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A Comparative Study on Time-cost Trade-off Approaches within Critical Path Method

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Abstract: In this study we present two approaches of time-cost trade-off to complete the project within T (the shortest possible duration to complete the project at least cost within the maximum available budget). The aim of this study is to discuss a comparison between the approach of Crashing Critical Activities (CCA) and the approach of Stretching Noncritical Activities (SNA) to illustrate the gauge of which approach should be used to complete the project within T in the least number of steps. Based on the five projects of varying levels and difficulties we can observe the obvious difference between the number of steps required to complete each project within T by CCA and the number of steps required to complete each project within T by SNA, i.e., SNA could complete the project in the least number of steps than CCA.

Key words: Crashing conditions, stretching conditions, optimum scheduling, faster and easier approach

INTRODUCTION

Time cost trade off problem is one of the highly important issues in project accomplishment and has been ever taken into consideration by project managers. Trade-off between project duration and total cost are extensively discussed in the project scheduling literature because of its practical relevance. It is generally realized that when project duration is compressed, the project will call for an increase in labor and more productive equipment and require more demanding procurement and construction management and then the cost will increase (Abbasnia *et al*., 2008).

The objective of CPM is to establish a feasible and desirable relationship between the time and cost of the project by reducing the target time and taking into account the cost of expediting (Nicholas, 2004; O'Brien and Plotnick, 2006; Taha, 2007). Several studies conducted on CPM for time-cost trade-off. Anagnostopoulos (2002) discussed the effective procedure for time-cost trade off in CPM networks when discrete time-cost combinations are allowed on the project activities is developed. Ipsilandis (2006) supposed that CPM focuses mostly on project's duration and critical activities, while RSM focuses on resource continuity. Ke *et al*. (2009) claimed in their study that in real-life projects, both the trade-off between the project cost and the project completion time, and the uncertainty of the environment are considerable aspects for decision-makers.

In this study we present two approaches for time-cost trade-off to complete the project in shortest possible duration at least cost within maximum available budget. The first approach is crashing critical activities (CCA) was originally developed along with the CPM for planning and controlling large scale projects. Crashing in critical path method means selecting the lowest cost slope activity or activities, which will shorten the critical path (s). This procedure is repeated until the project has been reduced sufficiently or the cost to shorten the project exceeds the benefits to the derived. The second approach is stretching non critical activities (SNA) which at first crashing all activities in the project simultaneously to directly find the shortest possible project duration and avoid the intermediate steps. So it is possible to reduce cost of the project by stretch or increase any noncritical activity by a certain amount without extending the project. In fact, the noncritical activities can be stretched until all the slack in the network is used up.

APPROACH OF STRECHING NONCRITICAL ACTIVITIES (SNA)

Step 1: Draw the project network

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Step 2: Determine the normal time and normal cost for each activity to determine the critical and noncritical activities
Step 3: Compute the normal total cost and normal duration of the project completion. If F equals the normal duration of completion then we stop the procedure
Step 4: Determine the crash time and crash cost for each activity to compute the cost slope
Step 5: Crash all activities in the project simultaneously
Step 6: Draw the project network after crashing all activities
Step 7: Determine the critical path and noncritical paths Also, identify the critical activities
Step 8: Compute the new total cost after crashing all activities in the project simultaneously
Step 9: Stretch noncritical activities: Start with those noncritical activities that will yield the greatest savings-those with the greatest cost slope. The noncritical activities can be stretched up to their normal time until all the slack in the different noncritical paths network is used up and then the saving cost of stretching all noncritical activities is found
Step 10: The final project cost of project completion within T is computed by subtracting the saving cost of stretching all noncritical activities from the cost of crashing all activities

PRACTICAL EXAMPLES

This study has been applied on five construction projects (Table 1) where data were taken from the 7 Nissan General Company in Iraq, the company is specialized in designing and execution of Bridges, Housing complexes, Concrete towers, Commercial and Industrial Buildings, Seeds Silos, Water and Water Treatment, Plants, Dams, …, etc.

The company accepted to construct new projects. The maximum budgeting available for the client is \$B. The client desires to complete the building within T.

To establish and confirm the results of this study, we will take all the five projects. But for the sake of brevity and non-repetition we will choose Project 4 to clarify some of the exact details, the results of the other projects will be presented as well.

Numerical data of project 1: The data of Project 1 is summarized in Table 2 in which there are 11 activities, where: $A_i = (A_1, A_2, \ldots, A_{11})$, B = \$78,500, T= 17, F= 18.

Numerical data of project 2: The data of Project 2 is summarized in Table 3 in which there 18 activities, where: $A_i = (A_1, A_2, \ldots, A_{18})$, \$B = \$246,000, T= 45, F= 50.

Numerical data of project 3: The data of Project 3 is summarized in Table 4 in which there 18 activities, where: $A_i = (A_1, A_2, \ldots, A_{18})$, B = \$216,000, T= 24, F= 26.

Numerical data of project 4: The data of Project 4 is summarized in Table 5 in which there are 23 activities, where A_i= (A, B, C,…, W), \$B = \$6,830,000, T= 46, F= 54.

