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Chain Sampling Plan Using Fuzzy Probability Theory

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Abstract: The acceptance sampling plan problem is one of the most important components in statistical quality control. One of these methods is chain sampling. This paper extends the concept of chain sampling plan when the proportion defective products are a trapezoidal fuzzy number for wider application. This paper has provided the definition, calculation and draw of the Operating Characteristics (OC) curve by using the concept of fuzzy probability. It is shown that the Operating Characteristic (OC) curve of the plan is like a band having high and low bounds, their width depends on the ambiguity proportion parameter in the lot and parameter i. Finally, many examples were used and then compared the OC bands for some value of i and for Poisson and binomial distributions.

Key words: Statistical quality control, chain sampling plan, fuzzy number, fuzzy probability of acceptance, fuzzy distribution

INTRODUCTION

Statistical Quality Control (SQC) is the most popular application of statistical methods. Acceptance sampling plan is a method of measuring random samples of products applied in SQC and improvement. Acceptance sampling plan is an important part of quality management for industrial and business purposes helping decision making process. It is used to improve quality of products during production. Several sampling procedures are available in the literature of acceptance sampling for the application of attribute quality characteristics (Schiling, 1982). Chain sampling plan (Chsp) is one of the sampling methods for acceptance or rejection with classical attributes of quality characteristics.

For situation in which testing is destructive or very expensive, sampling plans with small sample sizes are usually selected. These small sample size plans often have acceptance numbers of zero. Plans with zero acceptance numbers are often undesirable. However, in that their OC curves are convex throughout. This means that the probability of lot acceptance begins to drop very rapidly as the lot proportion defective becomes greater than zero. This is often unfair to the producer and in situation where rectifying inspection is used, it can require the consumer to screen a large number of lots that are essentially of acceptable quality (Montgomery, 1991). Dodge (1955) suggested an alternate procedure, known as chain sampling that might be a substitute for ordinary single sampling plans with zero acceptance numbers in certain circumstances. This is especially desirable in a situation in which small samples are demanded because of economic or physical difficulties for obtaining a sample. The chain sampling plan is characterized by the parameters n and i, where n is the sample size and i is the number of preceding samples with zero defective. Chain sampling plan will be useful when testing is costly or destructive. Chain sampling plan allows significant reduction in sample size under conditions of a continuing succession of lot from a stable producer. Chain sampling plan, in traditional form, is based on the crispness of parameter but problems of sampling plan have both random and fuzzy nature. Fuzzy acceptance sampling procedures mentioned in the previous section have been proposed for working with imprecise parameter. Karwowski and Evans (1986) identified three key reasons why fuzzy theory is relevant to production management, that are as follows:

- Imprecision and vagueness are inherent to the decision maker's mental model
- In the production management environment, the information required formulating a model's objective, decision variables, constraints and parameters may be vague or not precisely measurable
- Imprecision and vagueness as a result of personal bias and subjective opinion may further dampen the quality and quantity of available information

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In traditional sampling plan, the proportion of defective is generally assumed to be a crisp value. However, real parameters are usually vague and the assumptions are too rigid so working by such traditional methods is inaccurate. Recently, neural networks, genetic algorithms and fuzzy logic have attracted more attention and have been successfully employed in manufacturing.

Using the OC curve, Dodge (1955) has studied the properties of the chain sampling plan. Clark (1960) has presented additional OC curves which cover most of the situations. Soundararajan (1978) has described procedures and tables for construction and selection of chain sampling plans indexed by specified parameters. In early research, Fuzzy Logic was mainly used in acceptance sampling and statistical process control (Ohta and Ichihashi, 1998; Chakraborty, 1992, 1994; Kanagawa and Ohta, 1990; Kanagawa *et al.*,1993; Wang and Chen, 1995; Tamaki *et al.*, 1991; Kanagawa and Ohta, 1990; Grzegorzewski, 1998, 2001; Grzegorzewski, 2001, 2002) also considered sampling plans by variables with fuzzy requirements. Sampling plans by attributes for vague data were considered by Hryniewisz (1992, 1994). Hryniewisz. (2008) provided a short overview of basic problems of statistical quality control that have been solved by using of the probability theory and the fuzzy sets theory. Statistical Quality control considered by Al-Nasser and Al-Rawwash. (2007), Mahabubuzzaman *et al.* (2002), (Subramaniam and Arumugam (2006) Srinivasa Rao. (2011). Finally the properties of sampling plan under situations involving both impreciseness and randomness by using the theory of chance was studied by Sundaram (2009). Jamkhaneh *et al.* (2011a and b) considered acceptance sampling plan under the conditions of the fuzzy parameter. They showed that the OC curve of the plan for every alpha-cut are like a band having high and low bounds whose width depends on the ambiguity proportion parameter. The aim of this paper is to generalize the classical framework of chain sampling plan by attributes to chain sampling plan based on fuzzy quality. In the acceptance sampling plan, it is well known that the probability distribution plays a crucial role. Here, the fuzzy probability theory attributed to Buckley (2003) is used to explore the possibility of introducing a suitable sampling plan for a situation having impreciseness and the operating characteristic of a chain sampling plan calculating have been by using the concept of fuzzy probability. According to Buckley's definition, the number of defective items in the sample with fuzzy parameter $\tilde{p}$ has a fuzzy binomial probability mass function.

