Journal of Applied Sciences

ISSN 1812-5654

science alert

ANSInet
an open access publisher
http://ansinet.com

Journal of Applied Sciences 11 (24): 3821-3829, 2011
ISSN 1812-5654 / DOI: 10.3923/jas.2011.3821.3829

A Shallow Water Model for the Coast of Bangladesh and Applied to Estimate Water Levels for 'AILA'

[1]M.M. Rahman, [2]G.C. Paul and [2]A. Hoque
[1]Department of Mathematics, Shahjalal University of Science and Technology,
Sylhet 3114, Bangladesh
[2]Department of Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh

Abstract: A cyclone induced storm surge forecasting system has been developed for the coast of Bangladesh. The coast of Bangladesh has a specialty in terms of high bending and many off-shore islands. Incorporation of the coastline and island boundaries properly in the numerical scheme is essential for accurate estimation of water levels due to tide and surge. For that purpose, a model with doubly nested numerical schemes is developed in Cartesian coordinate system using vertically integrated shallow water equations. The developed system is applied on a severe cyclonic storm 'AILA' that hit Bangladesh on 25 May 2009. The computed peak sea-surface elevations along the coastal belt are in reasonable agreement with those observed.

Key words: Coast of Bangladesh, Meghna estuary, cyclone AILA, semi-implicit finite difference method, water level

INTRODUCTION

The Bay of Bengal is a region of the world frequently affected by storm surges associated with tropical cyclones. Statistics show that above 5% of the global tropical cyclones form over the Bay of Bengal. On average, five to six storms form in this region every year, but with 80% of the global casualties (Debsarma, 2009). Bangladesh is situated at the northern tip of the Bay of Bengal. The coastal belt of Bangladesh is also frequently lashed by tropical storms along with surges causing loss of life and property. The surge level along the coast of Bangladesh may be high even for a less intense storm. The factors responsible for this are extreme bending of the coastline, shallowness of water, off-shore islands, huge discharge through the Meghna and other rivers, low lying islands and coastal regions. Moreover, the head Bay of Bengal is a large tidal range (difference between high and low tides) area with the highest range around the Sandwip Island. Since the astronomical tidal oscillation is a continuous process in the sea, it becomes the initial dynamical condition in the tidal-surge interaction. The water level due to the tide-surge interaction becomes significantly high if a storm approaches the coast during a high tide period.

Many analyses on prediction of surge due to tropical storm have been made all over the world. Most of the works have been done for the Atlantic, North Sea and North-West European Continental Shelf, Australian region and the Bay of Bangal. Thacker (1977, 1979) incorporated the curving coastline and island boundaries with irregular grid finite difference technique as a substitute of the finite element method; but with less computational overheads. The studies on the development of storm surge models at the Institute of Oceanographic Sciences, Bidstone, England were documented by Heaps (1977). The studies of Arnold (1987), Bills and Noye (1987) and Tang and Grimshaw (1996) for the Australian region are mainly based on investigation of various open sea boundary conditions. A considerable number of works on modeling have so far been done for the Bay of Bengal region covering the coast of Bangladesh and the East Coast of India. Out of them the models of Das *et al.* (1974) and Jhons *et al.* (1981, 1985) are designed for the East Coast of India. Considering the existence of many small and big islands along the offshore region of Bangladesh, Roy (1999) developed a mathematical technique to incorporate islands of special shapes. Based on the functions defined for two opposite curved boundaries of the analysis area, one generalized function is derived to represent approximately two opposite boundaries of each island. Then the transformation of a spatial coordinate was considered so that the whole analysis area and the islands became rectangles in the transformed domain. But the limitation of the transformed coordinates models is that, if we want to incorporate the coastal and boundaries accurately in the numerical scheme, the grid size in the

---

Corresponding Author: M.M. Rahman, Department of Mathematics, University of Rajshahi, Rajshahi-6205, Bangladesh physical domain must be reduced. So, the fine meshes are required near the coast but it is unnecessary away from the coast and, in general, this may be achieved through nested numerical scheme (Rahman *et al*., 2011). But until now no attempt has been made to develop an efficient model suitable for operational forecasting purpose for the coast of Bangladesh. The present study attempts to develop a surge forecasting model based on the Cartesian coordinate system that incorporates the whole coastal belt and off-shore islands very accurately. The model is verified using water-level data from the severe cyclonic storm AILA that hit south western coastal districts of Bangladesh on 25 May 2009. For analysis and verification of the results, some locations along the coastal belt are considered. The locations are Hiron Point, Tiger Point, Kuakata, Char Madras (South Bhola), Char Chenga (South Hatiya), Char Jabbar, Companigonj, Sandwip, Chittagong and Bashkhali.

