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Numerical Simulation of Natural Convection in an Inclined Square Cavity

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Abstract: In this study, we report the fluid flow behavior and the characteristic of heat transfer from a differentially heated enclosure walls and tilted at various inclination angles. The double distribution function lattice boltzmann numerical scheme with the same lattice structure is applied to predict the velocity and temperature fields in the system. Two different types of boundary condition are applied at the top and bottom walls of the enclosure. In current study, we found that the vortex formation, size and flow characteristics are significantly affected by the magnitude of inclination angles. We also found that the convection mode of heat transfer dominates the heat transfer mechanism for every simulation condition due to relatively high rayleigh number condition applied in this study.

Key words: Inclination angle, lattice boltzmann, natural convection, double population

INTRODUCTION

The subject of natural convection heat transfer has been studied for quite extensively, mainly because of its important application in nuclear, aerospace technology and environmental applications. Recently, numerical observations at particle level of natural convection thermal fluid flow behavior have increased the depth of understanding of the underlying mechanism. Although, the natural convection phenomenon can be widely seen in industrial operations, much remain to be done due to the complexity of the interaction between the heated surface and the surrounding fluids. In order to gain fundamental knowledge about the complex mass and heat transfer mechanism, research is still needed.

In present study, we carried out numerical investigation of natural convection in a differentially heated enclosure's walls and inclined at various angles from the horizontal axis. Compared to the conventional problem of natural convection in an enclosure (90° of enclosure with differentially heated left and right walls), the current problem need to consider both component of buoyancy forces which influence strongly the flow structure and heat transfer therein.

The effect of inclination angle on natural convection in an enclosure has been discussed by Hart (1971), Ozoe *et al*. (1975) and Kuyper *et al*. (1993). The earliest result reported the stability analysis of the flow (Hart, 1971). An experimental investigation by Hiroyuki *et al*. (1974) to estimate the flow characteristic, Nusselt number and critical Rayleigh number revealed that the Rayleigh number at 10^4 gives the minimum and the maximum heat transfer rate as the angle increases. Besides that, experimental work by Kuyper *et al*. (1993) claimed that the heat flux is strongly dependent on the Rayleigh number and inclination angle.

In recent years, the Lattice Boltzmann Method (LBM) has received considerable attention as an alternative approach for simulating wide range of fluid flow. Unlike other numerical methods, LBM predicts the evolution of particle distribution function and calculates the macroscopic variables by taking moment to the distribution function. The LBM has a number of advantages over other conventional computational fluid dynamics approaches. The algorithm is simple and can be implemented with a kernel of just a few hundred lines. The algorithm can also be easily modified to allow for the application of other, more complex simulation components. For example, the LBM can be extended to describe the evolution of binary mixtures (Azwadi and Tanahashi, 2006), or extended to allow for more complex boundary conditions (Azwadi and Syahrullail, 2009). Thus the LBM is an ideal tool in thermal and fluid simulation.

The objective of this study is to predict the thermal and fluid flow behavior in a differentially heated, inclined square enclosure. Two different types of boundary

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conditions are applied at the top and bottom of the enclosure. To the best of authors' knowledge, there are very few lattice boltzmann predictions for the case in hand. Therefore, present study solved the governing continuity, momentum and energy equations indirectly, i.e., by using the lattice Boltzmann formulation with second order accuracy in space and time.

PROBLEM PHYSICS AND BOUNDARY CONDITIONS

The physical domain of the problem is represented in Fig. 1. The temperature difference between the left and right walls introduces a temperature gradient in a fluid and a consequent density difference induces a fluid motion, that is, convection.

The dynamical similarity depends on two dimensionless parameters: the Prandtl number Pr and Rayleigh number Ra:

$$Pr = \frac{\upsilon}{\chi} \tag{1}$$

$$Ra = \frac{\rho\beta(T_H - T_C)L^3}{\upsilon\chi} \tag{2}$$

where, υ, χ and L are the fluid kinematic viscosity, thermal diffusivity and width of the square cavity.