DEFINITIONS

Let $X = \{\chi_1,..., \chi_n\}$ be a finite set and **P** be a probability function defined on all subsets of X with

$$P(\{xi\}) = K_i, 1 \le i \le n, 0 < K_i < 1$$

and:

$$\sum_{i=1}^{n} K_i = 1$$

X together with **P** is a discrete (finite) probability function. If B be a subset of X, we have:

$$P(B) = \sum_{x_i \in B} P(\{x_i\})$$

In practice all the K_i 's values must be known exactly. Many times these values are estimated, or they are provided by experts. We now assume that the K_i 's values are uncertain and we will model this uncertainty using fuzzy set theory (Buckley, 2003).

Definition 1: (Dubis and Prade, 1978): The fuzzy subset $\tilde{A}$ of real line R, with the membership function $\mu_{\tilde{A}} : R \to [0,1]$ is a fuzzy number if and only if (a) $\tilde{A}$ is normal, (b) $\tilde{A}$ is convex (c) $\mu_{\tilde{A}}$ is upper semi continuous and (d) supp ($\tilde{A}$) is bounded.

Definition 2: (Dubis and Prade, 1978): The α-cut of a fuzzy number $\tilde{A}$ is a non-fuzzy set defined as $\tilde{A}[\alpha] = \{x \in \mathbb{R}; \mu_{\tilde{A}}(x) \ge \alpha\}$. Hence, we have $\tilde{A}[\alpha] = [\tilde{A}^L[\alpha], \tilde{A}^U[\alpha]]$ where $\tilde{A}^L[\alpha] = \inf\{x \in \mathbb{R}, \mu_{\tilde{A}}(x) \ge \alpha\}$, $\tilde{A}^U[\alpha] = \sup\{x \in \mathbb{R}, \mu_{\tilde{A}}(x) \ge \alpha\}$.

Definition 3: (Dubis and Prade, 1978; Hatami-Marbini *et al.*, 2009): A trapezoida fuzzy number is a fuzzy number that its membership function defined by four values, $a_1 \le a_2 \le a_3 \le a_4$ where the base of the trapezoid is the interval $[a_1,a_4]$ and its top (where the membership equals one) is over $[a_2,a_3]$ such that we can describe a membership function in the following manner:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x < a_2 \\ 1, & a_2 \le x \le a_3 \\ \dfrac{a_4 - x}{a_4 - a_3}, & a_3 < x \le a_4 \end{cases} \quad (1)$$

Trapezoidal fuzzy numbers with $a_2 = a_3$ are called triangular fuzzy numbers.

Definition 4: (Buckley, 2003): Let $\tilde{k}_i$'s, i=1,...,n are fuzzy numbers and probability of event X = x is uncertain, then random variable X together with the $\tilde{k}_i$ value is a discrete fuzzy probability function. We write $\tilde{P}$ for fuzzy P and $\tilde{P}(\{x_i\})=\tilde{k}_i$, where the reference of $\tilde{k}_i$ is [0,1]. Let B = {x_1, ..., x_l}be a subset of X and then define:

$$\tilde{P}(B)[\alpha]=\{\sum_{i=1}^{l}k_i \mid S\}, 0\leq\alpha\leq 1 \quad (2)$$

where, stands for the statement:

$$K_i\in\tilde{K}_i[\alpha], 1\leq i\leq n, \sum_{i=1}^{n}k_i=1$$

this is our restricted fuzzy arithmetic.

Definition 5: (Buckley, 2003): Let p be the probability of a "success" in each Bernoulli trial, is not known precisely. We substitute $\tilde{p}$ instead of P and $\tilde{q}$ for q, so that there is a $p\in\tilde{p}[1]$ and a $q\in\tilde{q}[1]$ with p + q = 1. Now let $\tilde{P}(r)$ be the fuzzy probability of r successes in m independent trials of the experiment. Under our restricted fuzzy algebra the fuzzy binomial probability mass function is defined as:

$$\tilde{P}(r)[\alpha]=\{C_m^r p^r q^{m-r} \mid S\}, 0\leq\alpha\leq 1, \quad (3)$$

where, now S is the statement, "$p\in\tilde{p}[\alpha], q\in\tilde{q}[\alpha], p+q=1$".
if $\tilde{P}(r)[\alpha]=[P^L[\alpha], P^U[\alpha]]$ then:

$$P^L[\alpha]=\min\{C_m^r p^r q^{m-r} \mid S\}, P^U[\alpha]=\max\{C_m^r p^r q^{m-r} \mid S\} \quad (4)$$

