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Entropy of Stochastic Intuitionistic Fuzzy Sets

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Abstract: In this study entropy, cross-entropy and discrimination information measures of stochastic intuitionistic fuzzy sets which result from hesitancy and randomness are proposed. It is an extension of De Luca and Termini's formula for SIFSs. It is also intended to extend the measures proposed by Vlachos and Sergiadis. We will prove that in special cases these measures lead to the same results as the measures proposed by others. We also suggest the application of these measures in image processing in future researches.

Key words: Intuitionistic fuzzy sets, stochastic, entropy, cross-entropy, discrimination information

INTRODUCTION

For the first time Zadeh (1965) introduced the concept of fuzzy sets which he defined in as the following:

Definition 1: Fuzzy set A is given by:

$$A = \{(x, \mu_A(x)) | x \in X\} \tag{1}$$

where, μ_A: $X \rightarrow (0, 1)$ is the membership function of the fuzzy set A.

Entropy measures in the stochastic systems are the degree of uncertainty which result from randomness, while in the theory of fuzzy sets uncertainty is due to the vagueness and complexity of the system. Thus the fuzzy entropy can be considered as the degree of uncertainty caused by personal judgment. Three decades after Shannon (1948) introduced the entropy of stochastic systems, De Luca and Termini (1972) for the first time introduced the concept of fuzzy entropy. The concept of entropy has had many implication for statistics and many people have applied it to their statistical studies (Ciavolino and AL-Nasser, 2009; Ciavolino and Dahlgaard, 2009; Al-Shalabi *et al.*, 2005). Their measure of fuzzy entropy was based on the fuzzy sets with a finite reference set. This measure is based on Shannon's measure of entropy. They regard the following four axioms as the underlying principles of fuzzy entropy function:

- **P1:** H(A) = 0 if and only if $\mu_A(x) = 0$or 1, $\forall x \in X$
- **P2:** H(A) takes the maximum value if and only if $\mu_A(x) = 1/2$, $\forall x \in X$
- **P3:** if A is a sharp set of B then $H(A) \leq H(B)$
- **P4:** $H(A) = A(A^c)$, where A^c is complementary set of A

Then, building on these four principles and Shannon's function, they introduced the following formula:

$$H(A) = -\frac{1}{n}\sum_{i=1}^{n}[\mu_A(x_i)\log\mu_A(x_i) + (1-\mu_A(x_i))\log(1-\mu_A(x_i))] \tag{2}$$

Kaufmann (1975) defined the fuzzy entropy by the Euclidean or Hamming distance of the fuzzy set and the nearest crisp set. Yager (1979) considers the relationship between the fuzzy set A and its compliment A^c as the essence of fuzziness. In spite of the apparent structural difference between Kaufmann and Yager's measures with De Luca's entropy, they satisfy the four axioms introduced by De Luca and Termini (1972). Kosko (1992) gave a new version of Kaufmann's entropy (Kaufmann, 1975). Considering the degree of hesitancy Atanassov (1986) defined intuitionistic fuzzy sets as the generalization of fuzzy sets. Szmidt and Kacprzyk (2001) introduced the entropy for intuitionistic fuzzy sets and its axioms.

Intuitionistic fuzzy sets and entropy: The concept of an Intuitionistic Fuzzy Set (IFS), which is an extention of the concept of a Fuzzy Set (FS), was introduced by Atanassov (1986).

Preliminaries

Definition 2: Intuitionistic fuzzy set B is given by:

$$B = \{(x, \mu_B(x), V_B(x)) | x \in X\} \tag{3}$$

where, $\mu_B : X \rightarrow (0, 1)$ and $v_B : X \rightarrow (0, 1)$ are such that:

$$0 \leq \mu_B(x) + v_B(x) \leq 1 \tag{4}$$

and $\mu_B(x)$ and $v_B(x)$ belong to (0, 1); $\mu_B(x)$ and $v_B(x)$ denote the degree of membership and the degree of non-membership of x, respectively.

For each intuitionistic fuzzy set in X we have a hesitation margin π_B (c). It is an intuitionistic fuzzy index of x, expressing a hesitation degree of whether x belongs to B or not. It is obvious that $0 \le \pi_B(x) \le 1$, for each $x \in X$:

$$\pi_B(x) = 1 - \mu_B(x) - v_B(x) \tag{5}$$

Therefore, if we want to fully describe an intuitionistic fuzzy set, we must have any two of the following functions:

- Membership function
- Non-membership function
- Hesitation margin

There are many applications of this concept in clustering and image processing (Pasha and Fatemi, 2006).

