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An Interpretation of the Cost Model in Data Envelopment Analysis

G.R. Jahanshahloo, S.M. Mirdehghan and J. Vakili
Faculty of Mathematical and Computer Sciences,
Tarbiat Moallem University, Tehran, Iran

Abstract: Cost efficiency evaluates the ability of a decision making unit to produce the current outputs at minimal cost, given its input prices. In this study, we present an interpretation of the cost models and also we propose a new model to evaluate the cost efficiency measure of the decision making units when all input prices of each DMU are available. This model is more desirable than the traditional cost model in terms of complexity and the numbers of constraints and variables are reduced in the new proposed model. Moreover, the proposed model is compared with the standard DEA model to determine their relations. By this comparison an interesting relation between the cost efficiency and the technical efficiency is found.

Key words: Data envelopment analysis, cost model, cost efficiency, envelopment form, multiplier form

INTRODUCTION

Performance evaluation is an important task for a Decision Making Unit (DMU) to find its weaknesses so that subsequent improvements can be made. Since, the pioneering work of Charnes *et al.* (1978), Data Envelopment Analysis (DEA) has demonstrated to be an effective technique for measuring the relative efficiency of a set of DMUs which utilize the same types of inputs to produce the same types of outputs. There are different types of efficiency. One of them is Cost Efficiency (CE). Cost efficiency evaluates the ability of a DMU to produce the current outputs at minimal cost, given its input prices. The concept of CE can be traced back to Farrell (1957), who originated many of the ideas underlying efficiency assessment. The CE can be interpreted as a measure of the potential cost reduction achievable given the outputs produced and the current input prices at each DMU. Farrell s concept was further developed by Fare *et al.* (1985), who formulated a Linear Programming (LP) model for CE assessment. This LP model requires input and output quantity data as well as input prices at each DMU. Camanho and Dyson (2005) presented a weight-restricted DEA model for measuring the cost efficiency of DMUs and Jahanshahloo *et al.* (2008) simplified this version of the cost efficiency model.

In this study, we propose a model to evaluate the cost efficiency measure which is more desirable than the traditional cost model formulated by Fare *et al.* (1985) in terms of complexity and we compare the presented model with the CCR models (multiplier and envelopment forms).

DISCUSSION

Suppose that we have n DMUs with activity vectors (x^t_j, y^t_j); j = 1, ..., n, where x_j and y_j are nonnegative and nonzero column vectors in $\Re^m$ and $\Re^s$, respectively. All DMUs, DMU_j (j = 1,. . . , n), use the same number, m, of inputs (x_{ij} (i = 1,..., m)) to produce the same number, s, of outputs (y_{rj} (r = 1, ..., s)). Note that the input and output vectors of all DMUs are the same in type but different in quantity.

The CCR model, which was suggested by Charnes *et al.* (1978), is a fractional linear programming problem and can be solved by being transformed into an equivalent linear programming problem. This linear programming problem is the input-oriented multiplier CCR model and is as follows:

$$\begin{aligned} \max \quad & \sum_{r=1}^{s} u_r y_{ro} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{io} = 1 \\ & \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le 0 \quad && j = 1,\dots,n \\ & v_i \ge 0 && i = 1,\dots,m \\ & u_r \ge 0 && r = 1,\dots,s \end{aligned} \tag{1}$$

The dual of the above problem is the input-oriented envelopment CCR model and is as follows:

Corresponding Author: Seyyed Morteza Mirdehghan, Faculty of Mathematical and Computer Sciences, Tarbiat Moallem University, Tehran, Iran Tel: +98-21-7750-7772

$$\begin{aligned} \min\ & \theta \\ \text{s.t.}\ & \sum_{j=1}^{n}\lambda_j x_{ij} \le \theta x_{io} && i=1,\ldots,m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} && r=1,\ldots,s \\ & \lambda_j \ge 0 && j=1,\ldots,n \\ & \theta \text{ is unrestricted} \end{aligned} \tag{2}$$