And if $\tilde{P}[a,b]$ be the fuzzy probability of χ successes so that a ≤ χ ≥ b, then:

$$\tilde{P}([a,b])[\alpha]=\{\sum_{x=a}^{b} C_m^x p^x q^{m-x} \mid S\}$$

$$\text{if } \tilde{P}([a,b])[\alpha]=[P^L([a,b])[\alpha], P^U([a,b])[\alpha]]$$

then:

$$P^L([a,b])[\alpha] = \min\left\{\sum_{x=a}^{b} C_m^x p^x q^{m-x} \mid S\right\}$$

$$P^U([a,b])[\alpha] = \max\left\{\sum_{x=a}^{b} C_m^x p^x q^{m-x} \mid S\right\}$$

where, S is the same with past case.

Definition 6: (Buckley, 2006): Let χ be a random variable having the Poisson mass function. If P (χ) stands for the probability that X = χ, then:

$$P(x)=\frac{e^{-\lambda}\lambda^x}{x!}$$

for χ = 0,1,2,... and parameter $\lambda > 0$.

Now substitute fuzzy number for λ to produce the fuzzy Poisson probability mass function. Let to be the fuzzy probability that X = χ. Then we find α-cut of this fuzzy number as:

$$\tilde{P}(x)[\alpha]=\left\{\frac{e^{-\lambda}\lambda^x}{x!} \middle| \lambda\in\tilde{\lambda}[\alpha]\right\}, 0\leq\alpha\leq 1$$

Let X be a random variable having the fuzzy binomial distribution and $\tilde{p}$ in the definition 5 be small which means that all $p\in\tilde{p}[0]$ are sufficiently small. Let $\tilde{P}[a,b][\alpha]$ be the fuzzy probability that a≤X≤b. Also set $\tilde{P}[a,b][\alpha]$ using the fuzzy Poisson approximation. Then:

$$\tilde{P}[a,b][\alpha]\approx\left\{\sum_{x=a}^{b}\frac{e^{-\lambda}\lambda^x}{x!} \middle| \lambda\in n\tilde{p}[\alpha]\right\}, 0\leq\alpha\leq 1 \quad (5)$$

Example 1: Suppose that m = 4, $\tilde{P}=(0.1, 0.18, 0.22, 0.3)$ ("about 0.18-0.22") for P and $\tilde{q}=(0.7, 0.78, 0.82, 0.9)$ for q. Now we will calculate the fuzzy probabilities $\tilde{P}(4)$ and $\tilde{P}(B)$, where B = {0,1}. By using the equations of Definition 5, we have:

$$\tilde{P}(4)[\alpha]=\{p^4 | s\}=[P^L[\alpha], P^U[\alpha]]$$

where:

$$P^L[\alpha]=\min\{p^4 | S\}, P^U[\alpha]=\max\{p^4 | S\}$$

Since:

$$\frac{\partial(p^4)}{\partial p}>0$$

on:

$$\tilde{p}[\alpha]=[0.1+0.08\alpha, 0.3-0.08\alpha]$$

We obtain:

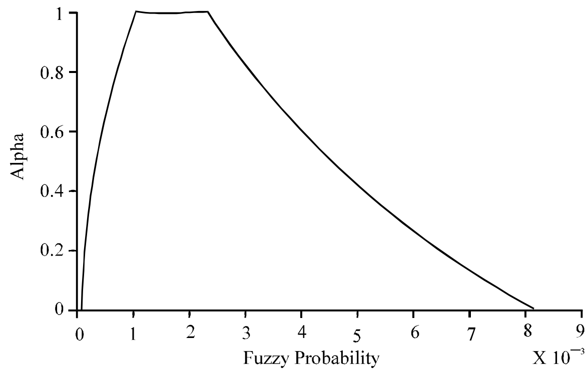

Fig. 1: The fuzzy probabilities $\tilde{P}(4)$

$$\tilde{P}(4)[\alpha]=[(0.1+0.08\alpha)^4,(0.3-0.08\alpha)^4]$$

And then:

$$\tilde{P}(4)[0]=[0.0001,0.0081]$$

$$\tilde{P}(4)[1]=[0.001,0.0023]$$

Figure 1 shows the membership function of fuzzy probabilities $\tilde{P}(4)$ in Example 1 by using fuzzy binomial distribution. We have also:

$$\tilde{P}(B)[\alpha]=[P^L[\alpha],P^U[\alpha]]$$

where:

$$P^L[\alpha]=\min\{(1-p)^4+4p(1-p)^3 \mid S\},$$
$$P^U[\alpha]=\max\{(1-p)^4+4p(1-p)^3 \mid S\}.$$

Since:

$$\frac{\partial((1-p)^4+4p(1-p)^3)}{\partial p}<0$$

on $\tilde{p}[\alpha]$ we obtain:

$$\tilde{P}(B)[\alpha]=[(0.7+0.08\alpha)^4+4(0.3-0.08\alpha)(0.7+0.08\alpha)^3,$$
$$(0.9-0.08\alpha)^4+4(0.1-0.08\alpha)(0.9-0.08\alpha)^3]$$

when α = 0, we obtain $\tilde{P}[B][0]$ = [0.6517,0.9477].

