtuous conduct (El-Awa, 2000).

Consistent with this prohibition, Islamic banks are not to engage in speculative trade in shares, short-selling or trading in unidentified items. Derivatives of various kinds such as futures contracts, options, swaps and forward foreign exchange transactions where the rates are determined by interest differentials and fluctuation in currencies are similarly prohibited due to their speculative nature (Anwar, 1994).

Maisir appears, for example, in the calculation of the surrender value in the client monthly mode of payment shown in Table 3. The surrender value is calculated by using the formula (Malaysian Insurance Institute, 2007).

Table 2a: The general client proposal of life insurance

| No. | Items | Value |
|---|---|---|
| 1 | Basic | RMx |
| 2 | Female | k year old non smoker |
| 3 | Period | n years |
| 4 | Premium | RMy (Monthly) = 12y |
| 5 | Value after maturity | RMz (Monthly) |
| 6 | Interest Rate | r per year |

Table 2b: Client proposal of particular life insurance company

| No. | Items | Value |
|---|---|---|
| 1 | Basic | RM18 000 |
| 2 | Female | 42 year old non smoker |
| 3 | Period | 20 years |
| 4 | Premium | RM125.65 (Monthly) |
| 5 | Value after maturity | RM29 802 |
| 6 | Interest rate | 5.738 per year |

Table 3: Client quotation of monthly mode of payment

| Year | Age | Premium (RM) | Payment layout (RM) | Surrender value (RM) | Death coverage (RM) |
|---|---|---|---|---|---|
| 1 | 5 | 125.65 | 1 507.80 | 0 | 18 000 |
| 2 | 6 | 125.65 | 3 015.60 | 0 | 18 000 |
| 3 | 7 | 125.65 | 4 523.40 | 2 359 | 20 359 |
| 4 | 8 | 125.65 | 6 031.20 | 2 739 | 20 739 |
| 5 | 9 | 125.65 | 7 530.00 | 3 179 | 21 179 |
| 6 | 10 | 125.65 | 9 046.80 | 3 691 | 21 691 |
| 7 | 11 | 125.65 | 10 554.60 | 4 285 | 22 285 |
| 8 | 12 | 125.65 | 12 062.40 | 4 974 | 22 974 |
| 9 | 13 | 125.65 | 13 570.20 | 5 774 | 23 774 |
| 10 | 14 | 125.65 | 15 078.00 | 6 703 | 24 703 |
| 11 | 15 | 125.65 | 16 585.80 | 7 782 | 25 785 |
| 12 | 16 | 125.65 | 18 093.60 | 9 034 | 27 034 |
| 13 | 17 | 125.65 | 19 601.40 | 10 488 | 28 488 |
| 14 | 18 | 125.65 | 21 109.20 | 12 175 | 30 175 |
| 15 | 19 | 125.65 | 22,617.00 | 14 134 | 32 134 |
| 16 | 20 | 125.65 | 24 124.80 | 16 408 | 34 408 |
| 17 | 21 | 125.65 | 25 632.60 | 19 049 | 37 049 |
| 18 | 22 | 125.65 | 27 140.40 | 22 113 | 40 113 |
| 19 | 23 | 125.65 | 28 648.20 | 25 672 | 43 672 |
| 20 | 24 | 125.65 | 30 156.00 | 29 802 | 47 802 |

$$F_v = P_v (1+i)^n$$

where, $F_v$ is the future value, $P_v$ is the premium paid value, i is the interest rate and n is the number of years. For Table 3, i = 0.1609 has been used.

In Table 3, the surrender value for the first two years is considerably zero. Therefore, the insurance company gambles for getting profit from its clients who means that the clients have to pay more value for premium but earn less in their profit.

## THE MODELS IN FAMILY TAKAFUL

As mentioned by Ali (2006), there exist two differences between conventional and takaful insurances which involve the investments of the assets and the treatment of expenses and surplus. In Takaful insurance all the investments are made in halal (permissible) assets and the treatment of the trading also are made in halal practices. The takaful operator's fund is based on two models of family takaful so-called Mudharabah and Wakala Models which will be described as follows.

