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A Novel Nonlinear System Modeling and Identification Method based on Modal Series

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Abstract: This study provides a novel modelling and identification approach to nonlinear systems. The model structure is based on modal series model of nonlinear systems. Modal series has been used for solving nonlinear differential equations. It provides an approximate solution for a nonlinear differential equation. Based on modal series, the paper proposed a model structure for nonlinear system identification. The proposed model structure introduces a linear subsystem and several Hammerstein subsystems for the main nonlinear system. Therefore, identification problem of a nonlinear system reduces to identification of a linear subsystem and several Hammerstein subsystems. Each Hammerstein subsystem contains a linear block and a nonlinear block. All Linear blocks of proposed model structure are the same and the structure of nonlinear block of each Hammerstein subsystem is distinct. The proposed model structure provides accessibility to different signals which may be used in the identification process of each block. Since modal series provides an approximate solution, in order to identify a more accurate model for the nonlinear system, one may use more calculation and more time to reach the satisfactory model. In order to identify the proposed model, an algorithm based on linear subspace identification methods and Hammerstein system identification has been proposed. Simulations are used to evaluate the results of what argued in this study.

Key words: Nonlinear system, modelling, identification, modal series

INTRODUCTION

Real-life systems almost always show nonlinear dynamical behavior. This behavior complicates the task of finding models that accurately describe these systems. While in a large number of applications a linear model shows already satisfactory results, there are numerous situations where linear models are not accurate enough; especially when we deal with very complex systems or require very high performance. Physical knowledge of the system can be a great aid in finding a nonlinear model. However, this knowledge is not always available. In these cases we have to determine a model from a finite number of measurements of the system's inputs and outputs.

The block-oriented approach to the nonlinear system modeling and identification assumes that the system consists of nonlinear memoryless and linear dynamic subsystems (Bendat, 1990). The approach has been used in many fields (Emerson *et al*., 1992; Eskinat *et al*., 1991; Haber and Unbehauen, 1990; Huebner *et al*., 1990; Kalafatis *et al*., 1997; Ralston *et al*., 1997; Zi-Qiang, 1993). Hammerstein system identification is one of the block oriented nonlinear system identification methods. The Hammerstein model consists of a static nonlinearity followed by a linear dynamic system and usually the signals between the nonlinear and linear blocks are inaccessible to measurements. The identification of the Hammerstein model involves estimating both the nonlinear and linear parts from the input-output measurements (Sjoberg *et al*., 1995). The Single-Input Single-Output (SISO) Hammerstein model has been successfully used to model (Leonessa and Luo, 2001), control (Haddad and Chellaboina, 2001). There are well-known algorithms for identification of this kind of nonlinear systems (Al-Duwaish and Nazmul Karim, 1997; Van Pelt and Bernstein, 2000).

Pariz *et al*. (2003) has presented a new approach for analysis and modeling of nonlinear systems. Pariz *et al.* (2003) and Abdollahi (2002) expand this approach for analyzing and modeling of continuous and discrete nonlinear systems. Modal series can expressed a nonlinear system in a new form which is more accurate than the linearized model of system and it expresses many nonlinear effects of main system. This new modeling structure of nonlinear systems can be used to model and identify a nonlinear system effectively.

In this study, we present a method to determine a novel modeling and identification approach for nonlinear systems using modal series state space model from a finite number of measurements of the inputs and outputs. An

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identification algorithm will also be introduced for this model which uses subspace and MIMO Hammerstein algorithms.

MODAL SERIES

As expressed by Pariz *et al*. (2003), Abdollahi (2002) and Modir Shanechi *et al*. (2003) any nonlinear system which is in the form of

$$\dot{x} = g(x,u) \tag{1}$$

where, $x = [x_1, x_2, \ldots, x_n]^T$ is the state vector, $u = [u_1, u_2, \ldots, u_m]^T$ is the input vector and $g : R^2 \times R^m \rightarrow R^n$ is a smooth vector function which $g(0,0)=0$, can be modelled by Eq. 2 and 3 called modal series.