Suppose that m = 40 in Example 1 then by using fuzzy Poisson distribution, we have:

$$\tilde{P}(0)[\alpha]=[P^L[\alpha],P^U[\alpha]]=\{e^{-40p} \mid S\}$$

where:

$$P^L[\alpha]=\min\{e^{-40p} \mid S\},P^U[\alpha]=\max\{e^{-40p} \mid S\},$$

since, $\frac{\partial e^{-40p}}{\partial p}<0$ on $\tilde{p}[\alpha]$, we obtain:

$$\tilde{P}(0)[\alpha]=[e^{-40(0.3-0.8\alpha)},e^{-40(0.1+0.8\alpha)}]$$

Chain sampling plan with fuzzy parameter: In this section, first we introduce the chain sampling plan for classical attributes characteristics. Suppose that we want to inspect a lot with a size of N. First, we choose and inspect a random sample of size n and then the number of defective items (D) will be counted. The procedure of Chsp-1 is as follows:

- If the number of observed defective items (d) is equal zero, then the lot will be accepted
- If the number of observed defective items is equal two or more, then the lot will be rejecting
- If the number of observed defective items is equal one, accept the lot provided there have been no defective items in the previous i lots. In practice, values of i are usually between three and five

If the size of the lot is very large, the random variable D has a binomial distribution with parameters n and p, in which p indicates the proportion of the defective items in the lot. So, the probability for the number of defective items to be exactly equal to d is:

$$P(d,n)=C^d_n\, p^d(1-p)^{n-d} \tag{6}$$

and the probability of acceptance of the lot is:

$$Pa(i)=P(0,n)+P(1,n)P(0,n)^i, i=1,2,3,4,5 \tag{7}$$

If p be small and n be great, then the random variable D has a Poisson distribution with parameter λ = np:

$$P(d,n)=\frac{e^{-\lambda}\lambda^d}{d!}$$

Suppose that we want to inspect a lot of size of N, where the proportion of defective items or the probability of defectiveness is not known precisely and which has some uncertain value. So we represent this parameter with a trapezoidal fuzzy number $\tilde{P}$ as follows:

$$\tilde{p}=(a_1,a_2,a_3,a_4),$$

$$\tilde{P}[\alpha]=[a_1+(a_2-a_1)\alpha, a_4-(a_4-a_3)\alpha]. \tag{8}$$

A chain sampling plan with a fuzzy parameter is defined by the sample size n and i that i is the number of preceding samples with zero defective. If conditions first and third is established then the lot will be accepted. If N is a large number, then the number of defective items in this sample has a fuzzy binomial probability distribution (Buckley, 2006). So the fuzzy probability acceptance is:

$$\begin{aligned}\tilde{p}_a(i)[\alpha] &= \{P(0,\, n) + P(1,n)P(0,n)^i \,|\, S\} \\ &= [P_a^L[\alpha], P_a^U[\alpha]]\end{aligned} \tag{9}$$

$$\begin{aligned}P_a^L[\alpha] &= \min\{P(0,\, n) + P(1,n)P(0,n)^i \,|\, S\}, \\ P_a^L[\alpha] &= \max\{P(0,\, n) + P(1,n)P(0,n)^i \,|\, S\},\end{aligned} \tag{10}$$

for $0 \le \alpha \le 1$, where, S stands for the statement:

$$\begin{aligned}&p \in \tilde{p}[\alpha], q \in \tilde{q}[\alpha], p + q = 1. \\ &\tilde{p}_a(i)[\alpha] = \\ &\{(1-p)^n + np(1-p)^{n(i+1)-1} \,|\, p \in \tilde{p}[\alpha]\}\end{aligned} \tag{11}$$

and if $\tilde{p}$ be small, then the random variable D has a fuzzy Poisson distribution with fuzzy parameter $\tilde{\lambda} = n\tilde{p}$ (Buckley, 2006). Then the probability of acceptance of the lot is:

$$\tilde{p}_a(i)[\alpha] = \{e^{-np} + npe^{-np(i+1)} \,|\, p \in \tilde{p}[\alpha]\} \tag{12}$$