**The Mudharah model:** In this model, the participant gets the amount accumulated in his participant account when he submits the early surrender. At his death, his beneficiaries will get the sum of the Death Coverage, in addition to the amount accumulated in his participant account. Mudharabah gives the right to the contracting parties to share the profit while liability for losses is borne by the participants. The product is based on profit sharing to cover the acquisition expenses, included commissions. The contract specifies; the profit (surplus) from the Takaful Company operators is to be shared between the providers of the capital and (participants) and the entrepreneur (takaful operator) (Billah, 2003). The sharing of such profit may be in a ratio 50:50, 60:40, 70: 30 and others as mutually agreed between the contracting parties. The principle of mudharabah (sleeping partnership) provides the specific details of the takaful contract (Syed, 1991). The cooperative of profit risk-sharing occurs among participants yet the Takafu Operator shares also in any operating surplus as a reward for its careful underwriting on behalf of participants. As an example, this mudharabah model has been practiced in Syarikat Takaful Malaysia (STM) (Zainol, 2005) as in Example 4.

**Example 4:** A person (A) wants to buy a house from a person (B) at a price of RM100,000.00 but he does not have the money to pay the entire RM100,000.00. So, the person A, goes to the bank and explains his situation. The bank will buy the house from the person B at the price of RM100,000.00 and then sell the house to the person A at a fixed selling price of RM271, 277.00, to be paid in instalment for 25 years. The price will be divided into 300 equal instalments of RM904.59 every month for the 25-years period.

**The Wakala model:** Takaful operator acts as an agency on behalf of the participants. In return for the services rendered, the operator is paid an agreed-upon predetermined fee. This fee can be a percentage of the contribution of total premium or an absolute amount. This is a fee on the portion of the contribution a general management fee and a fee on the investment of the contribution from the combination of general management fee and asset management fee (Warde, 2000).

Wakala is a form of representative relationship between Takaful Company and a participant, takaful operator earns a fee for services rendered while liability for losses is borne by the participants. The operating company does not share in the underwriting result but rather it is compensated by a fee deducted from contributions made by participants and/or investment profits generated by the takaful fund. The fee rate is fixed annually in advance in consultation with the Shariah committee of the company. As an example, this wakala model has been practiced in Takaful Etiqa, MAA Takaful and Ikhlas Takaful (Yaquby, 2001). This practice is shown in Example 5.

**Example 5:** A person (A) wants to buy a house from a person (B) at a price of RM100,000.00 but he does not have the cash to pay the entire RM100,000.00. The person A does not have time to go to the bank so he explains the situation to the bank's agent. The agent will buy the house from the person B at the price of RM100,000.00 and then sell it to the person A at a fixed selling price of RM271,277.00 to be paid in instalment over 25 years. However, the person A has to pay a management fee to the bank's agent at the amount of 1% of the selling price for his service. The selling price will be divided into 300 equal instalments of RM904.59 every month over the 25-year period.

## PROBLEM

Most of the people know what insurance is. They at least know if they pay a certain amount of money every month to an insurance company, they will get back their money plus more if accidents or deaths happen. However, that is the extent of their knowledge on insurance. They have no idea about the "inner workings" or the "fine prints" of the insurance transactions. Compared to the other forms of insurance, Family Takaful is definitely one

of the newer insurance families. This novelty attracts people. Thus, the clients' knowledge of Family Takaful is not extensive, to say the least, yet, there is a lot of clients' interest in Family Takaful Insurance.

The researchers has heard of cases where these clients purchase Family Takaful Insurance based solely on Takaful's agents' sales pitch and good words. It's clear that these clients need to be aware and informed of what Family Takaful Insurance is all about, specifically, in terms of surrender value, maturity value and mathematical formulation.

The surrender value and maturity value can be found in and explained by a premium life table but not the mathematical formulation. Actuaries and mathematicians know how to get the actual formula from premium life table. So, the derivation of both models is to find the general formula whether they have different concept between each others or not.

## THE PROPOSED METHOD

In this section, in order to find the comparison of mathematical analysis between two models of insurance in Family Takaful Insurance, the researchers to introduce two solution methods named LM1 and LM2 for the comparison as follows.

