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Zitterbewegung Anyons and Deformed Position-angular Momentum Uncertainty Principle Algebra

A. Farmany

Islamic Azad University, Ilam Branch, Ilam, Iran

Abstract: In this study, existence of a generalized uncertainty principle in the anyon Zitterbewegung model is explored. It is shown that the localization of anyon with high energy probes, receives a deformed algebra for position-angular momentum Heisenberg uncertainty principle.

Key words: Uncertainty principle, Zitterbewegung model, anyon

INTRODUCTION

In the particle physics, by using high energy probes, it could be possible to measure the location of a particle precisely and minimize the localization uncertainty. Since gravity couples to the energy-momentum tensor, the high energy probe would generate the large fluctuations on the metric. Because the situation of this probe is quantum mechanically, the metric fluctuations are indeterminable components. Thus, there exist a limitation to precise the localization; this is named generalized uncertainty principle. The heuristic derivations of generalized uncertainty principle are made in the black hole physics, microphysics, quantum mechanics and other area of physics (Witten, 1996; Yoneya, 2000; Kempf and Managano, 1997; Adler *et al.*, 2001; Maggiore, 1994; Adler and Santiago, 1999; Farmany and Dehghani, 2010; Farmany, 2010; Farmany *et al.*, 2007, 2008). In this letter we obtain a generalized uncertainty principle in the Zitterbewegung model of anyons. According to Chou *et al.* (1993), constructing a model for anyons as charged particles, is not economical way and a sensible model of anyons may be constructed as a model that focuses on anyons as spinning particle. Recently, this model received a much attention (Chou *et al.*, 1993; Plyushchay, 1990a, b; Jackiw and Nair, 1991, 1994; Wilczek, 1990; Jackiw and Pi, 1990; Farmany, 2005; Ghosh, 1994; Banerjee *et al.*, 1996; Farmany, 2011). This new model of anyons, exhibits the classical analogue of the quantum phenomenon Zitterbewegung. Also the model is derived from existing spinning particle model and retains the essential features of anyon in the non relativistic regime. In this letter, the existence of a generalized uncertainty principle in the Zitterbewegung model is explored.

Let we begin with the Lagrangian of an anyon (Ghosh, 1994):

$$L = (M^2 u^\mu u_\mu + J^2 \Sigma^a \Sigma_a + 2MJ u^\mu e_{\mu a} \Sigma^a)^{\frac{1}{2}} \quad (1)$$

where, $u^a = \frac{dx^a}{dt}$, $e_{\mu a} e_{\nu b} = \eta_{ab}$ and $e_{\mu a} e_{\nu b} = g_{\mu\nu}$, $u^\mu e_{\mu a} \Sigma^a = \text{const}$ and $g_{\mu\nu}$ is the space-time metric.

The canonical momenta are obtained from the Hamiltonian analysis as:

$$P_\mu = \frac{\partial L}{\partial u^\mu} = \left(\frac{M}{J} e_{\mu a} - w_{\mu a} \right) J_a \quad (2)$$

$$\frac{\partial L}{\partial \sigma^a} = 2MJ u^\mu e_{\mu a} + 2J^2 \Sigma_a = 2LJ_a \quad (3)$$

where, four primary constraints are:

$$J^a J_a = J^2 \quad (4)$$

$$\Pi_\mu = \frac{\partial L}{\partial u^\mu} - w_{\mu a} J^a = \lambda e_{\mu a} J_a \quad (5)$$

$$V^\mu = \varepsilon^{\mu\nu\lambda} \Pi_\nu e_{\lambda a} J_a \quad (6)$$

$$C = \Pi^\mu \Pi_\mu - M^2 \quad (7)$$

Note that $\Pi_\mu V^\mu = 0$ and λ is the angle variables. We consider the second class constraint set (V^μ, χ^ν) (Ghosh, 1994). The inverse of Poisson bracket is:

$$m^{-1} = \frac{1}{M^2} \begin{pmatrix} \frac{-1}{N} [(F - \lambda T)]^{\mu\nu} & -g^{\mu\nu} \\ g^{\mu\nu} & S^{\mu\nu} \end{pmatrix} \quad (8)$$

where, $m = \{V^\mu, \chi^\nu\}$, $N = M^2 - \frac{1}{2}\epsilon^{\mu\nu\lambda}J_\mu(F - \lambda T)_{\nu\lambda}$,

$F^a_{\mu\nu} = w^a_{\nu,\mu} - w^a_{\mu,\nu} - \epsilon^{abc}w_{\mu b}w_{\nu c}$ and $T_{\mu\nu}$ is the torsion term.