Basic operations on IFSs: Atanassov (1986) define a set of operations between two IFSs A and B.

Definition 3: The union operator ∪ between A and B is given by:

$$A \cup B = \{(x, \max\{\mu_A(x), \mu_B(x)\}, \min\{v_A(x), v_B(x)\}) \mid x \in X\}$$

Definition 4: The intersection operator n between A and B is given by:

$$A \cap B = \{(x, \min\{\mu_A(x), \mu_B(x)\}, \max\{v_A(x), v_B(x)\}) \mid x \in X\}$$

Definition 5: The complementary set of A is defined as:

$$A^c = \{(x, v(x), \mu_A(x)) \mid x \in X\}$$

Intuitionistic fuzzy entropy: De Luca and Termini (1972) proposed nonprobabilistic entropy for FSs and formulated the axiomatic requirements with which an entropy measure should comply. Szmidt and Kacprzyk (2001) extended the axioms of De Luca and Termini (1972), proposing the following definition for an entropy measure in the setting of IFSs theory.

Definition 6: (Szmidt and Kacprzyk, 2001): An entropy on IFS(X) is a real-valued functional E: IFS(X)→(0, 1), satisfying the following axiomatic requirements:

- E(A) = 0 iff A is a crisp set; that is $\mu_A(x_i) = 0$ or $\mu_A(x_i) = 1$ for all $x_i \in X$
- E(A) = 1 iff $\mu_A(x_i) = v_A(x_i)$ for all $x_i \in X$, that is $A = A^c$
- $E(A) \le E(B)$ if A is sharper than B, i.e.,

$$\begin{cases} \mu_A(x) \le \mu_B(x) \text{ and } v_A(x) \ge v_B(x), \text{ for } \mu_B(x) \le v_B(x) \\ \text{or} \\ \mu_A(x) \ge \mu_B(x) \text{ and } v_A(x) \le v_B(x), \text{ for } \mu_B(x) \ge v_B(x) \end{cases}$$

  for all $x_i \in X$

- $E(A) = E(A^c)$

Moreover, an intuitionistic fuzzy entropy measure was defined as:

$$E(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\max\mathrm{Count}(A_i \cap A_i^c)}{\max\mathrm{Count}(A_i \cup A_i^c)} \tag{6}$$

where, n is Cardinal (X) and A_i denotes the single-element IFS corresponding to the i th element of the universe X. In other words, Ai is the i th component of A. Moreover, max Count (A) denotes the biggest cardinality of A and is given by:

$$\max\mathrm{Count}(A) = \sum_{i=1}^{n}(\mu_A(x_i) + \pi_A(x_i)) \tag{7}$$

Zeng *et al*. (2008) introduced the concept of similarity measure of intuitionistic fuzzy sets and proposed a new method for describing entropy of intuitionistic fuzzy set based on similarity measure of intuitionistic fuzzy sets. They also introduced some formulas to calculate entropy of intuitionistic fuzzy sets.

ENTROPY OF STOCHASTIC INTUITIONISTIC FUZZY SETS

Here, first a simple form of intuitionistic fuzzy entropy measure is introduced; then it is extended to stochastic IFSs.

Definition 7: Simple intuitionistic fuzzy entropy for IFS A can be defined as:

$$E_u(A) = \frac{1}{n}\sum_{i=1}^{n} e(x_i)$$

where:

$$e(x_i) = 1 - \sqrt{(\mu_A(x_i) - v(x_i))^2} \tag{8}$$

It satisfies Szmidt and Kacprzyk (2001):

Definition 8: Stochastic intuitionistic fuzzy entropy for IFS A with a probability system $P = (p_1, p_2, \ldots, p_n)$ is defined as:

$$E_S(A) = \sum_{i=1}^{n} p_i e(x_i) \tag{9}$$

Property 1: In case of the uniform probability system the stochastic intuitionistic fuzzy entropy is the same as the simple intuitionist fuzzy entropy.

Definition 9: Total stochastic intuitionistic fuzzy entropy for IFS A with a probability system $P = (p_1, p_2, \ldots, p_n)$ is defined as:

$$E_T(A) = E_S(A)+H(P) \tag{10}$$

where,

$$H(P) = -\sum_{i=}^{n} p_i \log p_i$$

is Shannon's entropy.

Property 2: If probability system be a degenerate distribution system, $P = (0, \ldots, 1, \ldots, 0)$, then $E_T(A) = e(x_i)$.