NUMERICAL MODELS

In this study, the governing equation of incompressible, two-dimensional and laminar Navier-Stokes and energy equations were solved indirectly using the Lattice Boltzmann Method (LBM). Unlike other numerical methods, LBM predicts the evolution of particle distribution function and calculates the macroscopic variables by taking moment to the distribution function. LBM starts with the Boltzmann equation, discretised in velocity space, given as:

$$\frac{\partial f_i}{\partial t} + c_i\frac{\partial f_i}{\partial x} = -\frac{1}{\tau_f}\left(f_i - f_i^{eq}\right) \tag{3}$$

$$\frac{\partial g_i}{\partial t} + c_i\frac{\partial g_i}{\partial x} = -\frac{1}{\tau_g}\left(g_i - g_i^{eq}\right) \tag{4}$$

where, distribution function f is used to calculate density and velocity fields and distribution function g is used to calculate temperature field. F is the external force and τ_f and τ_g are the relaxation times carried by the momentum and energy, respectively.

The equilibrium distribution functions f_i^{eq} and g_i^{eq} are chosen so that they satisfy the macroscopic equations via Chapman-Enskog expansion. They can be written as (Azwadi and Tanahashi, 2007):

$$f_i^{eq} = \omega_i\rho\left[1 + 3c_i \cdot u + 4.5(c_i \cdot u)^2 - 1.5u^2\right] \tag{5}$$

$$g_i^{eq} = \omega_i T\left[1 + 3c_i \cdot u + 4.5(c_i \cdot u)^2 - 1.5u^2\right] \tag{6}$$

The values of the weight ω_i depend on the chosen lattice model. In present study, we chose nine-velocity lattice model to represent both f and g distribution functions. The lattice configurations are shown in Fig. 2.

The macroscopic variables such as density, velocity and temperature can be calculated by taking moment to the distribution functions as follow:

$$\rho = \sum_{i=0}^{8} f_i,\ \rho u = \sum_{i=0}^{8} c_i f_i,\ T = \sum_{i=0}^{8} g_i \tag{7}$$

The time relaxations can be related to the macroscopic fluid viscosity and thermal diffusivity using the following equations:

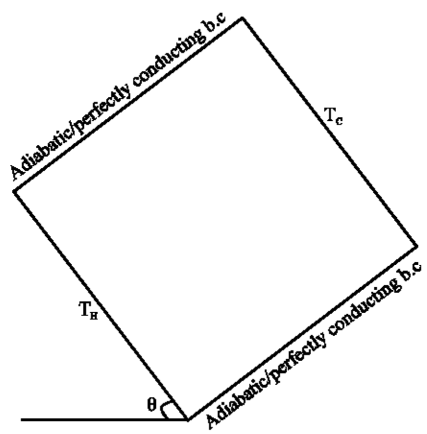

Fig. 1: Physical domain of the problem

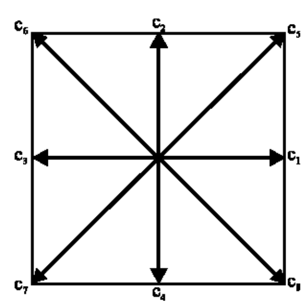

Fig. 2: Nine-velocity lattice model with ω_0 = 4/9, $\omega_{1,2,3,4}$ = 1/9 and $\omega_{5,6,7,8}$ for f and g distribution function

$$\tau_f = 3\upsilon + 1/2 \quad (8)$$

$$\tau_g = 3\chi + 1/2 \quad (9)$$

Via the so-called chapmann-enskog expansion, the evolution equations of distribution function could recover the macroscopic governing equations. Detail derivation will not be shown here and can be found in Azwadi and Tanahashi (2008).

The Boussinesq approximation is applied to the buoyancy force term. With this approximation, it is assumed that all fluid properties can be considered as constant in the body force term except for the temperature dependence of the density in the gravity term. So the external force in Eq. 1 is:

$$F = 3G(c-u)f_i^{eq} \quad (10)$$

where, G is the contribution from buoyancy force.

RESULTS AND DISCUSSION

Here, we shall apply the method and demonstrate the obtained results in terms of streamlines, isotherms and average nusselt number.

Here, we begin with the validation of the thermal lattice boltzmann model at inclination angle of 90°. Table 1 shows the average Nusselt number computed by the LBM for Ra = 10^3 to 10^4 and comparisons with those by Davis (1983) and Nithiarasu *et al*. (1997).