MATHEMATICAL FORMULATION

Vertically integrated shallow water equations: A system of rectangular Cartesian coordinates is used in which the origin, O, is in the undisturbed level of the sea surface (MSL). OX points towards the south, OY points towards the east and OZ is directed vertically upwards. The displaced position of the free surface is considered as $z = \zeta\ (x, y, t)$ and position of the sea floor as $z = -h\ (x, y)$, so that, the total depth of the fluid layer is $\zeta+h$. Then the vertically integrated shallow water equations given by Roy and Hussain (2001) are:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left[(\zeta + h)u\right] + \frac{\partial}{\partial y}\left[(\zeta + h)v\right] = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -g\frac{\partial \zeta}{\partial x} + \frac{T_x}{\rho(\zeta + h)} - \frac{C_f u\sqrt{u^2 + v^2}}{(\zeta + h)} \qquad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -g\frac{\partial \zeta}{\partial y} + \frac{T_y}{\rho(\zeta + h)} - \frac{C_f v\sqrt{u^2 + v^2}}{(\zeta + h)} \qquad (3)$$

where u, v are the velocity components of sea water in x and y directions, respectively, f is Coriolis parameter, g is the gravitational acceleration, T_x, T_y are the components of wind stress, h is the ocean depth from the mean sea level, C_f is the friction coefficient and ρ denotes the water density.

Also, in the above equations u and v are the vertically integrated components given by:

$$(u,\ v) = \frac{1}{\zeta + h}\int_{-h}^{\zeta} \left(\bar{u},\ \bar{v}\right) dz \qquad (4)$$

where, $\bar{u}$ and $\bar{v}$ are x and y components of the Reynolds averaged velocity.

Using Eq. 1 we may express the Eqs. 2 and 3 in the flux form and thus, Eqs. 1 and 3 may be written as:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial \tilde{u}}{\partial x} + \frac{\partial \tilde{v}}{\partial y} = 0 \qquad (5)$$

$$\frac{\partial \tilde{u}}{\partial t} + \frac{\partial (u\tilde{u})}{\partial x} + \frac{\partial (v\tilde{u})}{\partial y} - f\tilde{v} = -g(\zeta + h)\frac{\partial \zeta}{\partial x} + \frac{T_x}{\rho} - \frac{C_f\ \tilde{u}\sqrt{(u^2 + v^2)}}{\zeta + h} \qquad (6)$$

$$\frac{\partial \tilde{v}}{\partial t} + \frac{\partial (u\tilde{v})}{\partial x} + \frac{\partial (v\tilde{v})}{\partial y} + f\tilde{u} = -g(\zeta + h)\frac{\partial \zeta}{\partial y} + \frac{T_y}{\rho} - \frac{C_f\ \tilde{v}\sqrt{(u^2 + v^2)}}{\zeta + h} \qquad (7)$$

where:

$$(\tilde{u}, \tilde{v}) = (\zeta + h)(u, v)$$

where, u and v in the bottom stress terms of Eq. 2 and 3 have been replaced by $\tilde{u}$ and $\tilde{v}$ in Eqs. (6) and (7) in order to solve the equations numerically in a semi-implicit manner.

Boundary conditions: The normal component of the depth averaged velocity at the closed boundary is set to zero and for open boundaries, radiation type of boundary conditions are used to allow the disturbance, generated within the model area, to go out through the open boundary. Apart from the coast of Bangladesh, the boundaries are treated as straight lines in the open sea. Following Jhons *et al*. (1981), the western (85^0E), eastern (95^0E) and southern (15^0N) boundaries of the model are, respectively given by:

$$v + \left(\frac{g}{h}\right)^{\frac{1}{2}}\zeta = 0 \qquad (8)$$

$$v - \left(\frac{g}{h}\right)^{\frac{1}{2}}\zeta = 0 \qquad (9)$$

$$u - \left(\frac{g}{h}\right)^{\frac{1}{2}}\zeta = 0 \qquad (10)$$

Generation of wind stress: In this study, wind stresses are parameterized by the conventional quadratic law:

$$(T_x, T_y) = \rho_a C_D (u_a^2 + v_a^2)^{1/2} (u_a, v_a)$$

where, C_D is the drag coefficient, ρ_a is the air density and u_a and v_a are the x and y components of the surface wind.