CROSS-ENTROPY AND DISCRIMINATION INFORMATION OF STOCHASTIC INTUITIONISTIC FUZZY SETS

Definition 10: (Shang and Jiang 1997): Let A and B be two FSs defined on X. Then:

$$I_{FS}(A,B) = \frac{1}{n}\sum\left[\mu_A(x)\log\frac{\mu_A(x)}{\frac{1}{2}(\mu_A(x)+\mu_B(x))} + (1-\mu_A(x))\log\frac{1-\mu_A(x)}{1-\frac{1}{2}(\mu_A(x)+\mu_B(x))}\right] \tag{11}$$

is called fuzzy cross-entropy, where n is the cardinality of the finite universe X.

Definition 11: (Vlachos and Sergiadis, 2007): Let A and B be two IFSs defined on X. Then:

$$I_{IFS}(A,B) = \frac{1}{n}\sum\left[\mu_A(x)\log\frac{\mu_A(x)}{\frac{1}{2}(\mu_A(x)+\mu_B(x))} + \nu_A(x)\log\frac{\nu_A(x)}{\frac{1}{2}(\nu_A(x)+\nu_B(x))}\right] \tag{12}$$

is called intuitionistic fuzzy cross-entropy, where n is the cardinality of the finite universe X.

Formula (12) is the degree of discrimination of A from B. However, this is not symmetric.

Definition 12: (Vlachos and Sergiadis, 2007): For two IFSs A and B:

$$D_{IFS}(A, B) = I_{IFS}(A, B)+I_{IFS}(B, A) \tag{13}$$

is called a symmetric discrimination information measure for IFSs.

Property 3:

- DIFS (A, B)≥
- $D_{IFS}(A, B) = 0 \Leftrightarrow A = B$
- $D_{IFS}(A, B) = D_{IFS}(B, A)$

Definition 13: Stochastic intuitionistic fuzzy cross-entropy for two IFSs A and B with a probability system $P = (p_1, p_2, \ldots, p_n)$ is defined as:

$$I_{SIFS}(A,B) = \sum\left[p_i(\mu_A(x)\log\frac{\mu_A(x)}{\frac{1}{2}(\mu_A(x)+\mu_B(x))} + \nu_A(x)\log\frac{\nu_A(x)}{\frac{1}{2}(\nu_A(x)+\nu_B(x))})\right] \tag{14}$$

and stochastic intuitionistic fuzzy discrimination information is defined as:

$$D_{SIFS}(A, B) = I_{SIFS}(A, B)+I_{SIFS}(B, A) \tag{15}$$

Furthermore property 3 holds for $D_{SIFS}(A, B)$.

SUGGESTING AN APPLICATION IN IMAGE PROCESSING

We recommend applying the theory of fuzzy sets to image processing. In this method images should be considered as fuzzy subsets of a plane, in which the membership degree of the pixels to the image is in proportion with their gray level, while the non-membership degree of the pixels to the image is in inverse proportion with their gray level. The original image, as well as the degraded image and the restructured image will be considered as fuzzy sets A, B and C in reference set X, where X is a plan sheet. The aim is to transform the degraded image B to a denoised image which we call C via an algorithm. The algorithm first finds the noised pixels and then replaces them. The problem will be to choose a jumping cut point h as the unexpected jumping of the gray level in the algorithm to find the noised pixels. This jumping cut point depends on the individual image.

The cost function is introduced as the summation of the distance between A, C and the fuzzy entropy of C, because in addition to the distance of the original image and the denoised image, the entropy of the denoised image should be considered as the blurring caused by the replacement of noised pixeles. In our paper (Pasha *et al*., 2006) the Euclidian distance and Kaufmann's entropy (Kaufmann, 1975) were used for this goal. Since in most images the center of the image plane includes more important information than its margins, in this paper we have developed our previous idea, replacing stochastic fuzzy entropy for fuzzy entropy and replacing stochastic fuzzy discrimination information for the Euclidian distance. Thus we have the following formula for the cost function, introduced as stochastic fuzzy cost function:

$$C(A) = D_{SFS}\,(A, C) + E_T\,(A) \tag{16}$$

In the above formula we choose the probability density function according to structure of the background and the object in an image.

CONCLUSIONS

This study presented total entropy, cross-entropy and discrimination information measures for stochastic intuitionistic fuzzy sets. This is a development of other entropy measures introduced by De Luca and Termini (1972), Szmidt and Kacperzik (2001) and Vlachos and Sergiadis (2007). Also, an application of this idea to image processing has been suggested for future researches.

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