As can be seen from the Table 1, the results predicted by current model agree well with the previous studies. This gives us confidence to apply the proposed method for simulation of thermal fluid flow at various inclination angles.

In present study, we carried out numerical investigations at Rayleigh number of 5×10^5 and Prandtl number of 0.71. Two sets of boundary conditions, adiabatic and perfectly conducting boundary conditions, were applied at the bottom and top walls before the enclosure was tilted at various angles respect to horizontal axis.

The predicted isotherms and streamlines for various inclination angles are demonstrated in Fig. 3a-d and 4a-d. As can be seen from Fig. 3 and 4, the plots of isotherms are almost identical computed from two different boundary conditions applied at the top and bottom of the enclosure. At low value of inclination angle (θ = 40°), the isotherms show a good mixing occurring in the center and relatively small gradient indicating small value of the local nusselt number along the differentially heated walls. Further increment of inclination angle leads to the isotherms lines become parallel to the top and bottom walls indicates that the main heat transfer mechanism is by convection. At the highest inclination angle in present study, the isotherms lines are equally spaced indicate low average Nusselt number in the system.

Table 1: Comparison among Navier Stokes solver, finite element method and present model

Rayleigh Number	Davis (1983)	Nithiarasu *et al.* (1997)	Present
10^3	1.116	1.127	1.117
10^4	2.238	2.245	2.236

From the view of streamlines plots, they also demonstrate identical characteristic except for inclination of θ = 40°. At this value of inclination angle, for the case of perfectly conducting boundary condition, two corner vortices appear in the system and compress the central vortex to become more rounded. On the other hand, for the case of adiabatic boundary condition, no corner vortex appears and the central vortex presents in the form of square shaped indicates low and approximately uniform value of flow velocity near all four enclosure's walls. As we increase the inclination angle, the central vortex splits in to two smaller vortices. The velocity boundary layer can be clearly seen and this high flow velocity drags the central vortex pointing upper and lower corner of the enclosure. Further increment of inclination angle lead to central vortex grow in size indicates that some fluids from the hot or cold wall returns back to the same wall. All of these phenomena are in good agreement with previous studies (Ravnik *et al*., 2008; Azwadi *et al*., 2010; Rasoul and Prinos, 1997).

As can be seen from the Fig. 5, the computed nusselt numbers computed from perfectly conducting boundary condition are lower than those for the case of adiabatic types of boundary condition because the heat is allowed to pass through the top and bottom walls. Interestingly, the minimum value of average Nusselt number for both conditions is found converging to the same value and when the inclination angle approaching 180° for both types of boundary conditions. On the other hand, the maximum value of average Nusselt number is determined at inclination angle between 60° to 80°. These can be explained by analyzing the isotherms plots which demonstrating relatively denser lines near hot and cold walls leading to high temperature gradient near these regions. Lower value of average nusselt number at lower inclination angle was due to the presence of small corner vortices which contributes smaller local heat transfer along the hot and cold walls. For the computation at higher inclination angles, where the hot wall is close to the top position, the magnitude of the gravity vector is reduced results in low magnitude of flow velocity along the hot wall. Due to this reason, the heat transfer rates are small resulted from the reduction in the driving potential for free convection.