Numerical data of project 5: The data of Project 5 is summarized in Table 6 in which there are 36 activities, where: $A_i = (A_1, A_2, \ldots, A_{36})$, B = \$462,000, T = 69, F = 78.

Definition of terms:

CCA	:	Approach of Crashing Critical Activities
SNA	:	Approach of Stretching Noncritical Activities
A_i	:	Project's activities, where, i = (1, 2, 3,…, n)
B	:	Maximum available budget
T	:	Shortest possible duration to complete the project at least cost within the maximum available budget
F	:	The desired project completion time at least cost
$T_{N,I}$	:	Normal time for activity i
$T_{C,I}$	:	Crash time for activity i
$C_{N,I}$	:	Normal cost for activity i
$C_{C,I}$	:	Crash cost for activity i
U_i	:	Cost slope for activity i
CP	:	Critical path (longest path in the project network)
$D_{N,q}$	:	Normal time for critical activity q, where q = (1, 2, 3,…, L)
$D_{C,q}$	:	Crash time for critical activity q
$C_{N,j}$	:	Normal cost for noncritical activity j, where j = (1, 2, 3,…, m)
$C_{C,q}$	:	Crash cost for critical activity q
U_S	:	Cost slope for critical activity S, where S begin with critical activity that has the smallest cost slope = (1, 2, 3,…, Y)
U_x	:	Cost slope for noncritical activity x, where x begin with noncritical activity that has the biggest cost slope = (1, 2, 3,…, z)
$D_{r,S}$	:	Max reduction in duration for critical activity S
$D_{r,x}$	:	Max stretching in duration for noncritical activity x
$T.C_N$	:	Total cost to complete the project in normal condition
$T.C_a$	:	Total cost to complete the project by crashing all activities (crash condition)
$T.C_C$	:	Total cost to complete the project by CCA
$T.E_C$	:	Extra cost that adding to crash critical activities

Table 1: Data of the five construction projects

Project name	Project type	Default start date for the project	The expected date of completion of the project in normal conditions
1 (Al-Saidia)	House construction	1-1-2011	30 weeks
2 (Al-Badia)	Plant construction	20-1-2011	27 weeks
3 (Al-Bounok)	House construction	10-2-2011	29 weeks
4 (Al-Salam)	Plant construction	1-1-2011	80 weeks
5 (Al-Karama)	Plant construction	11-3-2011	70 weeks

Table 2: Activities data in normal and crash conditions for Project 1

Activities code	Activity predecessor	Normal time (T_n)	Crash time (T_c)	Normal cost (C_n)	Crash cost (C_c)	Max reduction in time	Cost slope
A_1	---	3	2	5,000	7,000	1	2,000
A_2	A_1	4	2	4,000	5,000	2	500
A_3	A_2	4	4	7,000	7,000	0	---
A_4	A_2	3	1	3,000	5,000	2	1,000
A_5	A_2	5	2	6,000	10,500	3	1,500
A_6	A_5 , A_3	4	3	8,000	10,000	1	2,000
A_7	A_4	3	1	4,000	5,500	2	750
A_8	A_7	6	4	6,000	9,000	2	1,500
A_9	A_6	7	4	5,000	8,000	3	1,000
A_{10}	A_8, A_9	4	2	6,000	7,500	2	750
A_{11}	A_3, A_5	9	7	3,000	4,000	2	500
Total cost				$57,000	$78,500		

Table 3: Activities data in normal and crash conditions for Project 2

Activity code	Activity predecessor	Normal time (T_n)	Crash times (T_c)	Normal cost (C_n)	Crash cost (C_c)	Max reduction in time	Cost slope
A_1	---	10	8	20,000	28,000	2	4,000
A_2	---	3	2	8,000	10,000	1	2,000
A_3	A_1	5	3	10,000	15,000	2	2,500
A_4	A_3	5	3	8,500	12,000	2	1,750
A_5	A_2	4	3	4,000	5,000	1	1,000
A_6	A_5	10	8	14,000	19,000	2	2,500
A_7	A_4	10	8	14,000	16,000	2	1,000
A_8	A_7	5	3	5,000	6,000	2	500
A_9	A_7	5	3	9,000	11,000	2	1,000
A_{10}	A_6	3	3	2,500	2,500	0	0
A_{11}	A_7	3	3	2,500	2,500	0	0
A_{12}	A_{10}	5	4	13,000	15,000	1	2,000
A_{13}	A_{11}	5	3	12,000	15,000	2	1,500
A_{14}	A_{13}	5	3	10,000	12,000	2	1,000
A_{15}	A_{12}	5	2	8,000	14,000	3	2,000
A_{16}	A_8,A_9,A_{14}	10	7	15,000	18,000	3	1,000
A_{17}	A_{15}	10	6	13,000	19,000	4	1,500
A_{18}	A_{16},A_{17}	10	7	18,000	26,000	3	2,666.66
Total cost				$186,500	$246,000		

$T.S.C_n$: Total saving cost to complete the project within T by SNA

$T.C.S_n$: Total cost to complete the project within T by SNA

$\theta_{CrashForT}$: Number of steps to meet F (considering F>T) or obtain T (considering F = T) by CCA

