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Enhanced Stutzer Index Optimization Using Hybrid Genetic Algorithm and Sequential Quadratic Programming

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Abstract: This study presents a hybrid approach by associating the Genetic Algorithm (GA) and the Sequential Quadratic Programming (SQP) to improve the Stutzer Index optimization. The Stutzer Index is a well-recognized portfolio performance measure that provides unbiased estimates of risk-adjusted performance. However, the tasks in optimizing and determining a good starting point for the constrained optimization of Stutzer Index are challenging, especially with the additional constraint on the negative term θ. By integrating GA and SQP, this study anticipates the hybrid model to improve the efficiency and the performance of the optimization. The optimal indices obtained from both the SQP and the hybrid GA-SQP that used the initial guess recommended by Stutzer and the optimal index acquired via the hybrid GA-SQP with random starting point, for different period of data and number of assets respectively, are utilized for the comparative study. The results revealed that the hybrid model is superior in the Stutzer Index optimization, owing to the consistent capability of GA to locate the global optimum region and SQP to reach the optimal solution. The results also attested that the hybrid model enhanced the efficiency of the optimization as it does not required user-defined starting point and can sufficiently attained the optimal solution by utilizing a randomly generated starting point. In general, the hybrid model is competent in improving the efficiency and the performance of the Stutzer Index optimization, albeit the enhancement is not statistically significant in smaller number of observations.

Key words: Genetic algorithm, initial guess, optimal solution, starting point, Stutzer Index

INTRODUCTION

The most widely used approaches in portfolio construction and risk-adjusted performance measurement are founded principally on the mean-variance framework of Markowitz (1952) and the reward-to-variability (or Sharpe Ratio) of Sharpe (1966). However, the necessity for alternative performance measurements escalates as the facts of non-normality in asset returns surface. In response to the inappropriate assumption of normality, Stutzer (2000) proposed a portfolio performance index that uses the quantifiable exponential decay rate of the likelihood that a portfolio will underperform a benchmark, i.e. the Stutzer Index. The Stutzer Index, also known as the Stutzer's Portfolio Performance Index, is an alternative risk-adjusted performance measurement that is sensitive to the shape and the higher moments of the returns distribution. Stutzer Index rewards those portfolios that have a lower probability of underperforming a specified benchmark on average, or portfolios with returns distribution that is positively skewed. The indexis established on the assumption that fund managers have an aversion towards excess returns that underperformed a pre-designated benchmark and hence, will favor portfolios with positive expected excess returns, i.e., portfolios with higher probability of decay rates.

Stutzer Index is a consistent generalization of the Sharpe Ratio that provides unbiased estimates of risk-adjusted performance, even with the presence of skewness and kurtosis in the asset returns and it was formerly used by the Morningstar, Inc. in its Star Ratings of mutual funds. Stutzer (2000) has shown that the Stutzer Index can be estimated by:

$$\hat{I} = \max_{\theta} -\log \frac{1}{N}\sum_{t=1}^{N} \exp\left(\theta R_t\right) \tag{1}$$

where, θ is a negative value, N is the total number of observations over the evaluation period, R_t is the excess returns of a portfolio over the predetermined benchmark at time t. Thus, for n assets under consideration, Stutzer (2000) has also attested that the optimal value of θ and the optimal weights, that maximizes the Stutzer Index can be obtained by:

$$I_m = \max_{w} \max_{\theta} -\log \frac{1}{N}\sum_{t=1}^{N} \exp\left(\theta \sum_{i=1}^{n} w_i R_{it}\right) \tag{2}$$

Notwithstanding the advantages demonstrated by the Stutzer Index, the optimization of Stutzer Index is slightly more challenging than other portfolio performance measures such as Sharpe Ratio and Sortino Ratio (Sortino and Price, 1994), due to the extra constraint applied on the negative value θ, in addition to the constraints in weights.