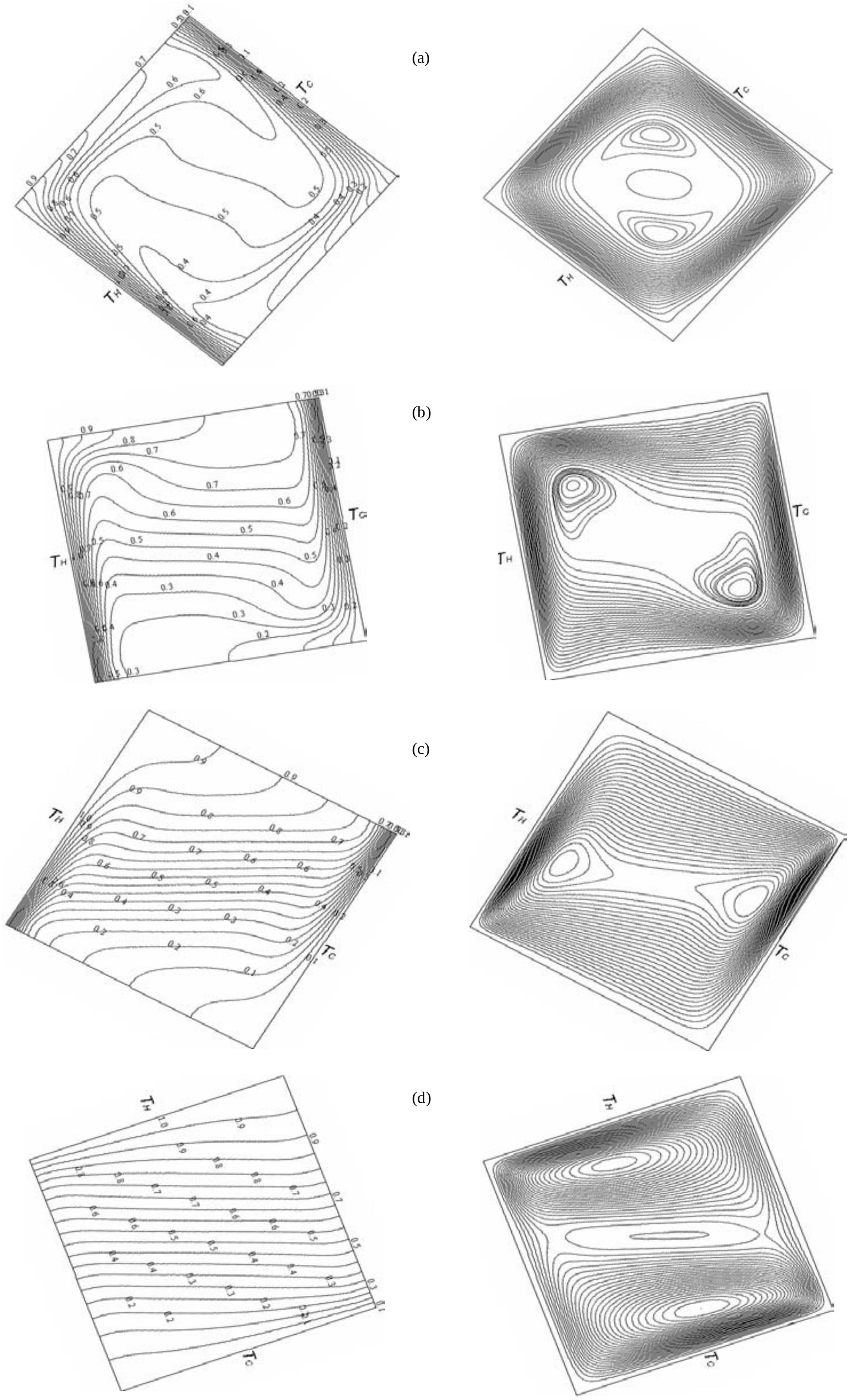

Fig. 3: Plots of isotherms (left) and streamline (right) with adiabatic boundary condition. (a) θ = 40°, (b) θ = 80°, (c) θ = 120° and (d) θ = 160°