Generally, the storm information is available in terms of the maximum sustained wind speed (V_0) and the corresponding radius (R). The circulatory wind field is then generated by the empirical formula due to Jelesnianski (1965), which is given by:

$$V_a = \begin{cases} V_0 \sqrt{(r_a/R)^3} & \text{for all} \quad r_a \le R \\ V_0 \sqrt{(R/r_a)} & \text{for all} \quad r_a > R \end{cases}$$

where, r_a is the radial distance at which the wind field is desired. The x and y components (u_a, v_a) of the wind field are derived from V_a given by the above empirical formula.

NUMERICAL METHOD

Set-Up of the nested schemes: In present study, we have taken the study area considerably big so that, a storm can move over the area at least for 3 days before crossing the coast. This is because the surge response along the coast becomes significant well before a storm reaching the coast. On the other hand, in order to include the major islands in the estuary the mesh size (the distance between two consecutive grid points) should be considerably smaller whereas, this is unnecessary away from the estuary. Consideration of very fine mesh over the whole analysis area involves, unnecessarily, more memory and more CPU time in the solution process and invites problem of numerical instability. Considering the above facts, a high-resolution numerical scheme (FMS) is nested into a coarse mesh scheme (CMS). In the fine mesh scheme, all the major islands are incorporated through proper stair step representation. For the existence of so many small and big islands and also for high bending of the coastline along the Meghna estuary (Fig. 1), a very fine mesh scheme (VFMS) for the region between Barisal and Chittagong is again nested into the fine mesh scheme. Information on the grids is given in Table 1.

Table 1: Model grid specifications

			Grid spacing	
Schemes	Scheme area	Grid size	North-South (Δx)	East-West (Δy)
CMS	15^0N to 23^0N (latitude) 85^0E to 95^0E (longitude)	60×61	15.08 km	17.52 Km
FMS	21^0-15′N to 23^0N (latitude) 89^0E to 92^0E (longitude)	92×95	2.15 Km	3.29 Km
VFMS	21.77^{0}N to 23^0N (latitude) 90.40^{0}E to 92^0E (longitude)	190×145	720.73 m	1142.39 m

Governing equations and boundary conditions for inner schemes: All of the schemes have the same dynamical Eq. 5-7 with different boundary conditions. An important feature of this doubly nesting is that the CMS is completely independent. On the other hand, along the open boundaries of the FMS the parameter ξ is prescribed from those obtained in CMS in each time step of the solution process. Similarly for the VFMS, the parameter ξ is prescribed from those obtained in FMS in each time step of the solution process.

Numerical procedure (with grid generation and model data set-up): We define discrete co-ordinate points in x-y plane by:

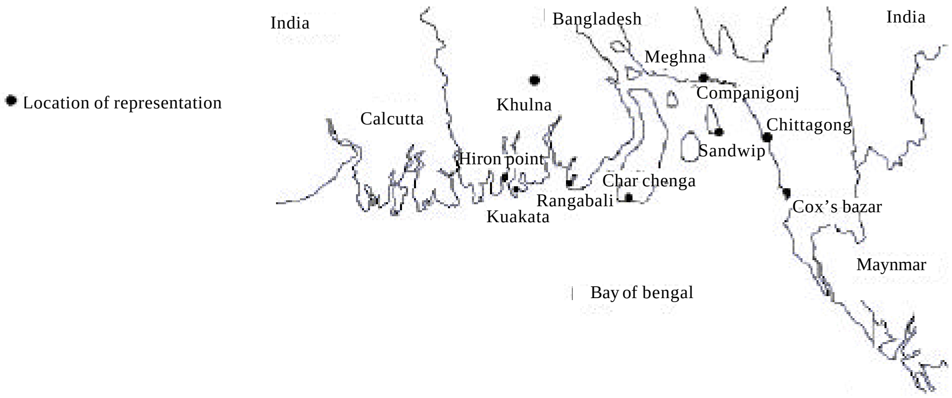

Fig. 1: The model domain and the locations of representation of 'Head Bay of Bengal'