Remarks:

- Equations 2 are categorized in three classes' v, w and z
- Class v is affected by the initial condition and is the zero input response of the system
- Class w is affected by the input and is the zero state response of the system
- Class z is affected by both initial condition and input. It is the interaction between initial condition and input and differs from zero when both of them do exist
- In linear systems the complete response of a system is equal to sum of its zero input and zero state responses, but this is not the case for nonlinear systems, because of the existence of equations class z
- Modal series method provides a solution for the system in terms of the modes of the system and the input. This can be better seen if we apply the transformation x=Ty, where T is the matrix of the right eigenvectors of B_{10}, use modal series approach to yield the solution and use back transformation $y=T^{-1}x$ to obtain the solution of Eq. 2

Extension to discrete modal series is straight and it will bring us to similar equations (Abdollahi, 2002).

$$x(t) = \sum_{i=1}^{\infty} v_i(t) + \sum_{j=1}^{\infty} w_j(t) + \sum_{i=1}^{\infty}\sum_{j=1}^{\infty} z_{ij}(t) \tag{2}$$

$$\begin{cases} v_1(t+1) = B_{10} v_1(t) & \text{(2-v1)} \\ v_2(t+1) = B_{10} v_2(t) + \dfrac{1}{2}\begin{bmatrix} v_1(t)^T B_{20}^1 v_1(t) \\ \vdots \\ v_1(t)^T B_{20}^n v_1(t) \end{bmatrix} & \text{(2-v2)} \\ \vdots \end{cases}$$

$$\begin{cases} w_1(t+1) = B_1 w_1(t) + B_1 u(t) & \text{(2-w1)} \\ w_2(t+1) = B_1 w_2(t) + \dfrac{1}{2}\begin{bmatrix} w_1(t)^T B_{20}^1 w_1(t) \\ \vdots \\ w_1(t)^T B_{20}^n w_1(t) \end{bmatrix} + \begin{bmatrix} w_1(t)^T B_{11}^1 u(t) \\ \vdots \\ w_1(t)^T B_{11}^n u(t) \end{bmatrix} + \\ \vdots \end{cases}$$

$$\frac{1}{2}\begin{bmatrix} u(t)^T B_{02}^1 u(t) \\ \vdots \\ u(t)^T B_{02}^n u(t) \end{bmatrix} \tag{2-w2}$$

$$\begin{cases} z_{11}(t+1) = B_{10} z_{11}(t) + \dfrac{1}{2}\begin{bmatrix} v_1(t)^T B_{20}^1 w_1(t) + w_1(t)^T B_{20}^1 v_1(t) \\ \vdots \\ v_1(t)^T B_{20}^n w_1(t) + w_1(t)^T B_{20}^n v_1(t) \end{bmatrix} + \\ \vdots \end{cases}$$

$$\begin{bmatrix} v_1(t)^T B_{11}^1 u(t) \\ \vdots \\ v_1(t)^T B_{11}^n u(t) \end{bmatrix} \tag{2-z1}$$

$$\begin{cases} v_1(t=0) = x(0) \\ v_i(t=0) = 0 \quad i = 2,3,\ldots \\ w_j(t=0) = 0 \quad j = 1,2,3,\ldots \\ z_{ij}(t=0) = 0 \quad i = 1,2,3,\ldots,\ j = 1,2,3,\ldots \end{cases} \tag{3}$$

where $B_{10} = \frac{\partial g}{\partial x}\Big|_{\substack{x=0\\u=0}}$, $B_{01} = \frac{\partial g}{\partial u}\Big|_{\substack{x=0\\u=0}}$, $B_{20}^j = \frac{\partial^2 g_j}{\partial x \partial x}\Big|_{\substack{x=0\\u=0}}$, $B_{11}^j = \frac{\partial^2 g_j}{\partial x \partial u}\Big|_{\substack{x=0\\u=0}}$, $B_{02}^j = \frac{\partial^2 g_j}{\partial u \partial u}\Big|_{\substack{x=0\\u=0}}$ and so on.

A MODIFIED REPRESENTATION OF MODAL SERIES MODEL OF NONLINEAR SYSTEMS

Definition: For matrices P and Q with dimensions $n_p \times m_p$ and $n_q \times m_q$, respectively, the Kronecker product is defined as a $(n_p n_q) \times (m_p m_q)$ matrix:

$$P \otimes Q = \begin{bmatrix} P_{11}Q & \cdots & P_{1m_p}Q \\ \vdots & \vdots & \vdots \\ P_{n_p 1}Q & \cdots & P_{n_p m_p}Q \end{bmatrix} \tag{4}$$

We define the superscript notation $\otimes$ and (p) for referring to Kronecker product and the repetitive application of the Kronecker product, respectively.