Optimization technique is generally used to find a set of optimal parameters of some objective function and it might be subject to some parameter bounds, equality and/or inequality constraints. A good starting point for the optimizer is essential in ensuring the accomplishment of the optimization, especially in a global optimization problem to avoid local optimum. Stutzer (2000) solved the maximization of Stutzer Index by using numerical optimization and he proposed that the estimated optimal portfolio's weights of the Sharpe Ratio and the -1 times the portfolios mean excess return divided by its variance, is a good initial guess for the portfolio weights and θ, respectively in optimizing the Stutzer Index. Following his work, Benson *et al.* (2008) employed numerical optimization and the initial guess recommended by him to solve the Stutzer Index optimization problem in their study. In a more recent study, Lye and Ng (2010) also followed Stutzer's suggestion in the choice of starting point, although they applied SQP to optimize the Stutzer Index. However, most of the standard optimization methods such as numerical optimization and SQP are sensitive to the starting point and their solutions are more likely to be trapped in a local minimum or converge prematurely. Motivated by the study conducted by El-Mihoub *et al.* (2006) which highlighted the merits of hybrid genetic algorithms, and the fact that optimizing and determining a good starting point for the constrained optimization of the Stutzer Index is a challenging task, therefore, this study proposes to use the integrated genetic algorithm and the sequential quadratic programming (GA-SQP) to:

- Enhance the efficiency in optimizing the Stutzer Index by using randomly generated starting point, in which it diminished the difficulty and the necessity to determine the best starting point preceding the optimization
- Improve the performance of the Stutzer Index optimization by utilizing the advantages of GA andSQP in finding the global optimum and reaching the optimal solution, respectively

By combining the GA (a well-known global search algorithm) with the SQP (an efficient local search method), it is needless to decide the best starting point for the hybrid GA-SQP and it is anticipated to improve the performance of the Stutzer Index optimization as well.

MATERIALS AND METHODS

This study was conducted from January 2010 to September 2010, in the Multimedia University of Malaysia. Figure 1 displays the basic steps in the hybrid GA-SQP. The first component of the hybrid model uses the GA to search for the region in which the global optimum is located. Genetic algorithms (Goldberg, 1989; Mitchell, 1998) apply an evolutionary process via the genetic operators (selection, crossover, and mutation) to perform global search in a solution space gradually, to discover the best solution for the problem. Genetic algorithms are well-recognized effective global optimization tools in solving both constrained and unconstrained optimization problems because of their simplicity, derivative-free, inbuilt parallel processing capability and most importantly, it can perform equally well even without user-defined starting point. In this study, the search for the best solution is continued by SQP in the second part of the hybrid model. Sequential Quadratic Programming (Fletcher, 1987) is a well-known method in solving optimization of nonlinear continuous objective function, in which it iteratively solves a series of quadratic programming subproblems that are subject to some linear constraints. Even though the SQP is proven in finding the local optimum of an optimization problem, the starting point provided to the method is very crucial in locating the desired optimal solution.

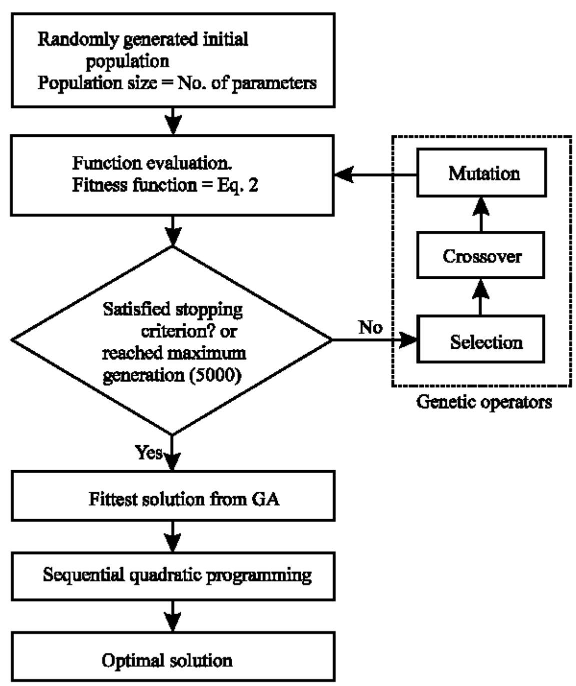

Fig. 1: The basic steps in the hybrid Genetic algorithm-sequential quadratic programming

Hybrid GA-SQP is applied in various fields and its competency to solve various real-world problems is verified (Da-kuo *et al*., 2008; He *et al*., 2008; Mansoornejad *et al*., 2008; Nisar *et al*., 2008; Rentizelas and Tatsiopoulos 2010; Wang *et al*., 2006; Zeeshan *et al*., 2010). By retaining the merits of GA and SQP, respectively, in locating the most promising region of convergence and in obtaining the desired optimal solution, the hybrid GA-SQP applied in this study aims to enhance the efficiency and the performance of the optimization of the Stutzer Index. This study used the Genetic Algorithm and Direct Search Toolbox and the Optimization Toolbox of the MATLAB for the constrained optimizations. The optimization of the Stutzer Index is realized by using the Eq. 2 and it is subject to the following constraints:

- The total weights of the portfolio for n assets under consideration is equal to one ($\sum_{i=1}^{n} w_i = 1$)
- The weight in the portfolio is non-negative ($w_i, \geq 0$), by assuming short selling is not permitted
- The value of θ is negative (θ<0)

The data employed for the empirical study are the adjusted daily returns of the top 30 constituent stocks (by market capitalization as of 31st December 2008) of the Kuala Lumpur Composite Index (KLCI) in Bursa Malaysia, Malaysia, dated from 2nd January 1996 to 31st December 2008, that are available throughout the period of study. All the data, including the market benchmark index KLCI, are retrieved from the DataStream. Table 1 presents the summary statistics of the stock returns and it is verified from the Jarque-Bera test that the adjusted daily returns are not normally distributed. The empirical study in this paper is segmented into two main parts. The first part of the study utilizes the entire period of data to maximize theStutzer Index for different number of assets (10, 20 and 30). The optimal Stutzer Index acquired from the SQP by using the starting point suggested by Stutzer (2000) is compared with the optimal index attained by the hybrid GA-SQP. The study on the sufficiency of randomly generated starting point in contrast to the initial guess suggested by Stutzer (2000) is also carry out via the hybrid GA-SQP and the S-GA-SQP (hybrid GA-SQP with initial guess proposed by Stutzer) respectively. As for the latter part of the empirical study, the data are divided into a series of subperiods (3-month and 6-month) in which each corresponding portfolio is revised after every subperiod. The respective optimal index acquired from the S-SQP, hybrid GA-SQP and the S-GA-SQP are consequently utilized in the Mann-Whitney nonparametric test for the comparative study.

Table 1: Descriptive statistics for stock daily returns and test for normality

Stock	N	Mean	SD.	Skewness	Kurtosis	JB
MAYBANK	3265	0.0001	0.0228	1.21	35.23	176841.8
SIME	3265	0.0003	0.0238	1.30	19.41	51740.14
TENAGA	3265	0.0004	0.0294	0.73	14.78	28960.82
MISC	3265	0.0005	0.0212	0.82	29.36	114563.3
PBBANK	3265	0.0006	0.0207	1.28	19.04	49533.06
COMMERZ	3265	0.0007	0.0318	2.40	34.12	159819.5
PETGAS	3265	0.0002	0.0208	0.37	44.73	258694.1
IOICORP	3265	0.0010	0.0285	0.46	11.82	19171.76
GENTING	3265	0.0002	0.0234	0.42	7.01	6629.92
BAT	3265	0.0004	0.0182	-0.64	29.72	115627.4
YTL	3265	0.0004	0.0263	1.38	32.17	135142.9
TM	3265	0.0002	0.0235	0.70	16.41	35799.19
PPB	3265	0.0006	0.0189	0.56	10.39	14288.46
KLK	3265	0.0006	0.0220	0.85	22.54	71979.46
HLBANK	3265	0.0004	0.0233	1.08	18.97	48657.27
RHBCAP	3265	0.0003	0.0348	2.78	37.81	192216.6
PETDAG	3265	0.0005	0.0194	-0.35	24.57	79949.99
AMMB	3265	0.0004	0.0319	1.64	24.44	79727.94
BJTOTO	3265	0.0005	0.0264	0.87	22.19	64653.85
TANJONG	3265	0.0005	0.0254	0.40	12.57	21283.03
UMW	3265	0.0007	0.0289	4.20	86.04	972708.5
MAS	3265	0.0002	0.0303	0.37	18.88	49040.12
MMCCORP	3265	0.0006	0.0341	1.20	28.75	112378
SARAWAK	3265	0.0001	0.0290	1.16	11.71	18957.27
GAMUDA	3265	0.0006	0.0313	1.58	35.63	172750.8
SPSETIA	3265	0.0008	0.0285	1.25	19.22	50140.43
LMCEMNT	3265	0.0005	0.0318	0.45	21.27	61010.41
ORIENT	3265	0.0003	0.0233	2.82	70.79	660700.9
SHELL	3265	0.0003	0.0216	0.99	38.88	199412.2
AFG	3265	0.0005	0.0357	1.01	18.83	47087.9