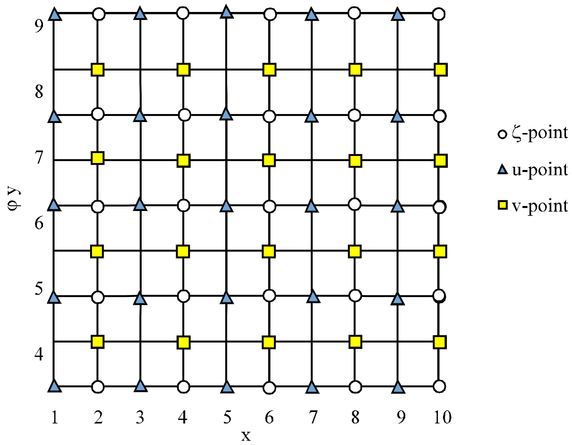

Fig. 2: Staggered grid system

$$x_i = (i-1)\,\Delta x,\ i = 1, 2, 3, ..., m$$
$$y_j = (j-1)\,\Delta y,\ j = 1, 2, 3, ..., n$$

A sequence of time instants is defined by:

$$t_k = k\Delta t,\ k = 1, 2, 3, ...$$

It is seen in Ali (1979) that the unstaggered grid cannot handle properly the high frequency waves, particularly two grid waves. So, we have used a suitable arrangement of grid points (Staggered grid system) in x-y plane is shown in Fig. 2. The grid lines are parallel to the co-ordinate axes and form a uniform network with a rectangular mesh having sides of length Δx in the x direction and Δy in the y direction. In the Staggered grid system, there are three distinct type of computational points. These three type of points are defined as follows: For a grid point (x_i, y_j), if i is even and j is odd, the point is a ζ-point at which ζ is calculated. If i is odd and j is odd, the point is a u-point at which u is calculated and finally, if i is even and j is even, the point is a v-point, at which v is calculated. We choose m, the number of grid line along x- axis, as even so that at the open sea boundary parallel to y-axis, there are ζ-points and v-points. Also n, the number of grid line along y- axis, as odd so that there are ζ-points and u-points at the open boundaries along and parallel to x-axis. As the computational method uses a rectangular grid, the coastline and the island boundaries have been approximated following the grid lines using stair step representation.

The governing equations given by 5-7 and the boundary conditions given by 8-10 are discretised by finite-difference (forward in time and central in space) and are solved by conditionally stable semi-implicit method using staggered grid system. For numerical stability, the x and y components of the momentum equations are modeled in a semi-implicit manner. For example, from Eq. 6 the term:

$$\tilde{u}\sqrt{(u^2 + v^2)}$$

is discretised as:

$$\tilde{u}^{k+1}\sqrt{u^{k2} + v^{k2}}$$

where, the subscript k+1 indicate that ũ is to be evaluated in advanced time level. Moreover, the CFL criterion has been followed in order to ensure the stability of the numerical scheme. Along the closed boundary, the normal component of the velocity is considered as zero and this is easily achieved through appropriate stair step representation as mentioned earlier. The initial values of ζ, u and v are taken as zero. The time step is taken as 60 sec that ensures stability of the numerical scheme. In the solution process, the values of the friction coefficient C_f and the drag coefficient C_D are taken as uniform throughout the physical domain, which are 0.0026 and 0.0028, respectively. The depth data are collected form the Admiralty bathymetric chats.

RESULTS AND DISCUSSION

Analysis of the computed surge response: For the purpose of analysis, the computed results are presented at some coastal and island locations of Bangladesh for the severe cyclonic storm 'AILA', which is one of the most severe tropical storm that hit the cost of Bangladesh recently. AILA hit south western coastal districts of Bangladesh on 25 May 2009, killing 190 people, affecting more than 3.9 million people across the 11 coastal districts, disrupting their livelihoods and destroying infrastructure. Most of the computed results are shown in Fig. 5-7. The results are computed for 80 hours and are presented for the last 48 hours.

Records show that AILA was one of the most severe of this century having maximum wind speeds 120 km h^{-1} (75 miles h^{-1}). History about AILA is shown in Table 2, the data are received from the Bangladesh Meteorological Department (BMD). According to Bangladesh Meteorological Department, a depression formed over the southeast Bay of Bengal at 0900 UTC of 23 May, 2009 and this depression developed into a cyclonic storm AILA at 1200 UTC of 24 May. AILA moved approximately northwards and at 0300 UTC of 25 May it gradually turned towards left and then towards north-northwest. It then moved northwardly and finally towards the coast of Bangladesh. At about 0800 UTC of 25 May 2009, it started