Example 2: A Company is packaging its products in boxes. In each box there are twenty products. The management of this firm believes that approximately between 1% and 2% of the products have problems packaging. A customer who wants to buy one of these lots of boxes, must choose one box randomly and then investigates, if all the products of that box are good then he will buy the lot or if the number of observed defective items is equal one, accept the lot provided there have been no defective items in the previous 3 lots, otherwise this lot will be rejected. Because the proportion of defective products was explained linguistically, we can consider that as a fuzzy number $\tilde{p}$ = (0,0.01,0.02,0.03) and accordingly we will have $\tilde{q}$ = (0.97,0.98,0.99,1). Then the α-cut of the fuzzy probability of lot acceptance is:

$$\begin{aligned}&\tilde{p}[\alpha] = [0.01\alpha, 0.03 - 0.01\alpha], \\ &\tilde{p}_a(3)[\alpha] = \{(1-p)^{20} + 20p(1-p)^{79} \,|\, S\}, \\ &\qquad = [P_a^L[\alpha], P_a^U[\alpha]], \\ &P_a^L[\alpha] = (0.97 + 0.01\alpha)^{20} + 20(0.03 - 0.01\alpha)(0.97 + 0.01\alpha)^{79}, \\ &P_a^U[\alpha] = (1 - 0.01\alpha)^{20} + 0.2\alpha(1 - 0.01\alpha)^{79},\end{aligned}$$

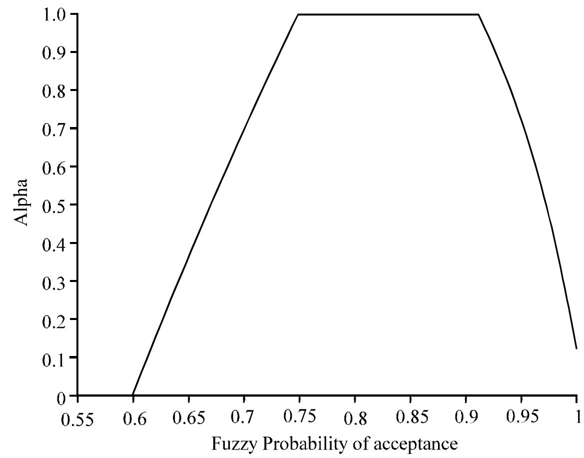

Fig. 2: The fuzzy probability of lot acceptance for a Chsp with fuzzy parameter $\tilde{p} = (0, 0.01, 0.02, 0.03)$

Under α = 0 we obtain $\tilde{p}_a(3)[0]$ = [0.5979,1], i.e., it is expected that for every 100 lots in such a process, 60 to 100 lots will be accepted. And under α = 1 we obtain $\tilde{p}_a(3)[1]$ = [0.7487,0.9083]. Figure 2 shows the membership function of fuzzy probability of lot acceptance in Example 2 by using fuzzy binomial distribution. By using fuzzy Poisson distribution the α-cut of the fuzzy probability of lot acceptance is:

$$\begin{aligned}\tilde{p}_a(3)[\alpha] &= \{e^{-20p} + 20pe^{-80p} \,|\, S\} \\ &= [P_a^L[\alpha], P_a^U[\alpha]]\end{aligned}$$

$$\begin{aligned}&P_a^L[\alpha] = e^{-(0.6-0.2\alpha)} + (0.60 - 0.2\alpha)e^{-(2.4-0.8\alpha)}, \\ &P_a^U[\alpha] = e^{-0.2\alpha} + 0.2\alpha e^{-0.8\alpha}\end{aligned}$$

under α = 0 we obtain $\tilde{p}_a$ (3)[0] = [0.6032, 1], i.e., it is expected that for every 100 lots in such a process, 60 to 100 lots will be accepted. And under α = 1 we obtain $\tilde{p}_a$ (3)[0] = [0.7511, 0.9085].

Fuzzy operating characteristic (FOC) band: Operating characteristic curve plots the probability of accepting the lot (Y-axis) versus the lot fraction defectives (X-axis) under the specified sampling plan considered during management of a project. The OC curve is the primary tool for displaying and investigating the properties of a sampling plan. Other applications of the OC curve are:

- Operating characteristic curve describes how well an acceptance plan discriminates between good and bad lots
- Operating characteristic curve aids in the selection of plans that are effective in reducing risks
- The critical points, producer's risk and customer's risk are determined by the OC curve