Now we can express that the class v deals with transient states of nonlinear system which depends on initial conditions. We can neglect transient effects and assume zero initial condition for class v when we want to identify nonlinear system. Since class z equations depends on class v and w and class v states are assumed zero, so class z equations are zero, too. Then we can rewrite the discrete modal series in the form of Eq. 5. Where A, B_1, B_2 ... and u_1, u_2 ... are defined by Eq. 6 and 7.

$$x(t) = \sum_{j=1}^{\infty} w_j(t) \tag{5}$$

$$\begin{cases} w_1(t+1) = Aw_1(t) + B_1u_1(t) & (5\text{-}1) \\ w_2(t+1) = Aw_2(t) + B_2u_2(t) & (5\text{-}2) \\ \vdots \end{cases}$$

$$u_1(t) = u(t) \tag{6-1}$$

$$\zeta_2(t) = \begin{bmatrix} u(t) \\ w_1(t) \end{bmatrix}$$
$$u_2(t) = \zeta_2(t) \otimes \zeta_2(t) = \zeta_2^{(2)}(t) \tag{6-2}$$
$$\vdots$$

$$A = B_{10} \tag{7-1}$$

$$B_1 = B_{01} \tag{7-2}$$

$$B_2 = \text{Coefficients Matrix of } \zeta_2^{(2)} \tag{7-3}$$
$$\vdots$$

It is supposed that the nonlinear system is linear in output equations. It means

$$\begin{aligned} y(t) &= Cx(t) + Du(t) \\ &= C\sum_{k=1}^{K_m} w_k(t) + Du(t) \\ &= \underbrace{Cw_1(t) + Du(t)}_{y_1(t)} + \sum_{k=2}^{K_m} \underbrace{Cw_k(t)}_{y_k(t)} \end{aligned} \tag{8}$$

where, K_m is the maximum number of modal series terms. So output of each w_k equation is as follows:

$$y_1(t) = Cw_1(t) + Du(t) \tag{9-1}$$
$$y_k(t) = Cw_k(t) \qquad k = 2,3,... \tag{9-2}$$

Looking at Eq. 5-1, 5-2, ... implies that inputs ($u_k(t)$, k=2,3,...) for $w_k(t)$ (k=2,3,...) state equations, rely on $w_j(t-1)$ (j=k-1,k-2,...1) and $u_1(t-1)$.

Since it is possible to construct $\zeta_k(t)$ and then the input vector u_k of every w_k equation for every sample time, it is clear that Eq. 5-1, 5-2 ... are linear. w_k is in fact the linear model of nonlinear system and all other terms try to model nonlinear dynamics of the main nonlinear system.

Now we can express that there are a set of equations in the form of (10) which can approximate the main nonlinear system.

$$w_k(t+1) = Aw_k(t) + B_ku_k(t) \tag{10-1}$$
$$y_k(t) = Cw_k(t) + D_ku(t) \tag{10-2}$$

where, D_1 = D and D_k = 0 for k = 2,3,.... And the main state vector can be computed by Eq. 5.

In attention to Eq. 5 to 10, one can proposed a model structure as shown in Fig. 1. Figure 1 expresses a common linear block and several nonlinear blocks. It also presents r the Hammerstein structure of subsystems.

In an identification problem for nonlinear systems, we can identify the structure proposed in Fig. 1 as an approximation of nonlinear systems. This would be very flexible method, since we can choose arbitrary number of modal series terms (w_k) to be identified.

Because of the state space form of this model of nonlinear systems, we proposed to use a modified version of a subspace algorithm as presented next.

A NOVEL ALGORITHM FOR IDENTIFICATION OF A NONLINEAR SYSTEM USING THE PROPOSED MODEL

Hammerstein systems are known structure for nonlinear system identification. There are lots of literatures devoted to identification problem of Hammerstein systems (Gomez and Baeyens, 2000). Identification of MIMO Hammerstein systems are still an active field but there are well practiced works around (Michel and Westwick, 1996). A straight and interesting approach to identification of MIMO systems are subspace identification algorithms (Van Overschee and de Moor, 1996). Linear subspace algorithms have shown their ability in linear system identification and considering the structure introduced earlier we can proposed an algorithm for MIMO nonlinear system identification.