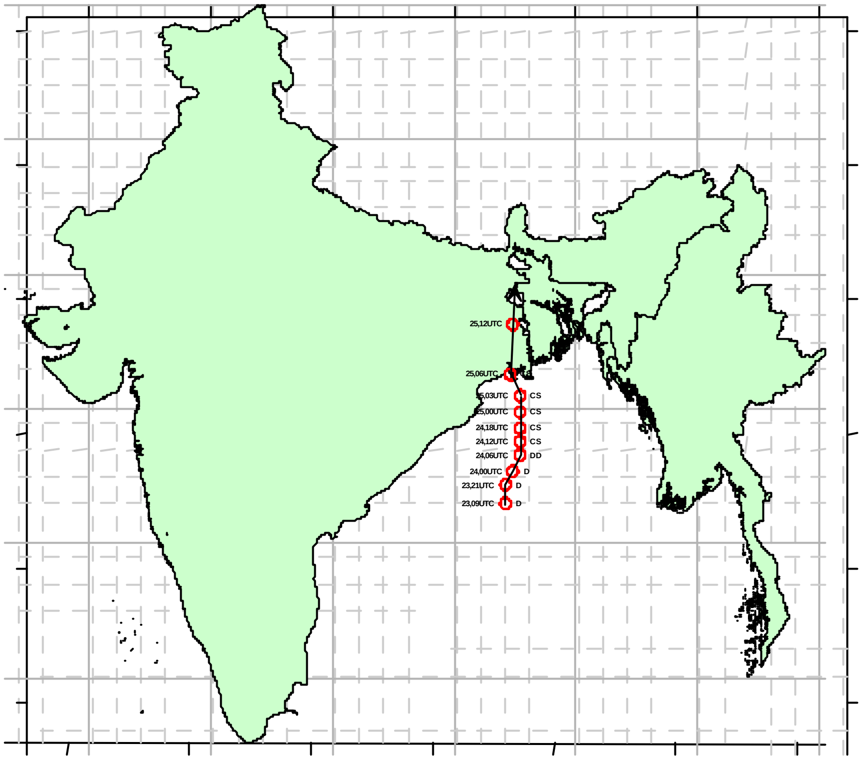

Fig. 3: Observed track of cyclonic storm "AILA"

Table 2: History of the storm AILA

	AILA 2009		
Date	Hour	Latitude	Longitude
23-05	09:00	16.00^0N	80.00^0E
23-05	15:00	16.50^0N	80.00^0E
23-05	21:00	16.70^0N	80.00^0E
24-05	00:00	17.20^0N	88.30^0E
24-05	09:00	17.80^0N	88.60^0E
24-05	12:00	18.30^0N	88.60^0E
24-05	15:00	18.60^0N	88.60^0E
24-05	18:00	18.80^0N	88.60^0E
24-05	21:00	19.20^0N	88.60^0E
25-05	00:00	19.40^0N	88.60^0E
25-05	03:00	20.00^0N	88.60^0E
25-05	06:00	21.60^0N	88.30^0E
25-05	12:00	22.90^0N	88.30^0E
25-05	20:00	24.20^0N	88.50^0E

Maximum wind speed: 120 km h^{-1}, Maximum radius of sustained wind: 54 km

to cross the coast of Bangladesh and moved continuously northwards. During the next 07-08 h the system completed its crossing the coast. The track (path) of AILA is given in Fig. 3.

Figure 4 shows AILA at peak intensity, which is collected from Wikipedia wesite [http://en.wikipedia.org/wiki/Cyclone_Aila]. Figure 5 depicts the computed surge levels associated with AILA at Hiron Point, Char Madras and Chittagong. It may be observed that, the maximum