**Method LM1:** In this approach, the researchers has to find the life table premium from Takaful Company which uses the mudharabah model and then the researcher needs to derive a suitable formula for such life table. The researcher is allowed the use of Table 4a by Takaful Company. So, by using the general formulae from our derivation, the researcher can find the surrender value, maturity value and death coverage.

The numbers given under the Total column in Table 4a can be computed by using the formulae as shown in Table 4b. For example, by mathematical induction through child age 1 to child age 4, the expression for Total column in the child age row equal to 8(5) given in Table 4a, can be written as:

$$P_5 (1+r)+P_4 r (1+r)+P_3 r (1+r)^2+P_2 r (1+r)^3+P_1 r (1+r)^4$$

Therefore, the general expression for Total column given in Table 4a, can be written as:

$$P_n (1+r)+P_{n-1} r (1+r)+...+P_3 r (1+r)^{n-3}+P_2 r (1+r)^{n-2}+P_1 r (1+r)^{n-1}$$

for (n = 1,…, 14) where n = 1 is associated to child age 4 and continue with this way up to n = 14.

**Method LM2:** The researcher has to find the life table premium from takaful company which uses the wakala model and then derive a suitable formula for that particular life table.

Suppose that the researcher is given Table 5 a-c. Clearly, by a few steps of calculation, we obtain several general formulae given in Table 5d which can be used to produce all the numbers written in Table 5c. Therefore, the researcher has a method for finding the surrender and maturity values.

Table 4a: Client quotation

| Child age | Money outlay | Insured account (Pi) | Special account | Profit | Total | Child death | Insured death |
|---|---|---|---|---|---|---|---|
| 4 (1) | 600 | 544 | 56 | 27 | 571 | 1571 | 8800 |
| 5 (2) | 1200 | 1088 | 112 | 83 | 1171 | 2171 | 8200 |
| 6 (3) | 1800 | 1633 | 167 | 169 | 1801 | 2801 | 7600 |
| 7 (4) | 2400 | 2177 | 223 | 386 | 2463 | 3463 | 7000 |
| 8 (5) | 3000 | 2721 | 279 | 436 | 3157 | 4157 | 6400 |
| 9 (6) | 3600 | 3265 | 335 | 621 | 3887 | 4887 | 5800 |
| 10 (7) | 4200 | 3809 | 391 | 843 | 4652 | 5652 | 5200 |
| 11 (8) | 4800 | 4354 | 446 | 1103 | 5456 | 6456 | 4600 |
| 12 (9) | 5400 | 4898 | 502 | 1403 | 6301 | 7301 | 4000 |
| 13 (10) | 6000 | 5442 | 558 | 1745 | 7187 | 8187 | 3400 |
| 14 (11) | 6600 | 5986 | 614 | 2132 | 8118 | 9118 | 2800 |
| 15 (12) | 7200 | 6530 | 670 | 2565 | 9095 | 10095 | 2200 |
| 16 (13) | 7800 | 7075 | 725 | 3047 | 10121 | 11121 | 1600 |
| 17 (14) | 8400 | 7619 | 781 | 3580 | 11199 | 12199 | 1000 |

Table 4b: General client quotation

| Child age | Money outlay | Insured account | Special account |
|---|---|---|---|
| 1 | $x_1$ | $P_1$ | $P = P_1+P_1 r = P_1 (1+r)$ |
| 2 | $x_2$ | $P_2$ | $P_2+(P_2+P_1 (1+r)) r = P_2 (1+r)+P_1 r (1+r)$ $= (P_2+p_1 r) (1+r)$ $= (P_2+P_1 r) (1+r)$ |
| 3 | $x_3$ | $P_3$ | $P = P_3+[P_3+(P_2+P_1) (1+r)+P_1 (1+r)]r$ $P = [P_3+(P_1+P_2)r+P_1 r^2] (r+1)$ |
| 4 | $x_4$ | $P_4$ | $[P_4+[P_1+P_2+P_3)r+(2P_1+P_2)r^2+P_1 r^3] (r+1)$ |

Table 5a: Table of particular participant items

| Items | Value |
|---|---|
| Participant age | 43 years old non smoker |
| Sex | Female |
| Child Age | 5 year old |
| Term | 16 years |
| Monthly Payment | RM150 |
| Total yearly Payment | RM1800 (P) |
| Profit rate | 5.50% |