Suppose that the event B is the event of lot acceptance. Then the fuzzy probability of lot acceptance in terms of fuzzy proportion of defective items would be a band with upper and lower bounds. Hence, we call it Fuzzy Operating Characteristic (FOC) band. The uncertainty value of a proportion parameter is one of the factors that the bandwidth depends on. The less uncertainty value results in less bandwidth and if the proportion parameter gets a crisp value, the lower and upper bounds will become equal and the OC curve will be in classical state. Knowing the uncertainty value of the proportion parameter (a_1, a_2, a_3, a_4) and the variation of its position on the horizontal axis, we have a different fuzzy number ($\tilde{p}$) on which the FOC band is plotted in terms of it. To achieve this aim we consider the structure of $\tilde{p}$ as follows:

$$\begin{aligned}&\tilde{p}_k=(k, b_2+k, b_3+k, b_4+k),\\&p_k\in\tilde{p}_k[\alpha], q_k\in\tilde{q}_k[\alpha],\quad p_k+q_k=1\end{aligned}\tag{13}$$

where, $b_i = a_i - a_1$, i = 2, 3, 4 and $k\in[0,1-b_4]$. The α-cut of FOC band is plotted according to the values of the following fuzzy probability:

$$\tilde{p}_k[\alpha]=\left[P_k^L[\alpha], P_k^U[\alpha]\right]=[k+b_2\alpha, b_4+k-(b_4-b_2)\alpha]\tag{14}$$

$$\tilde{p}_{a,k}(i)=\tilde{p}_k(B)[\alpha]=\left[P_k^L[\alpha], P_k^U[\alpha]\right]\tag{15}$$

$$\begin{aligned}&P_k^L[\alpha]=\min\{P(0,n)+P(1,n)P(0,n)^i \mid p\in\tilde{p}_k[\alpha]\},\\&P_k^U[\alpha]=\max\{P(0,n)+P(1,n)P(0,n)^i \mid p\in\tilde{p}_k[\alpha]\}\end{aligned}\tag{16}$$

Example 3: In Company related to Example 2, we have assumed that $\tilde{p}=(0,005,0.01,0.15,0.02)$ then:

$$0\le k\le 0.985+0.005\alpha, 0\le\alpha,\le 1,$$

$$P_k^L[\alpha]=\min\{P(0,n)+P(1,n)P(0,n)^i \mid p\in\tilde{p}_k[\alpha]\},$$

$$P_k^U[\alpha]=$$
$$\max\{P(0,n)+P(1,n)P(0,n)^i \mid p\in\tilde{p}_k[\alpha]$$

and:

$$\tilde{p}_{a(b),k}(i)[\alpha]=\left[P_k^L(i)[\alpha], P_k^U(i)[\alpha]\right],$$

$$P_k^L(i)[\alpha]=(0.985-k+0.005\alpha)^n+n(0.015+$$
$$k-0.005\alpha)(0.985-k-0.005\alpha)^{n(i+1)-1},$$

$$P_k^U(i)[\alpha]=(1-k-0.005\alpha)^n+n(k+0.005\alpha)(1-k-0.005\alpha)^{n(i+1)-1}$$

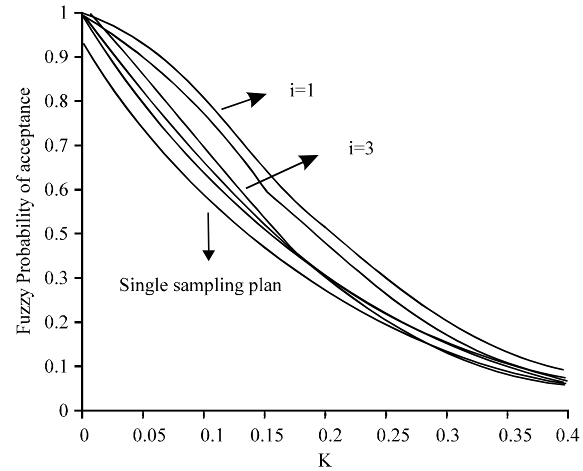

Fig. 3: FOC band for a chain sampling plan with fuzzy parameter

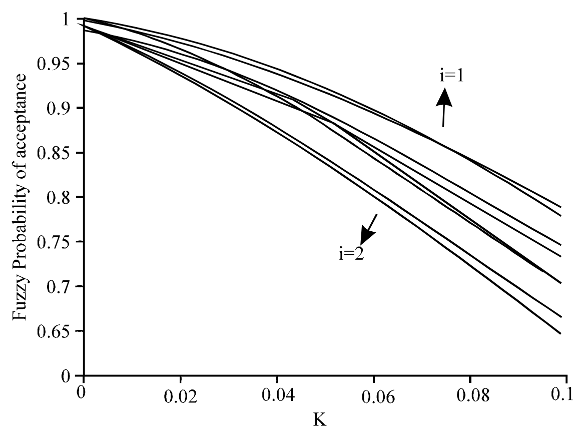

Fig. 4: FOC band for a chain sampling plan with fuzzy binomial and Poisson distribution