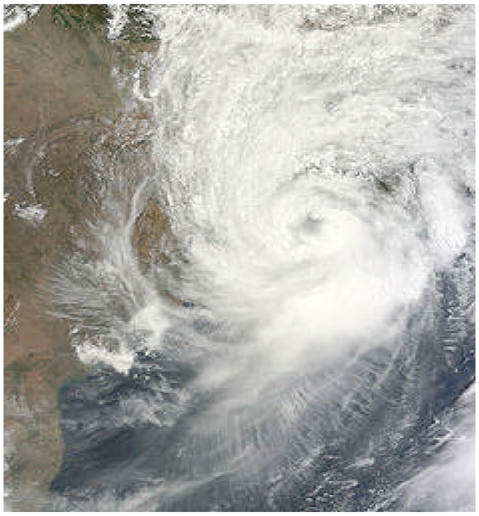

Fig. 4: Severe Cyclonic Storm Aila at peak intensity

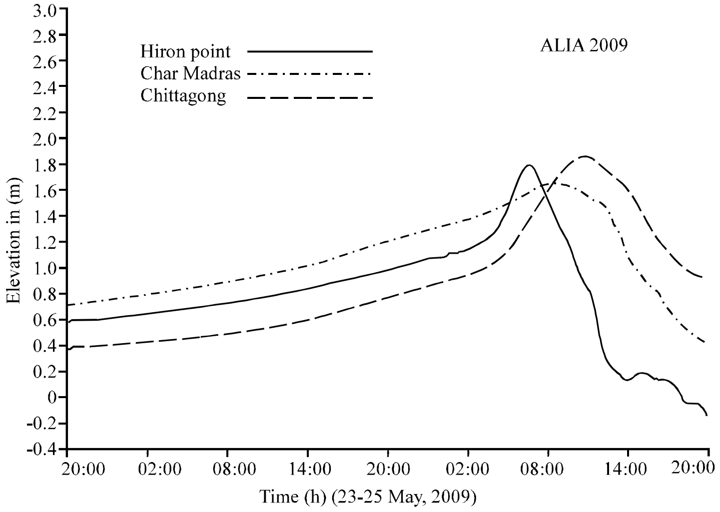

Fig. 5: Computed water level (w.r.t. MSL) due to surge at different locations associated with Cyclonic storm Aila 2009

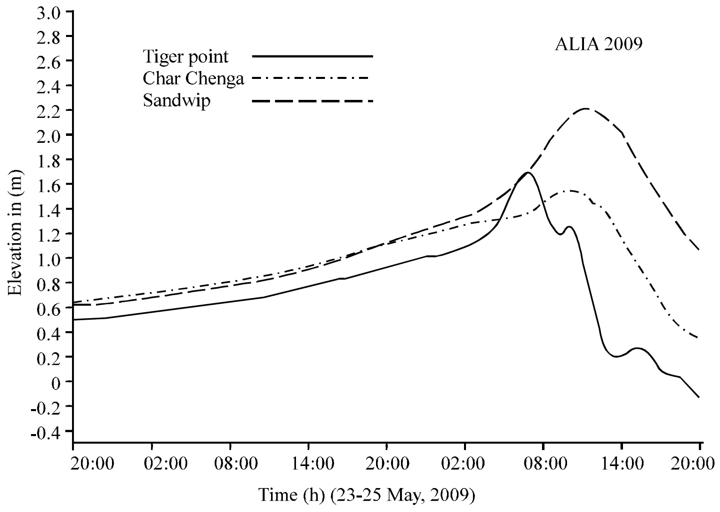

Fig. 6: Computed water level (w.r.t. MSL) due to surge at different locations associated with Cyclonic storm Aila 2009

surge level is increasing with time as the storm approaches towards the coast and finally there is recession. At Hiron Point a strong recession is occurred after 07 h of 25th May, earlier than in any other locations (Fig. 5). The recession takes place due to backwash of water from the shore towards the sea. The recession reaches up to -0.02 m at 2000 hrs of 25th May. It may be noticed that the recession at Kuakata, Char Madras, Char Chenga, Char Jabbar, Companigonj, Sandwip and Chittagong began approximately at 0700, 0830, 1000, 1130, 1200, 1200 and 1130 h (Fig. 5-7), respectively. Thus, the beginning of recession delays as we proceed towards east as is expected. At every location, the peak surge is attaining before the land falls time of the storm. This is expected, as the circulatory wind intensity is highest along the coast when the storm reaches near the coast.

The maximum surge responses associated with AILA at ten representative locations from west to east along the coastal belt of Bangladesh are shown in Fig. 8. The maximum elevation varies between 1.40 m (at Kuakata) to 2.95 m (at Companigonj). It is observed that the