Table 5b: Table of fraction of distribution basic plan

| Year | Percentage of cumulative personal account | Percentage of personal account |
|---|---|---|
| 1 | 100 | 0 |
| 2 | 90 | 10 |
| 3 | 50 | 50 |
| 4-16 | 25 | 75 |

Table 5c: Table of illustration plan

| Year | Total monthly payment in a year (M) | Comulative payment outlay | Comulative profit of personal account ($C_n$) | Basic death coverage (Y) | Total death coverage (T) |
|---|---|---|---|---|---|
| 1 | 1 800 | 1 800 | 0 | 141 000 | 141 000 |
| 2 | 1 800 | 3 600 | 190 | 141 000 | 141 190 |
| 3 | 1 800 | 5 400 | 1 150 | 141 000 | 142 150 |
| 4 | 1 800 | 7 200 | 2 637 | 141 000 | 143 637 |
| 5 | 1 800 | 9 000 | 4 207 | 141 000 | 145 207 |
| 6 | 1 800 | 10 800 | 5 862 | 141 000 | 146 862 |
| 7 | 1 800 | 12 600 | 7 609 | 141 000 | 148 609 |
| 8 | 1 800 | 14 400 | 9 452 | 141 000 | 150 452 |
| 9 | 1 800 | 16 200 | 11 396 | 141 000 | 152 396 |
| 10 | 1 800 | 18 000 | 13 447 | 141 000 | 154 447 |
| 11 | 1 800 | 19 800 | 15 611 | 141 000 | 156 611 |
| 12 | 1 800 | 21 600 | 17 894 | 141 000 | 158 894 |
| 13 | 1 800 | 23 400 | 20 302 | 141 000 | 161 302 |
| 14 | 1 800 | 25 200 | 22 843 | 141 000 | 163 843 |
| 15 | 1 800 | 27 000 | 25 523 | 141 000 | 166 523 |
| 16 | 1 800 | 28800 | 28 351 | 141 000 | 169 351 |

Table 5d: Table of Derivation of the General Formulae

| Year | Monthly payment in a year (Total) | Cumulative payment outlay | Cumulative profit of personal account | Total death coverage |
|---|---|---|---|---|
| n | $M_n$ | $\sum_{i=1}^{n} M_i$ | $C_n = (M_nI+C_{n-1})(1+r)$ | $Y+C_n$ |

In Table 5d, $M_n$ is a total monthly payment in year n, I is a percentage of personal account in decimal, $C_n$ is cumulative profit of personal account for year n and r is an interest rate in a year.

## NUMERICAL RESULT

Given below are the numerical results as discussed. One is for a mudharabah model practiced by one takaful company and the other one is for a wakala model practiced by another takaful company.

**The calculation of particular family takaful in mudharabah model:** For simplification of the terms given in Table 4a, rewrite Table 4a as a new table called Table 5a with the associated columns are as follows. The researcher renamed child age as number of year (i), Total Insured Account as $Q_i$, Profit of the Total Insured Account as $Q_ir$, Total Profit per year as $T_i$ and Insured Death as $D_i$ where:

$$Q_i = P_i + \sum_{j=1}^{i-1} T_{i-j} \; (i = 2,3,...,14; Q_1 = P_1)$$

$$T_i = P_i + Q_i r \; (i = 1,2,3,....14)$$

$$C_i = T_i + 1000 \; (i = 1,2,3,...,14)$$

and:

$$D_i = D_{i-1} - 600 \; (i = 2, 3..., 14; D_1 = 8800)$$

**Surrender value:** Table 6 shows how much a participant can earn if he or she surrenders after 10 years that is an amount of RM13,447.00. It follows that after a deduction of a management fee of RM150.00, the net surrender value is RM7,187.00-RMRM150.00 which comes to RM7,037.00.

**Maturity value:** Based on the client quotation of a family takaful as shown in Table 6, the researcher finds that the table has detailed information and the total payment for 14 years is RM8,400.00. But the participant can earn RM11,199.00 for his child's education with the management fee of RM150.00. Therefore, the net maturity value is RM11,199.00-RM150.00 which comes to RM11,049.00.