In the coordinates system the generic Dirac bracket is defined by:

$$\{x^\mu, x^\nu\}^* = \{x^\mu, x^\nu\} - \left\{ \{x^\mu, V^\mu\} \{x^\nu, \chi^\nu\} \right\} m^{-1} \begin{pmatrix} \{V^\mu, x^\nu\} \\ \{\chi^\nu, x^\nu\} \end{pmatrix} \quad (9)$$

From Eq. 9 we can write (Farmany, 2005),

$$[x^\mu, x^\nu] = i\hbar\theta^{\mu\nu} \quad (10)$$

where, $i\hbar\theta^{\mu\nu} = -\frac{i}{N}S^{\mu\nu}$ and $S^{\mu\nu}$ is an anti-symmetric matrix.

According to quantum mechanics, the momentum and the position of a particle could not be measured precisely. This is due to the fact that operators \hat{p}_x and \hat{x} are not commute. In fact in the quantum mechanics, the act of measurement interferes with the system and modifies it, since there is a large uncertainty in the final velocity (or coordinate) of a particle as:

$$\Delta x^\mu \Delta p^\nu \geq i\hbar\delta^{\mu\nu} \quad (11)$$

So, an attempt to localize a particle with a minimum uncertainty in Δx , the momentum uncertainty Δp will be increase. According to Eq. 11 for an anyon to be observed by means of a photon with momentum p , the usual Heisenberg arguments leads to a position uncertainty given by Eq. 11 (Franke-Arnold *et al.*, 2004). But we should consider the effect of the non-commutativity on the localization process. Let us reconsider it. In this framework, there is a particle in a non-commutative space whose coordinates satisfies the Heisenberg algebra.

$$[x^\mu, x^\nu] = i\hbar\theta^{\mu\nu} \quad (12a)$$

$$[x^\mu, p^\nu] = i\hbar\delta^{\mu\nu} \quad (12b)$$

$$[p^\mu, p^\nu] = 0 \quad (12c)$$

From Eq. 12a-c, we can set up the Jacobi identity as:

$$[x^\mu, [x^\nu, p^\xi]] + \text{cyclic}(\mu, \nu, \xi) = 0 \quad (13)$$

As a result of Eq. 13, an important consequence of the non-commutative coordinates is that neither the position operator does satisfy the usual low:

$$[x^\mu, L^\nu] = i\hbar\epsilon^{\mu\nu\xi}x_\xi \quad (14)$$

and nor the angular momentum operator satisfy the standard Eq. 3 algebra

$$[L^\mu, L^\nu] = i\hbar\epsilon^{\mu\nu\xi}L_\xi \quad (15)$$

In fact we can write,

$$[x^\mu, L^\nu] = i\hbar\epsilon^{\mu\nu\xi}x_\xi + i\hbar\epsilon^{\nu\xi\eta}p^\eta\theta^{\mu\xi} \quad (16)$$

for position operator and

$$[L^\mu, L^\nu] = i\hbar\epsilon^{\mu\nu\xi}L_\xi^5 + i\hbar\epsilon^{\mu\nu\xi}\epsilon^{\nu\eta\zeta}p^\eta p^\zeta\theta^{\mu\xi} \quad (17)$$

for angular momentum operator, respectively. Comparing Eq. 14 with Eq. 16 we find a new term as $i\hbar\epsilon^{\nu\xi\eta}p^\eta\theta^{\mu\xi}$ in Eq. 16. From Eq. 16 we have,

$$x^\mu L^\nu = i\hbar(\epsilon^{\mu\nu\xi}x_\xi + \epsilon^{\nu\xi\eta}p^\eta\theta^{\mu\xi}) \quad (18)$$

that shows a deformed algebra of position-angular momentum uncertainty.

CONCLUSION

It is interesting that generalized uncertainty principle may be derived in the quantum field theory, string theory, black hole physics, quantum mechanics and other area of physics. In this essay, the existence of a generalized uncertainty principle in the anyon Zitterbewegung model is explored.

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