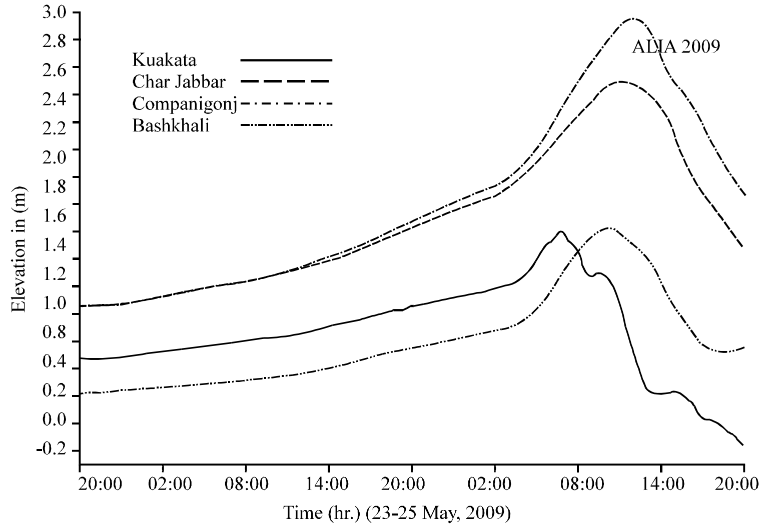

Fig. 7: Computed water level (w.r.t. MSL) due to surge at different locations associated with Cyclonic storm Aila 2009

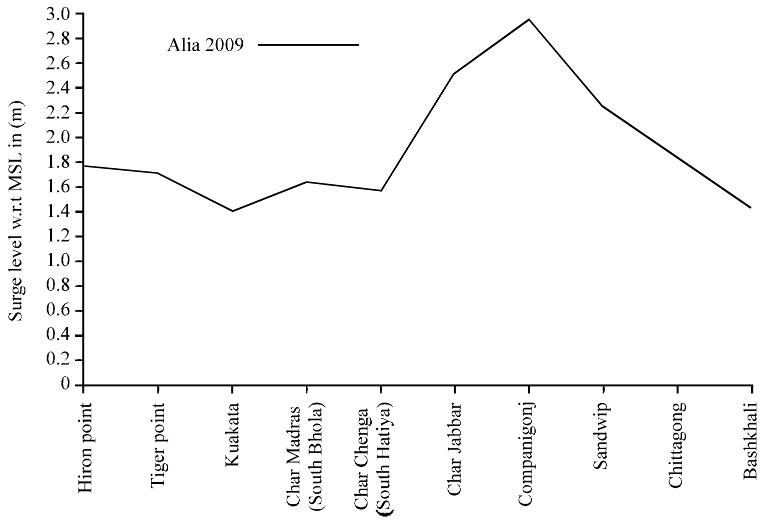

Fig. 8: Computed maximum surge levels associated with Aila 2009 along the coastal belt between Hiron Point and Bashkhali

maximum surge increases from west to east up to Companigonj and then gradually decrease reaching minimum at Bashkali.

Finally, Fig. 9 shows the contours of the recommend water level associated with AILA in the Meghna estuary region (in the VFMS region of our computed domain).

Comparison between computed and observed water level: The verification of a model is dependent on the correct observational data. But time series of observed authentic water level data are very limited. The Hydrographic Department of BIWTA (Bangladesh Inland Water Transport Authority) collects water level data at different coastal locations through manual gauge readers. But during a severe storm period it is not possible to stay in the gauge station to collect data. That is why water level data for severe storm periods are not available. However, some observed data were collected form Wikipedia website [http://en.wikipedia.org /wiki/Cyclone_Aila]. According to Wikipedia website, there was 3 m (10 ft) surge height at the western regions of Bangladesh. It is evident from the Fig. 8 that the computed surge heights are almost identical with the website data.