In order to be able to identify each subsystem of the proposed structure we need to know input and output of each subsystem. Inputs are distinct, since u_1 is defined by Eq. 6-1 and inputs of ith subsystem are $u_1(t-1)$ and states of all previously identified subsystems ($w_j(t-1)$, j=1, ..., i-1).

To produce outputs for each subsystem, consider following output equation expansion for a K_m modal series terms representation.

$$\begin{aligned} y(t) &= Cx(t) \\ &= C\sum_{k=1}^{K_m} w_k(t) = \sum_{k=1}^{K_m} Cw_k(t) \\ &= \sum_{k=1}^{K_m} y_k(t) \end{aligned} \quad (11)$$

So we proposed the following output equation (10-k) for each subsystem when we want to identify the kth subsystem:

$$y_1(t) = Cw_1(t) = y(t) \quad (12\text{-}1)$$

$$y_2(t) = Cw_2(t) = y(t) - \hat{y}_1(t) \quad (12\text{-}2)$$

$$\vdots$$

$$y_k(t) = Cw_k(t) = y(t) - \sum_{j=1}^{k-1} \hat{y}_j(t) \quad (12\text{-}k)$$

|Now, the following algorithm is proposed in order to identify such a model.

Algorithm:

- Let $y_1(t) = y(t)$, $u_1(t) = u(t)$, $k = 1$ and choose an arbitrary number of modal series terms (K_m)
- Identify linear subsystem of proposed structure (Fig. 1) using one of the MIMO linear system identification algorithms (preferably linear subspace algorithms). And produce $w_1(t)$ and A, B_1, C
- Determine $w_i(t)$, $i = 1, 2, \ldots, k$
- Let $k = k+1$ and use (12-k) to produce $y_k(t)$
- Use one of the MIMO Hammerstein system identification algorithms while you know that the linear block of this subsystem has been identified in step 2 and the nonlinear block structure has been defined in proposition A1. And determine (k-1)th MIMO Hammerstein subsystem (Fig. 1)
- if k = Km go to step 7 else go to step 3
- End of Algorithm

The algorithm uses subspace methods to identify each subsystem (Van Overschee and De Moor, 1996). We can apply N4SID, CVA, MOESP algorithms. The proposed algorithm has been applied to identify the example expressed in the next section.