**Death coverage:** In Mudharabah model, the participant will earn more than the surrender value if the child dies. For example, if the participant has invested more than 10 years, he will earn RM5652.00 at his child's death. If the participant dies after 10 years of investing in takaful, his child will earn RM5200.00 and also he will get the maturity value when he reaches 17 years old.

**Calculation and derivation of the formulae in wakala model:** In Table 7a, the researcher is given the data of cumulative profit of personal account and death coverage prepared by the one company. After observation of the data given in Table 7a, the researcher succeed to produce several formulae given in Table 7b from which the researcher knows how the company introduced its insurance to the clients.

In Table 7b, $M_n$ is a total monthly payment in year n, I is a percentage of personal account in decimal as shown in the Table 5b, $C_n$ is cumulative profit of personal account for year n and r is an interest rate in a year.

**Surrender value:** Table 7a shows how much a participant can earn if he or she surrenders after 10 years: an amount of RM13,447.00. However, he or she cannot earn the RM13,447.00 because he or she has to pay the wakala fee for earlier surrender. Therefore, it follows that after a

Table 6: Calculation of client quotation of particular family takaful

| Year (i) | Insured account ($P_i$) | Total of the insured account ($Q_i$) | Profit of the total insured account ($Q_i r$) | Total profit per year ($T_i$) | Child death ($T_i$+1000) | Insured Death ($D_i$) |
|---|---|---|---|---|---|---|
| 1 | 544 | 544 | $544\left(\frac{5}{100}\right)=27$ | 544+27 = 571 | 1 000 +571 = 1 571 | 8 800 |
| 2 | 1088 | 1 088+571 = 1 659 | $1\ 659\left(\frac{5}{100}\right)=83$ | 1 088+83 = 1 171 | 1 000+1 171= 2 171 | 8 800-600= 8 200 |
| 3 | 1633 | 1 633+1 171+571 = 3 375 | $3\ 375\left(\frac{5}{100}\right)=169$ | 1 699 +169 = 1 801 | 1 000+1 801 = 2 801 | 8 200-600 = 7 6004 |
| | 2177 | 2 177+1 801+1 171+571 = 5 720 | $5\ 720\left(\frac{5}{100}\right)=286$ | 2 177 +286 = 2 463 | 1 000+2 463 = 3 463 | 7 600-600 = 7 000 |
| 5 | 2722 | 2 721+2 463+1 801+1 171+ 571 = 8 727 | $8\ 727\left(\frac{5}{100}\right)=436$ | 27 21+436= 3 157 | 1 000+3 157 = 4 157 | 7 000-600 = 6 400 |
| 6 | 3265 | 3 265+3 157+2 463+1 801+1 171 +571 = 12 428 | $12\ 428\left(\frac{5}{100}\right)=621$ | 3 265+ 621 = 3 887 | 1 000+3 887 = 4 887 | 6 400-600 = 5 800 |
| 7 | 3809 | 3 809+3 887+3 157+2 463+ 1 801+1 171+571 = 16 859 | $16\ 859\left(\frac{5}{100}\right)=843$ | 3 809+843 = 4 652 | 1 000+4 652 = 5 652 | 5 800-600 = 5 200 |
| 8 | 4354 | 4 354+4 652+3 887+3 157+2 463 +1 801+1 171+571 = 22 056 | $22\ 056\left(\frac{5}{100}\right)=1\ 103$ | 4 354+1 103 = 5 456 | 1 000+5 456 = 6 456 | 5 200-600 = 4 600 |
| 9 | 4898 | 4 898+5 456+4 652+3 887+3 157 +2 463+1 801+1 171+571 = 28 056 | $28056\left(\frac{5}{100}\right)=1\ 403$ | 4 898+1 403 = 6 301 | 1 000+6 301 = 7 301 | 4 600-600 = 4 000 |
| 10 | 5442 | 5 442+6 301+5 456+4 652+3 887 +3 157+2 463+1 801+1 171+571 = 34 901 | $34\ 901+\left(\frac{5}{100}\right)=1\ 745$ | 5 442 +1 745 = 7 187 | 1 000+7 187 = 8 187 | 4 000-600 = 3 400 |
| 11 | 5986 | 5 986+7 187+6 301+5 456+4 652 +3 887+3 157+2 463+1 801+1 171 +571 = 42 632 | $42\ 632\left(\frac{5}{100}\right)=2\ 132$ | 5 986+2 132 = 8 118 | 1 000+8 118 = 9 118 | 3 400-600 = 2 800 |
| 12 | 6530 | 6 530+8 118+7 187+6 301+5 456 +4 652+3 887+3 157+2 463+1 801+ 1 171+571 = 51 294 | $51\ 294\left(\frac{5}{100}\right)=2\ 565$ | 6 530+2 565 = 9 095 | 1 000+9 095 = 10 095 | 2 800-600 = 2 200 |
| 13 | 7075 | 7 075+9 095+8 118+7 187+6 301+ 5 456+4 652+3 887+3 157+2 463+ 1 801+1 171+571 = 60 934 | $60\ 934\left(\frac{5}{100}\right)=3\ 047$ | 7 075+3 047 = 10 121 | 1 000+1 0121 = 11 121 | 2 200-600 = 1 600 |
| 14 | 7619 | 7 619+1 0121+9 095+8 118+7 187+ 6 301+5 456+4 652+3 887+3 157+ 2 463+1 801+1 171+571 = 71 599 | $71\ 599\left(\frac{5}{100}\right)=3\ 580$ | 7 619+3 580 = 11 199 | 1 000+11 199 = 12 199 | 1 600-600 = 1 000 |