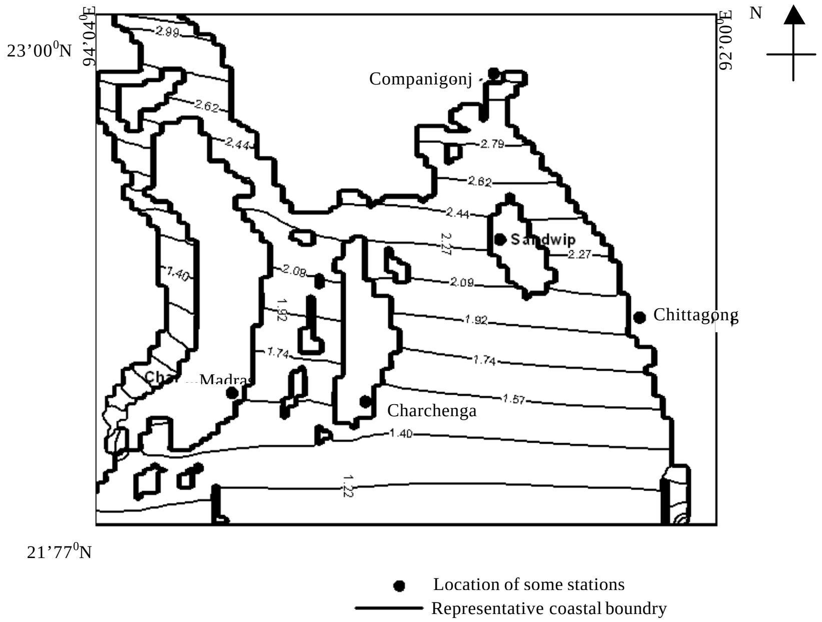

Fig. 9: Contours of the recommend water level, in the VFMS region of the computed domain associated with Aila 2009 Cyclone. The numbers on the contours refer surge height in meters

CONCLUSION

In this study we have developed an efficient numerical model to compute surge levels along the coast of Bangladesh. Use of nested numerical scheme ensures fine resolution near the coast and coarse resolution in the deep sea. The model is capable of incorporating bending of the coastline and island boundaries with a considerable accuracy. Here, we could not include fresh river discharge though lower Maghna. The study including that river discharge is in progress and will be reported in a subsequent paper.

REFERENCES

Ali, A., 1979. Storm surge in the Bay of Bengal and some related problems. Ph.D. Thesis, University of Reading, England.

Arnold, R.J., 1987. An Improved Boundary Condition for Tidal Model of Bass Strait. In: Numerical Modelling: Application to Marine Systems, Noye, J. (Ed.). Elseviers, North Holland, The Netherlands, pp: 145-158.

Bills, P. and J. Noye, 1987. An Investigation of Open Boundary Conditions for Tidal Models of Shallow Seas. In: Numerical Modelling: Application to Marine Systems, Noye, J. (Ed.). Elseviers, North Holland, The Netherlands, pp: 159-194.

Das, P.K., M.C. Sinha and V. Balasubrahmanyam, 1974. Storm surges in the Bay of Bengal. Q. J. Royal Meteorological Soc., 100: 437-449.

Debsarma, S.K., 2009. Simulations of storm surges in the Bay of Bengal. Mar. Geodesy, 32: 178-198.

Heaps, N.S., 1977. Development of storm surge models at Bidston. Institute of Oceanographic Sciences Report 53, Institute of Oceanographic Sciences, Wormley, UK., pp: 30. http://eprints.soton.ac.uk/14334/1/14334-01.pdf

Jelesnianski, C.P., 1965. A numerical calculation on of storm tides induced by a tropical storm impinging on a continental shelf. Mon. Wea. Rev., 93: 343-358.

Jhons, B., S.K. Dube, U.C. Mohanti and P.C. Sinha, 1981. Numerical simulation of the surge generated by the 1977 Andhra Cyclone. Q. J. Roy. Metrol. Soc. London, 107: 919-934.

Jhons, B., A.D. Rao, S. K. Dube and P.C. Sinha, 1985. Numerical modeling of tide surges interaction in the Bay of Bengal. Phil. Trans. R. Sco. Lond., 313: 507-535.

Rahman, M.M., A. Hoque, G.C. Paul and M.J. Alam, 2011. Nested numerical schemes to incorporate bending coastline and islands of Bangladesh and prediction of water levels due to surge. Asian J. Math. Stat., 4: 21-32.

Roy, G.D., 1999. Inclusion offshore islands in transformed coordinates shallow water model along the coast of Bangladesh. Environ. Int., 25: 67-74.

Roy, G.D. and F. Hussain, 2001. A nearly orthogonal 2D grid system in solving the shallow water equations in the head Bay of Bengal. The Abdus Salam International Centre for Theoretical Physics, Italy. http://streaming.ictp.it/preprints/P/01/162.pdf.

Tang, Y. and R. Grimshaw, 1996. Radiation boundary conditions in barotropic coastal ocean numerical models. J. Comput. Phy., 123: 96-110.

Thacker, W.C., 1977. Irregular grid finite-difference: Simulation of oscillations in shallow circular basine. J. Phys. Oceanogr., 7: 284-292.

Thacker, W.C., 1979. Irregular Grid Finite-Difference Techniques for Storm Surge Calculations for Curving Coastline. In: Marine Forecasting, Nihoul, J.C.J. (Ed.). Elsevier Science Publishers Ltd., England, pp:261-283.