JB indicates the Jarque-Bera normality test. If the JB test statistic is greater than the critical value 15.6372, the JB test rejects the null hypothesis that the data are from a normal distribution at 1% significance level

RESULTS AND DISCUSSION

The first part of the study employed the entire period of data and compared the performance of the S-SQP with the hybrid GA-SQP. Table 2 presents the optimal Stutzer Index and the daily returns (%) for different number of assets. The results revealed that the optimal Stutzer Index acquired from the hybrid GA-SQP is higher than the optimal index obtained by the S-SQP and it is consistent in all three different numbers of assets. The superiority of the hybrid GA-SQP attested the importance of starting point in standard optimization such as SQP and the capability of GA and its consistency in locating the global optimum region. The findings are consistent with some of the earlier studies conducted although the hybrid model was applied in other areas (Mansoornejad *et al*., 2008; Nisar *et al*., 2008; Rentizelas and Tatsiopoulos 2010; Wang *et al*., 2006). In addition, the results also disclosed that the hybrid GA-SQP has enhanced the efficiency of the optimization of Stutzer Index as it does not required user-defined starting point. Furthermore, the identical optimal indices yielded by the hybrid GA-SQP and the S-GA-SQP, as shown in Table 2, verified that a randomly generated starting point is sufficient for the hybrid GA-SQP in optimizing the Stutzer Index. The competency shown by the hybrid model in utilizing a random starting point is also highlighted in other applications (He *et al*., 2008; Zeeshan *et al*., 2010). Moreover, the optimal daily returns (the product of the asset's average daily return and its respective optimal weights) obtained from the hybrid GA-SQP also outperformed the S-SQP. No significant difference is evident between the daily returns acquired by the hybrid GA-SQP and the S-GA-SQP and this further confirmed that the two methods have successfully reached the optimal solution.

The weights and the value of θ in the optimal portfolio for different number of assets are disclosed in Table 3. The overall weights allocated in each stock are nearly comparable in all the three methods. On the other

Table 2: The optimal Stutzer Index and daily returns (%) for different number of assets

	Stutzer Index			Daily returns (%)		
No. of assets	S-SQP	GA-SQP	S-GA-SQP	S-SQP	GA-SQP	S-GA-SQP
10	0.0004	0.0028	0.0028	0.0492	0.0496	0.0497
20	0.0003	0.005	0.005	0.0426	0.0426	0.0426
30	0.0003	0.0068	0.0068	0.0449	0.0452	0.0451

Table 3: The weight (%) and the value θ in the optimal portfolio for different number of assets

	10 assets			20 assets			30 assets		
Stock	S-SQP	GA-SQP	S-GA-SQP	S-SQP	GA-SQP	S-GA-SQP	S-SQP	GA-SQP	S-GA-SQP
MAYBANK	8.00	7.66	7.63	7.14	7.04	7.08	7.47	7.26	7.27
SIME	7.48	7.35	7.26	5.73	5.73	5.68	5.13	5.16	5.10
TENAGA	4.71	4.73	4.75	2.61	2.52	2.48	0.89	0.74	0.76
MISC	10.98	10.93	10.97	6.16	6.15	6.10	5.74	5.73	5.76
PBBANK	18.73	18.98	19.16	8.15	8.21	8.21	6.34	6.49	6.52
COMMERZ	12.15	12.50	12.43	6.25	6.38	6.42	5.01	5.23	5.21
PETGAS	7.24	7.51	7.18	5.00	4.86	4.96	4.81	4.81	4.76
IOICORP	15.59	15.61	15.68	7.71	7.70	7.64	6.11	6.12	6.12
GENTING	6.10	5.69	6.01	2.77	2.57	2.60	2.69	2.60	2.62
BAT	9.02	9.05	8.94	4.08	4.09	4.16	4.35	4.44	4.48
YTL				2.30	2.47	2.40	1.31	1.34	1.34
TM				11.30	11.28	11.19	11.84	11.88	11.80
PPB				7.66	7.56	7.61	5.46	5.49	5.48
KLK				3.79	3.83	3.79	3.13	3.08	3.04
HLBANK				3.96	4.11	4.14	2.42	2.43	2.44
RHBCAP				3.10	3.15	3.17	2.02	2.11	2.12
PETDAG				2.68	2.56	2.58	1.65	1.54	1.55
AMMB				2.49	2.58	2.52	1.59	1.71	1.73
BJTOTO				3.37	3.45	3.43	2.54	2.45	2.44
TANJONG				3.73	3.77	3.83	3.05	3.09	3.10
UMW							3.85	4.00	4.00
MAS							1.35	1.21	1.20
MMCCORP							2.28	2.28	2.26
SARAWAK							0.01	0.01	0.01
GAMUDA							2.00	1.96	1.96
SPSETIA							2.43	2.45	2.44
LMCEMNT							1.52	1.52	1.48
ORIENT							0.00	0.00	0.00
SHELL							0.48	0.40	0.42
AFG							2.55	2.49	2.49
θ	-1.02	-12.08	-12.07	-0.88	-26.81	-26.81	-0.72	-35.98	-35.92