Table 7a: Calculation of client quotation of particular family takaful

| Year | Calculation of cumulative profit of personal account in Wakala model (Surrender value/maturity value) | Total of death coverage |
|---|---|---|
| 1 | [1 800(0.0)+0] (1.055) = 0 | 141 000+0 = 141 000 |
| 2 | [1 800(0.1)+0] (1.055) = 190 | 141 000+190 = 141 190 |
| 3 | [1 800(0.5)+190] (1.055) = 1 150 | 141 000+1 150 = 142 150 |
| 4 | [1 800(0.75)+1 150] (1.055) = 2 637 | 141 000+2 637 = 143 637 |
| 5 | [1 800(0.75)+2 637] (1.055) = 4 207 | 141 000+4 207 = 145 207 |
| 6 | [1 800(0.75)+4 207 ] (1.055) = 5 862 | 141 000+5 862 = 146 862 |
| 7 | [1 800(0.75)+5 862] (1.055) = 7 609 | 141 000+7 609 = 148 609 |
| 8 | [1 800(0.75) +7 609] (1.055) = 9 452 | 141 000+9 452 = 150 452 |
| 9 | [1 800(0.75) +9 452] (1.055) = 11 396 | 141 000+11 396 = 152 396 |
| 10 | [1 800(0.75) +11 396] (1.055) = 13 447 | 141 000+13 447 = 154 447 |
| 11 | [1 800(0.75)+13 447] (1.055) = 15 611 | 141 000 +15 611= 156 611 |
| 12 | [1 800(0.75)+15 611] (1.055) = 17 894 | 141 000+17 894 = 158 894 |
| 13 | [1 800(0.75)+17 894] (1.055) = 20 302 | 141 000+20 302 = 161 302 |
| 14 | [1 800(0.75)+20 302] (1.055) = 22 843 | 141 000+22 843 = 163 843 |
| 15 | [1 800(0.75)+22 843] (1.055) = 25 523 | 141 000+25 523 =166 523 |
| 16 | [1 800(0.75)+25 523] (1.055) = 28 351 | 141 000+28 351 = 169 351 |

Table 7b: Several formulae for obtaining the data given in table 7

| No. of year | Total monthly payment in a year | Cumulative payment outlay | Cumulative profit of personal account | Total death coverage |
|---|---|---|---|---|
| n | $M_n$ | $\sum_{i=1}^{n} M_i$ | $C_n = (M_n I + C_{n-1})\ (1+r)$ | $Y+C_n$ |

deduction of RM150.00 in the wakala fee, the net surrender value is RM13,447.00-RM150.00 which comes to RM13,297.00.