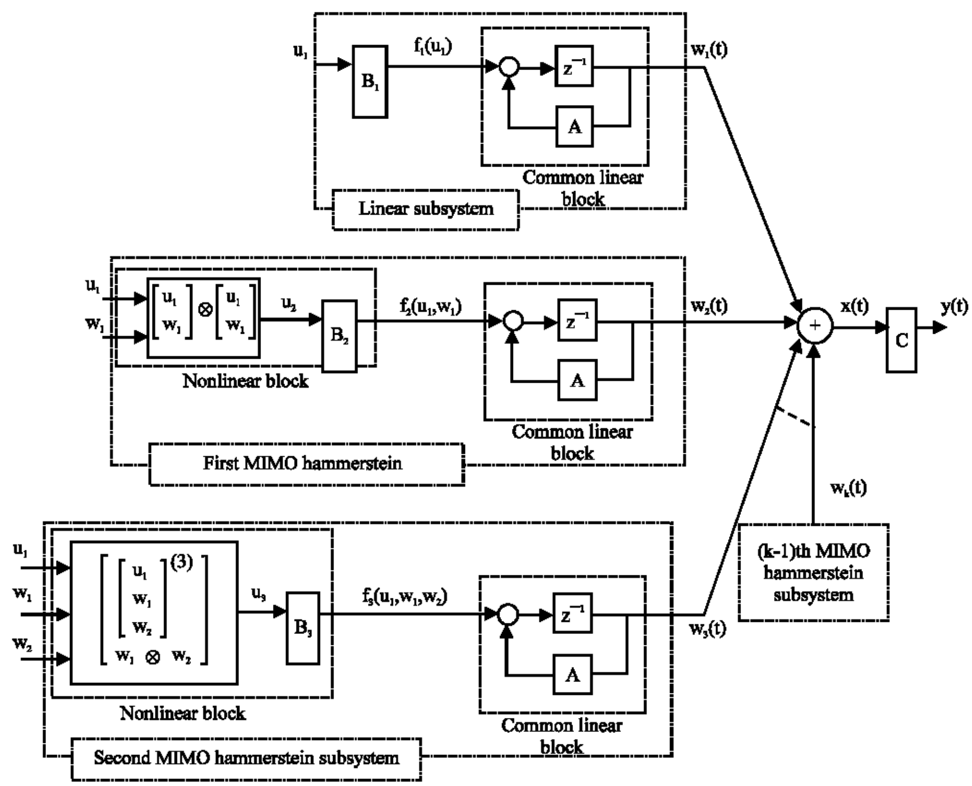

Fig. 1: Presentation of a nonlinear system as proposed by modal series model ($u_1=u_1(t-1)$, $w_1=w_1(t-1)$, $w_2=w_2(t-1)$)

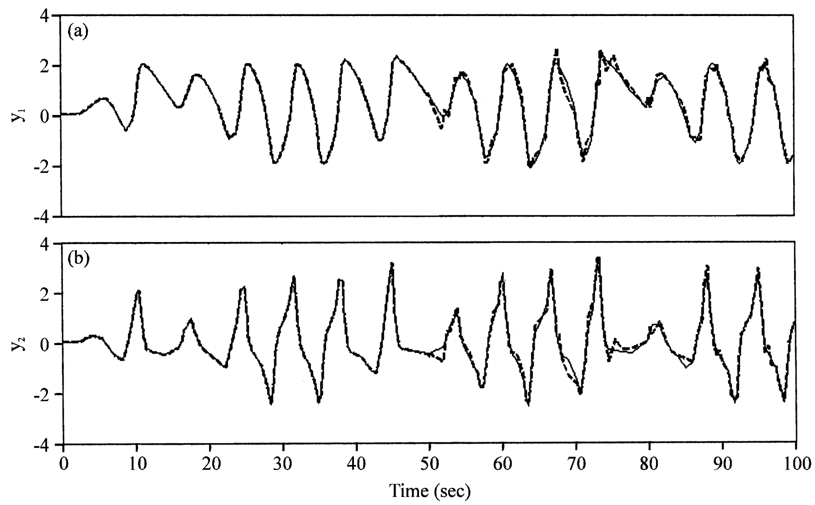

Fig. 2: Van der Pol Oscillator: the solid line represents the nonlinear simulation output and dashed line represents the predicted output

SIMULATIONS

In this simulation example we use N4SID subspace identification for linear system identification and the method proposed by Michel and Westwick (1996) to identify the MIMO Hammerstein subsystems.

The identification algorithm presented in the previous section is now tested using a sampled data Van der Pol Oscillator with nonlinear input.

System equations presented in 13:

$$\dot{x}_1 = x_2 + \omega_1 \quad (13\text{-}1)$$

$$\dot{x}_2 = -x_2 + (1 - x_1^2)x_2 + u^2 + \omega_2 \quad (13\text{-}2)$$

$$y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \quad (13\text{-}3)$$

where, w_1, w_2, v_1 and v_2 are white noise signals, with covariance 0.001, 0.0002, 0.02 and 0.03, respectively.

The chosen input signal u for identification process was a zero-mean, white noise sequence of length N = 1000 with a sampling period T = 0.05 sec uniformly distributed between -1 and 1.

In order to validate the proposed model structure and identification algorithm, additional 1000 samples are obtained from the simulation and the result is compared with the predicted output in Fig. 1.

In Fig. 2, the first 50 sec of data are used for identification and the remaining 50 sec of data are used for validation. The solid line represents the nonlinear simulation output and the dashed line represents the predicted output.

As it can be seen in Fig. 2, the algorithm could have done the identification of a severely nonlinear system in a satisfactory manner.

CONCLUSION

In this study, we proposed a new modeling approach for nonlinear systems based on modal series. New model contains linear and Hammerstein subsystems which are common in their linear blocks and the structure of nonlinear blocks are distinct. A MIMO nonlinear system identification algorithm based on subspace identification, Hammerstein modeling and proposed model has been presented. The proposed model and identification algorithm can find many applications in the context of nonlinear system identification and control.

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