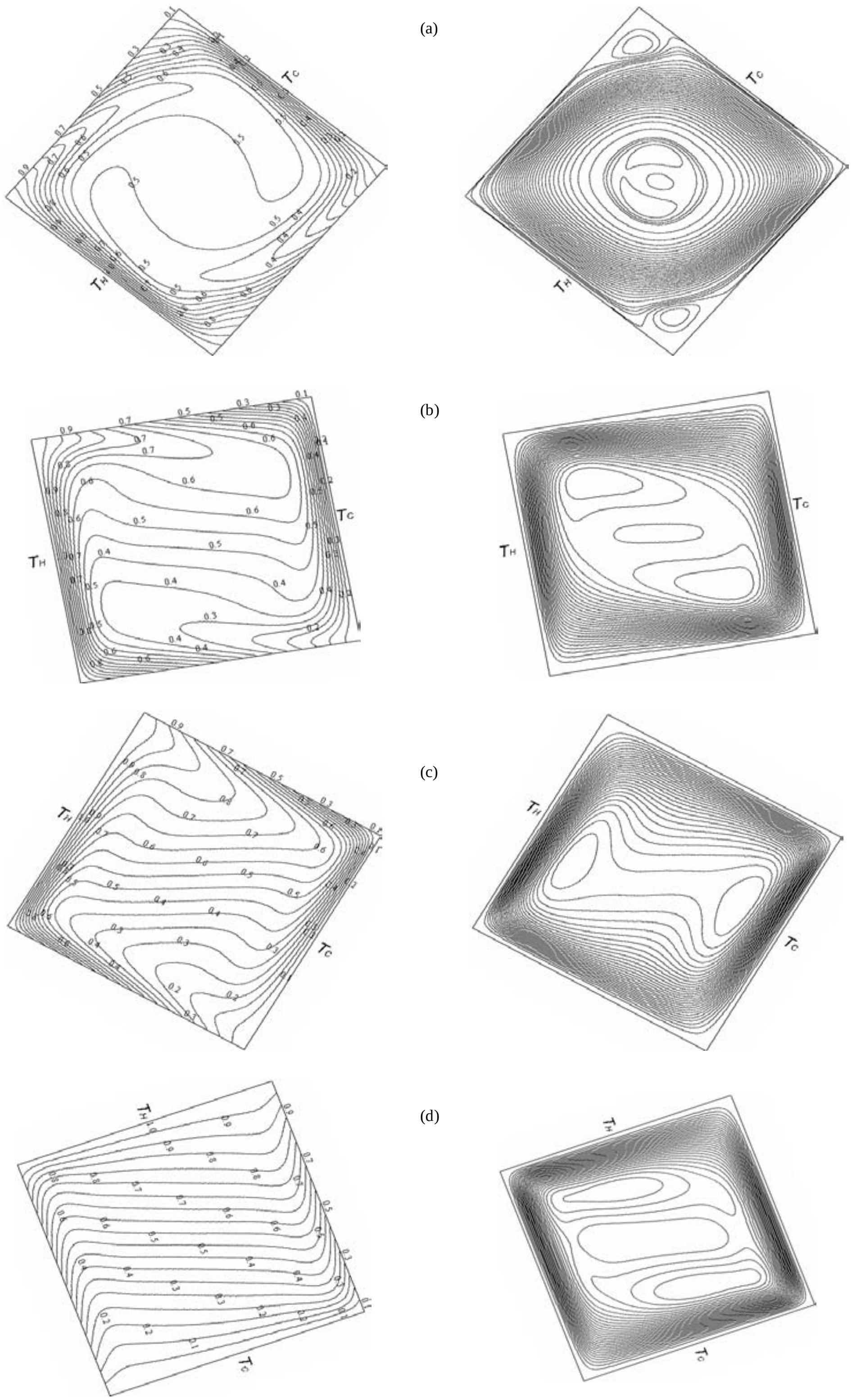

Fig. 4: Plots of isotherms (left) and streamline (right) and streamlines with perfectly conducting boundary condition. (a) θ = 40°, (b) θ = 80°, (c) θ = 120° and (d) θ = 160°

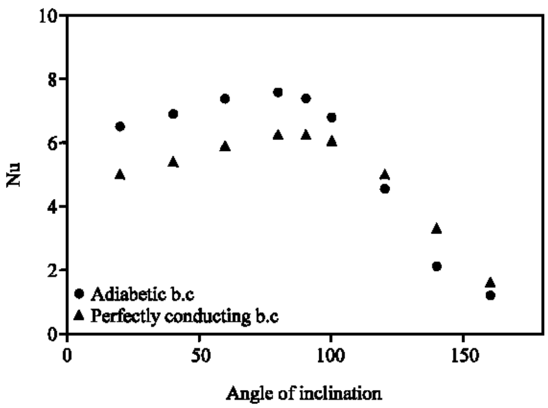

Fig. 5: Comparison of computed averaged Nusselt number using two different types of boundary conditions

CONCLUSION

The natural convection from a differentially heated enclosure's wall and tilted at various inclination angles has been studied using lattice Boltzmann approaches. The governing macroscopic momentum and energy equations have been solved indirectly, i.e., the double population method. From Fig. 3 and 4, the boundary layers for the velocities and temperature can be observed clearly. As expected, the flow pattern including the boundary layers and vortices with heat transfer mechanisms are significantly influenced by the magnitude of inclination angle of the square enclosure. The results obtained demonstrate that the lattice boltzmann formulations are reliable approaches to study flow and heat transfer in a differentially heated enclosure.

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