**Maturity value:** Table 7a shows how much a participant can earn if he or she investment until 16 years in family takaful. However, he or she cannot earn the RM28,351.00 because he or she has to pay the wakala fee. Therefore, it follows that after a deduction of RM150.00 in the wakala fee, the net maturity value is RM28,351.00-RM150.00 which comes to RM28,201.00.

**Death coverage:** In wakala model, the participant will earn a lot of money if his child dies before the maturity period. He will earn a regular surrender value of RM13,447.00. However, if his child dies after 10 years of investing in takaful, he will earn RM154,447.00. If the participant dies before the maturity value, his child will still earn RM28,351.00 after 17 years of maturity period.

## DISCUSSION

The researcher has discovered quite a number of researches in insurance such as the axiomatic characterization of price measures by Goovaerts and Laeven (2008) that are super additive and co monotonic additive for normally distributed random variables. To develop a pricing rule for life insurance by Virginia (2008) under stochastic mortality in an incomplete market by assuming that the insurance company requires compensation for its risk in the form of a pre-specified instantaneous Sharpe ratio.

In his paper, Wang *et al.* (1997) proposed axioms for pricing insurance that characterize the premium principle of Wang (1996). Under this premium principle, the price at which to insure a given risk is the expectation of the risk with respect to a distorted probability. With assumption that the prices are given by Wang's premium principle and it determine the optimal indemnity contract for a risk-averse buyer who acts to maximize expected utility. Furthermore, Dickson and Waters (1999) consider a classical surplus process modified by the action of a constant force of interest. The writers derive recursive algorithms for the calculation of the probability of ruin in finite time. The writers also discuss the numerical evaluation of the probability of ultimate ruin using methods proposed by De Vylder (1996) and Sundt and Teugels (1995). Finally, the writers consider the problem of recovery from ruin.

The determinants of demand for private health insurance among formal sector employees in Malawi using a multinomial logit by Makoka *et al.* (2007). However, the researcher hasn't been able to find any previous literature that compare the two models of Islamic Takaful Insurance; The mudharabah and wakala. Thus, in this part of the paper, the results are not interpreted in relation to previous literature whether these are in support or are in contradiction to the research.

In the mudharabah model, the researcher find that the maturity value is RM11,099.00 after the management fee and the payment outlay for the whole 14 years is RM8,400.00. So, the participant earns more in their maturity value. The researcher also find that the surrender value after 10 years of saving in the takaful education plan is RM7,037.00 and the total payment for 10 years is RM6,000.00. If the participant surrenders at the end of the first year, he still has his money but not in full due to the fact that he has to pay the management fee of early surrender.

In the wakala model, the researcher finds that the maturity value is RM28,351.00 and the payment outlay for the whole 16 years is RM28,800.00. The researcher also find that in the surrender value after 10 years of saving is RM13,447.00. However, the total payment after 10 years is RM18,000.00. So, the researcher finds that the participant earns less in maturity value or surrender value compared to his savings in takaful. On top of that, if the participant surrenders at the end of the first year, he cannot get his money back.

In comparison we can see that in the mudharabah model, the participant and the child will get the death coverage if either one of them dies even though it is not a big amount. In the wakala model, the participant will get a big amount in death coverage when his child dies before the maturity period. However if the participant dies the child will not get the death coverage at all other than the maturity value.

The results of the research clearly show that mudharabah model takaful insurance is a much better deal compare to wakala. Therefore, the results of this research can be published and promoted as proof that as far as the clients are concerned the mudharabah model offers them the best value for their money.

## CONCLUSION

Based on the mathematical analysis, we can form two conclusions. The mudharabah model of takaful insurance gives the clients more benefits in that they can earn more money in saving. On the other hand, wakala model of takaful insurance is more beneficial to the takaful company. The financial risk is lower, plus it doesn't have to spend money on the salary of the agents, who work entirely on commission. Unfortunately, quite a number of takaful companies which have been using the mudhabarah model, have switched to the wakala model. They do not want to pay their agents' fixed salary. The agents have to work hard to find more clients so they can make more commission. This translates into higher management fees which further translates into less money earned by the clients. This research paper will prove that clients will be better off if they choose a takaful company that sells the mudharabah model and not the wakala model.

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