By using fuzzy Poisson distribution the α-cut of the fuzzy probability of lot acceptance is:

$$\tilde{p}_{a(p),k}(i)[\alpha]=\left[P_k^L(i)[\alpha], P_k^U(i)[\alpha]\right],$$

$$P_k^L(i)[\alpha]=e^{-n(0.015+k-0.005a)}+n(0.015+k-0.005\alpha)e^{-n(0.015+k-0.005\alpha)(i+1)},$$

$$P_k^U(i)[\alpha]=e^{-n(k+0.005\alpha)}+n(k+0.005\alpha)e^{-n(k+0.005\alpha)(i+1)}$$

Figure 3 compared α-cut of FOC bands of fuzzy binomial distribution for Chsp-1 and SSP. Table 1 shows α-cut (α = 0) of the fuzzy probability of lot acceptance for some value of i and for Poisson and binomial distributions.

Figure 4 and 5 compared α-cut of FOC bands of fuzzy binomial distribution and fuzzy Poisson distribution for different i. Table 2 shows α-cut (α = 0) of the fuzzy probability of lot acceptance for some value of $\tilde{p}_k$ and for Poisson and binomial distributions.

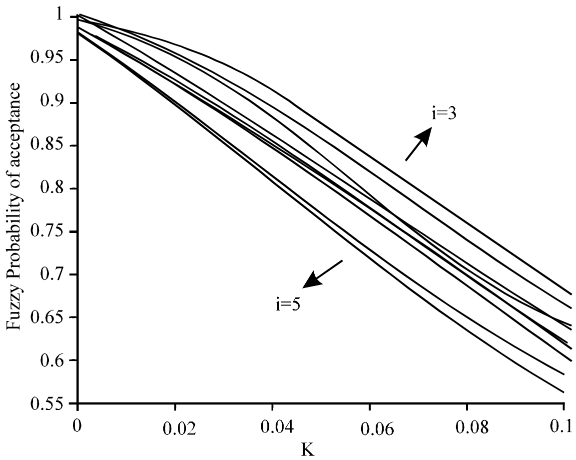

Fig. 5: FOC band for a chain sampling plan with fuzzy binomial and Poisson distribution

Table 1: Fuzzy probability of acceptance with k = 0.005, n = 5 and $\alpha = 0$

i	$\tilde{p}_{a(b),k}(i)[\alpha]$	$\tilde{p}_{a(p),k}(i)[\alpha]$
1	[0.9873,0.9991]	[0.9867,0.9991]
2	[0.9793,0.9985]	[0.9789,0.9985]
3	[0.9720,0.998]	[0.9719,0.9979]
4	[0.9655,0.9974]	[0.9655,0.9974]
5	[0.9596,0.9969]	[0.9597,0.9968]

Table 2: Fuzzy probability of acceptance with I = 3, n = 5 and $\alpha = 0$

$\tilde{p}_k[o]$	$\tilde{p}_{a(b),k}(3)[0]$	$\tilde{p}_{a(p),k}(3)[0]$
[0,0.015]	[0.9835,1]	[0.9833,1]
[0.01,0.025]	[0.9584,0.9923]	[0.9583,0.9922]
[0.02,0.035]	[0.9258,0.9720]	[0.9264,0.9719]
[0.03,0.045]	[0.8882,0.9428]	[0.89,0.9430]
[0.04,0.055]	[0.8475,0.9075]	[0.8511,0.9086]
[0.05,0.065]	[0.8052,0.8681]	[0.8111,0.8708]
[0.06,0.075]	[0.7624,0.8265]	[0.771,0.8312]

Table 1 and 2 shows that OC band with using from fuzzy Poisson distribution optimal approximate for OC band with using from fuzzy binomial distribution for the proportion of defective items with small fuzzy numbers. With regard this; such plan can be designed based on OC fuzzy Poisson distribution.

However, with increasing $\tilde{p}$ this approximation is weaker and it will be more with the increase of i.

Rectifying chain sampling inspection plan: Rectifying inspection serve to improve lot quality. Under rectifying sampling inspection plan whenever we accept a lot we replace all the defective items encountered in the sample by good items. Whereas rejected lot are sent for 100% inspection and all the defectives encountered are replaced by good items.

Fuzzy average outgoing quality: The random variable, Outgoing Quality (OQ), is defined as the ratio of the number of nonconforming items in the output of this process after sampling inspection and rectification to the total number of items in the lot. The fuzzy probability distribution of OQ is then:

$$\tilde{p}\left(OQ=\frac{j}{N}\right)[\alpha]=\{a_j(i)|\, p\in\tilde{p}[\alpha]\} \tag{17}$$

$$a_j(i)=\begin{cases}(1-p_a(i))+p_a(i)(1-p)^{N-n}, j=0\\ p_a(i)C^j_{N-n}\,p^j(1-p)^{N-n-j},\ j=1,2,\ldots.(N-n),\end{cases} \tag{18}$$

where:

$$p_a(i)=P(0,n)+P(1,n)P(0,n)^i \tag{19}$$

The fuzzy mathematical expectation of OQ is obtained from Eq. 17 as:

$$E(OQ)=\left\{\frac{(N-n)pp_a(i)}{N}|p\in\tilde{p}[\alpha]\right\} \tag{20}$$

The fuzzy mathematical expectation of OQ is named fuzzy average outgoing quality (FAOQ).