Farrell (1957) also proposed a measure of cost efficiency, which assumes that prices are fixed and known and maybe different among DMUs. Cost efficiency evaluates the ability of a decision making unit to produce the current outputs at minimal cost, given its input prices. In order to obtain a measure of cost efficiency for DMUs with multiple inputs and outputs, the minimum cost for the production of a DMU's current outputs with existing input prices is obtained by solving the following linear problem, as first formulated by Fare *et al.* (1985):

$$\begin{aligned} \min\ & \sum_{i=1}^{m} c_{io} x_i \\ \text{s.t.}\ & \sum_{j=1}^{n}\lambda_j x_{ij} \le x_i && i=1,\ldots,m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} && r=1,\ldots,s \\ & \lambda_j \ge 0 && j=1,\ldots,n \\ & x_i \ge 0 && i=1,\ldots,m. \end{aligned} \tag{3}$$

In the above model, $c_{io}>0$ is the price of input i of DMU_o where $o \in \{1, \ldots, n\}$ is the index of the DMU under assessment. x_i is a variable that, at the optimal solution, gives the amount of input i to be employed by DMU_o in order to produce the current outputs at minimal cost, subject to the technological restrictions imposed by the existing productivity possibility set. Note that this model assumes that the input prices at each DMU are fixed and known, although they can differ between DMUs.

Cost efficiency is then obtained as the ratio of minimum cost with current prices (the optimal value of (3)) to the current cost at DMU_o, as follows:

$$CE_o = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}}$$

in which * indicates the optimality and it is clear that $CE_o \le 1$.

Because the costs of inputs are nonnegative for each DMU, there exists an optimal solution such as (λ^{*t}, x^{*t}) such that all constraints

$$\sum_{j=1}^{n}\lambda_j x_{ij} \le x_i$$

i = 1, …, m, are binding on this solution, i.e.,

$$\sum_{j=1}^{n}\lambda_j^* x_{ij} = x_i^*$$

i = 1, …, m. This means that the cost efficiency of DMU_o is obtained from the following model:

$$\begin{aligned} CE_o = \frac{1}{\sum_{i=1}^{m} c_{io} x_{io}} \min\ & \sum_{i=1}^{m} c_{io} x_i \\ \text{s.t.}\ & \sum_{j=1}^{n}\lambda_j x_{ij} = x_i && i=1,\ldots,m \\ & \sum_{j=1}^{n}\lambda_j y_{rj} \le y_{ro} && r=1,\ldots,s \\ & \lambda_j \ge 0 && j=1,\ldots,n \\ & x_i \ge 0 && i=1,\ldots,m. \end{aligned} \tag{4}$$

In modal (4) let Eq.

$$x_i = \sum_{j=1}^{n}\lambda_j x_{ij}$$

i = 1, ..., m, in the objective function of it, so modal (4) is equivalent to:

$$\begin{aligned} CE_o = \min\ & \sum_{j=1}^{n}\lambda_j \frac{\sum_{i=1}^{m} c_{io} x_{ij}}{\sum_{i=1}^{m} c_{io} x_{io}} \\ \text{s.t.}\ & \sum_{j=1}^{n}\lambda_j y_{rj} \ge y_{ro} && r=1,\ldots,s \\ & \lambda_j \ge 0 && j=1,\ldots,n \end{aligned} \tag{5}$$

The dual of the above model is:

$$\begin{aligned} \max\ & \sum_{r=1}^{s} u_r y_{ro} \\ \text{s.t.}\ & \sum_{r=1}^{s} u_r y_{rj} \le \frac{\sum_{i=1}^{m} c_{io} x_{ij}}{\sum_{i=1}^{m} c_{io} x_{io}} && j=1,\ldots,n \\ & u_r \ge 0 && r=1,\ldots,s \end{aligned} \tag{6}$$

Now we compare Model (6) with the standard DEA model (Multiplier CCR model) to determine their relations. Consider the multiplier CCR model (Model (1)). If in the multiplier CCR model we let

$$v_i = \frac{c_{io}}{\sum_{i=1}^{m} c_{io} x_{io}}$$

then this model is equivalent to Model (6). That is, in the multiplier CCR model, if variables v_i are replaced with constants

$$\frac{c_{io}}{\sum_{i=1}^{m} c_{io} x_{io}}$$

i = 1, ..., m, then the optimal value of the objective function is the cost efficiency of DMU_o.

Here, we present another version of the cost model to evaluate the cost efficiency measure.