Table 4: The Mann-whitney nonparametric test on the optimal Stutzer Index for a series of subperiods with null hypothesis: the median of GA-SQP is not greater than the median of S-SQP

	3-month			6-month		
No. of assets	GA-SQP	S-SQP	p-value	GA-SQP	S-SQP	p-value
10	0.0392	0.0375	0.4624	0.0202	0.0202	0.3675
20	0.0941	0.0941	0.4948	0.0351	0.0351	0.489
30	0.1705	0.1705	0.5	0.071	0.071	0.4927

Table 5: The Mann-Whitney nonparametric test on the optimal Stutzer Index for a series of subperiods with null hypothesis: the median of GA-SQP is equal to the median of S-GA-SQP

	3-month			6-month		
No. of assets	GA-SQP	S-GA-SQP	p-value	GA-SQP	S-GA-SQP	p-value
10	0.0392	0.0392	1.0000	0.0202	0.0202	0.9927
20	0.0941	0.0941	1.0000	0.0351	0.0351	1.0000
30	0.1705	0.1705	1.0000	0.071	0.071	1.0000

hand, the divergence in the optimal value of θ is noticeable between the S-SQP and the other two approaches. With no significant variations in the weights between the three approaches, the smaller value of θ in S-SQP ought to be the factor why its performance is inferior to the hybrid GA-SQP and the S-GA-SQP. This further demonstrated the importance of a good starting point in a standard optimizer, particularly in the problem that involved wider search space such as the value of θ ($-\infty < \theta < 0$) in the Stutzer Index optimization.

In the second part of the study, the corresponding optimal Stutzer Index of each subperiod is acquired via the S-SQP, hybrid GA-SQP and S-GA-SQP. Table 4 exhibits the hypothesis test summary obtained from the Mann-Whitney nonparametric test. Even though the results revealed that the hybrid GA-SQPfaintly better than the S-SQP, the superiority is not statistically significant in smaller number of observations. However, the outperformance of the hybrid GA-SQP over the S-SQP becomes perceptible when the number of observations under consideration increased. This can be evident from the decreases in the p-value shown in the Table 4, when the range of the subperiod widen from 3 to 6 month. The factual deficiency of SQP in solving large-scale problem is verifiable by the inbuilt weakness of standard SQP algorithm, which was pointed out as well in the research of Murray (1997). On the other hand, the hybrid model attested its potential to overcome such limitations and the prospect to solve optimization problems particularly in finance, business and economics, which often have enormous number of data.

As for the comparative study between the hybrid GA-SQP and the S-GA-SQP, as shown in Table 5, the Mann-Whitney nonparametric test disclosed no significant difference between the optimal indices regardless of the number of observations. These outcomes further verified the sufficiency of the hybrid GA-SQP in optimizing the Stutzer Index by using random starting point and the capability of GA in conducting global search.

CONCLUSION

This study used a hybrid model that is incorporated by two well-recognized optimization methods that complement each other perfectly: the genetic algorithm and the sequential quadratic programming, to improve the optimization of the Stutzer Index. The results revealed that the hybrid model successfully enhanced the efficiency and the performance of the Stutzer Index optimization, in comparison to the standard sequential quadratic programming that employed the initial guess suggested by Stutzer (2000). The findings also attested that the hybrid model does not required any user-defined starting point as it can efficiently attained the optimal index by utilizing a randomly generated starting point, i.e., consumed less time. Even though the improvement is not statistically significant in smaller number of observations, the hybrid model, in general, demonstrated competency in improving the efficiency and the performance of the Stutzer Index optimization.

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