Example 4: Suppose that the size of lot be equal to 100 and n = 5, i = 3, $\tilde{p}$ = (0, 0.01, 0.02, 0.03), then fuzzy average outgoing quality is as follows:

$$\tilde{p}[\alpha]=[0.01\alpha,0.03-0.01\alpha],$$
$$\tilde{p}_a(3)[\alpha]=\{(1-p)^5+5p(1-p)^{19}\,|\,S\},$$
$$FAOQ=\{0.95p((1-p)^5+5p(1-p)^{19}))\,|p\in\tilde{p}[\alpha]\}$$

under $\alpha = 1$ we obtain:

$$FAOQ[1]=[0.0094,0.0185]$$

Thus, rectifying chain sampling inspection plan changes the quality of the lots in percent defective from "about 1 to 2 percent" to "about 0.94 to 1.85 percent" on the average. Figure 6 shows the membership function of FAOQ in Example 4 by using fuzzy binomial distribution.

Fuzzy average total inspection: The random variable, Total Inspection (TI), is defined as the number of inspection items in the output of this process after sampling inspection and rectification in the lot (Fig. 7). The fuzzy probability distribution of TI is then:

$$\tilde{p}(OQ=j)[a]=\{b_j(i)|p\in\tilde{p}[a]\} \tag{21}$$

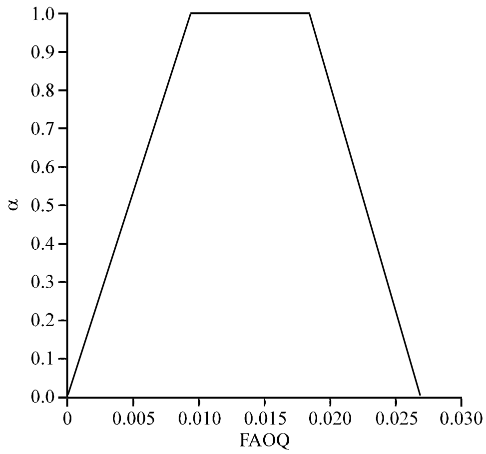

Fig. 6: Fuzzy average outgoing quality with N=100, n=5, i=3, $\tilde{p}$ = (0 ,0.01, 0.02, 0.03)

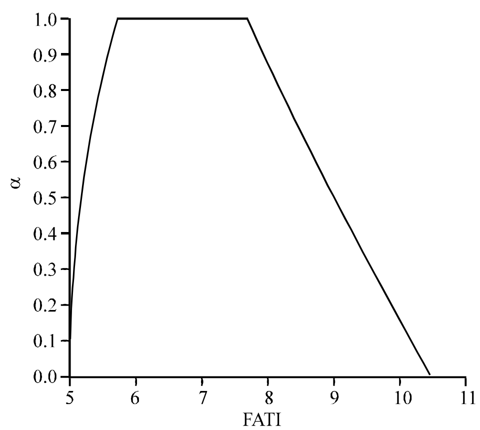

Fig. 7: Fuzzy average total inspection with N=200, n=5, i=3, $\tilde{p}$ = (0, 0.01, 0.02, 0.03)

$$b_j(i)=\begin{cases} p_a(i), & j=n \\ (1-p_a(i)), & j=N \end{cases} \tag{22}$$

The fuzzy mathematical expectation of TI is obtained from Eq. 22 as:

$$E(OQ)=\{n+(N-n)(1-p_a(i)) | p \in \tilde{p}[a]\} \tag{23}$$

The fuzzy mathematical expectation of TI is named fuzzy average total inspection (FATI).

In Example 4, fuzzy average total inspection is as follows:

$$FATI=\{5+95(1-p)^5-5p(1-p)^{19})| p\in \tilde{p}[\alpha]\}$$

under α = 0 we obtain FATI [0] = [5, 10.43] and FATI [1] = [5.7316, 7.6558].

CONCLUSION

We proposed a chain acceptance sampling plan based on a fuzzy parameter by using fuzzy probability theory. We modeled this parameter using trapezoidal fuzzy number. The main result is that this plan yields a fuzzy environment consistent with classical views thereof. We calculated the operating characteristic curve of a chain sampling plan and the acceptance probability by using the concept of fuzzy probability. Finally, we have shown that in our plan, the α-cut of FOC band is like a band having high and low bounds.

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