Consider Model (5). Let

$$\theta = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m} \lambda_j c_{io} x_{ij}}{\sum_{i=1}^{m} c_{io} x_{io}}$$

Then, Model (5) is converted to the following model:

$$\begin{aligned} \min \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \sum_{i=1}^{m} c_{io} x_{ij} = \theta \sum_{i=1}^{m} c_{io} x_{io} \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1,\ldots,s \\ & \lambda_j \geq 0 \qquad j = 1,\ldots,n \\ & \theta \text{ is unrestricted} \end{aligned} \tag{7}$$

The optimal value of the above model is equal to the optimal value of the following model:

$$\begin{aligned} \min \quad & \theta \\ \text{s.t.} \quad & \sum_{j=1}^{n} \lambda_j \sum_{i=1}^{m} c_{io} x_{ij} \leq \theta \sum_{i=1}^{m} c_{io} x_{io} \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} \qquad r = 1,\ldots,s \\ & \lambda_j \geq 0 \qquad j = 1,\ldots,n \\ & \theta \text{ is unrestricted} \end{aligned} \tag{8}$$

Note that at optimality of (8), constraint

$$\sum_{j=1}^{n} \lambda_j \sum_{i=1}^{m} c_{io} x_{ij} \leq \theta \sum_{i=1}^{m} c_{io} x_{io}$$

is binding.

The above model is another version of the cost model and its optimal value is the cost efficiency of DMU_o. In this model, there are n+1 variables and r+1 constraints (without non-negativity constraints), while the standard minimal cost model has n+m variables and m+r constraints (without non-negativity constraints); that is, the proposed model has m-1 less variables and m-1 less constraints than the standard cost model formulated by Fare *et al*. (1985). Therefore, the complexity of our proposed model to evaluate the cost efficiency measure is much less than the traditional version.

Also, it is clear that the feasible region of the input-oriented CCR envelopment form (Model (2)) is a subset of the feasible region of the proposed model. Therefore, the cost efficiency of a DMU is not greater than the technical efficiency of this DMU, i.e.,

$$\text{The technical efficiency of } DMU_o \geq \text{The cost efficiency of } DMU_o$$

With attention to Models (2) and (8), we observe that the cost efficiency of a DMU can be evaluated using the input-oriented envelopment CCR model such that the summation of the first m constraints

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}$$

multiplied by c_{io} for i = 1, ..., m

$$\left(\sum_{j=1}^{n} \lambda_j \sum_{i=1}^{m} c_{io} x_{ij} \leq \theta \sum_{i=1}^{m} c_{io} x_{io} \right)$$

is used instead of the first m constraints

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}$$

i = 1, ..., m.

Furthermore, the cost efficiency of a DMU with m inputs and s outputs, as defined at the beginning of this section, is evaluated by using input-oriented envelopment CCR model. Now we define n reduced DMUs D_j with activity vectors

$$\left(\tilde{x}_j, \tilde{y}_j^t\right)$$

where

$$\tilde{x}_j = \sum_{i=1}^{m} c_{io} x_{ij}$$

and

$$\tilde{y}_j^t = y_j^t$$

j = 1, 2, ..., n, are nonnegative real numbers and vectors in in $\Re$ and $\Re^s$, respectively.

The technical efficiency of DMU D_o using Model (2) is the cost efficiency of DMU_o, i.e..

The technical efficiency of the reduced DMU D_o = The cost efficiency of DMU_o

CONCLUSIONS

There are different types of efficiency measures such as technical efficiency and cost efficiency. These measures are different in terms of efficiency score values, because each uses different information obtained from decision making units, e.g., technical efficiency only uses the quantities of inputs and outputs and cost efficiency uses the quantities of inputs, outputs and all input prices. Any efficiency measure that uses more information obtained from DMUs is more realistic than the other efficiency measures; but because not all DMUs in the real world have the same type of information, we cannot use all DEA models for certain types of information. For example, to obtain the cost efficiency measure of a DMU, we need not only all input and output quantities but also all input prices and if one of the input prices is not available, we cannot obtain the cost efficiency of DMUs, but we can obtain the technical efficiency of these DMUs. In this study, a new model has been suggested to evaluate the cost efficiency measures of DMUs. This model is more desirable than the minimal cost model to evaluate the cost efficiency measure formulated by Fare *et al*. (1985) in terms of complexity. Also the relations between the proposed model and the envelopment and multiplier CCR models have been discussed.

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