$\theta_{Stretcht}$: Number of steps to obtain T (considering F=T) by SNA

General descriptions and formulations:

- Critical path of the project network in normal condition:

$$C.P._k = \sum_{q=1}^{L} D_{N,q} \quad (1)$$

- Total cost of the project in normal condition:

$$T.C_N = \sum_{i=1}^{n} C_{N,2} \quad (2)$$

- Total cost of the project by crashing all activities:

$$T.C_a = \sum_{i=1}^{n} C_{C,i} \quad (3)$$

- Critical path of the project network by crashing all activities:

$$C.P._c = \sum_{q=1}^{L} D_{C,q} \quad (4)$$

Table 4: Activities data in normal and crash conditions for Project 3

Activity code	Activity predecessor	Normal time (T_n)	Crash times (T_c)	Normal cost (C_n)	Crash cost (C_c)	Max reduction in time	Cost slope
A_1	---	3	2	2,500	3,500	1	1,000
A_2	A_1	2	1	4,000	6,000	1	2,000
A_3	A_2	2	1	6,000	9,000	1	3,000
A_4	A_3	15	10	25,000	35,000	5	2,000
A_5	A_3	4	2	3,000	5,000	2	1,000
A_6	A_3	10	6	40,000	60,000	4	5,000
A_7	A_3	1	1	5,000	5,000	0	0
A_8	A_3	6	3	7,500	14,000	3	2,166.66
A_9	A_4	2	2	5,000	5,000	0	0
A_{10}	A_5	10	7	8,000	10,000	3	666.66
A_{11}	A_6	5	5	6,000	6,000	0	0
A_{12}	A_7	5	3	4,000	5,000	2	500
A_{13}	A_9	8	6	12,000	14,000	2	1,000
A_{14}	A_{10}	10	6	10,000	16,000	4	1,500
A_{15}	A_{11},A_{12}	3	3	6,500	6,500	0	0
A_{16}	A_8,A_{15}	5	4	7,000	8,000	1	1,000
A_{17}	A_{13},A_{14}	2	1	4,000	5,000	1	1,000
A_{18}	A_{16},A_{17}	1	1	3,000	3,000	0	0
Total cost				\$158,500	\$216,000		

Table 5: Activity data in normal and crash conditions for Project 4

Activity code	Activity predecessor	Normal time (T_n)	Crash time (T_c)	Normal cost (C_n)	Crash cost (C_c)	Max reduction in time	Cost slope
A		2	1	20,000	30,000	1	10,000
B	A	3	1	60,000	100,000	2	20,000
C	B	2	1	30,000	40,000	1	10,000
D	C	2	1	20,000	30,000	1	10,000
E	D	4	2	100,000	150,000	2	25,000
F	B	3	2	150,000	180,000	1	30,000
G	E, F	2	1	200,000	300,000	1	100,000
H	G	10	7	500,000	620,000	3	40,000
I	H	15	10	650,000	850,000	5	40,000
J	H	7	5	250,000	300,000	2	25,000
K	I	2	1	20,000	25,000	1	5,000
L	J, K	9	6	300,000	420,000	3	40,000
M	H	2	1	20,000	25,000	1	5,000
N	H	3	2	30,000	40,000	1	10,000
O	H	6	4	120,000	150,000	2	15,000
P	I	7	4	450,000	570,000	3	40,000
Q	L, M	4	2	350,000	500,000	2	75,000
R	N, Q	6	3	550,000	760,000	3	70,000
S	O, R	7	5	450,000	600,000	2	75,000
T	J, P	5	3	350,000	450,000	2	50,000
U	S	5	3	250,000	320,000	2	35,000
V	T	4	2	150,000	220,000	2	35,000
W	U, V	4	2	100,000	150,000	2	25,000
Total cost				\$5,120,000	\$6,830, 000		

- Total extra cost that adding to crash critical activities:

$$TE_c = \sum_{s=1}^{Y} D_{r,s}.U_s \tag{5}$$

- Total cost of the project by CCA:

$$T.C_C = T.C_N + T.C_E \tag{6}$$

- Total saving cost obtained by SNA:

$$T.S.C_n = \sum_{x=1}^{Z} D_{r,x}.U_x \tag{7}$$

- Total cost to complete the project within T by SNA:

$$T.C.S_n = T.C_a - T.S.C_n \tag{8}$$

- Number of steps to meet F (considering F>T) or complete the project within T (considering F = T) by CCA:

$$\theta_{Crash\,Fort} = \sum_{s=1}^{Y} D_{r,s} \tag{9}$$

- Number of steps to complete the project within T (considering F = T) by SNA:

Table 6: Activities data in normal and crash conditions for Project 5

Activity code	Activity predecessor	Normal time (T_n)	Crash times (T_c)	Normal cost (C_n)	Crash cost (C_c)	Max reduction in time	Cost slope
A_1		3	2	2,500	3,500	1	1,000
A_2	A_1	2	1	4,000	6,000	1	2,000
A_3	A_2	2	1	6,000	9,000	1	3,000
A_4	A_3	15	10	25,000	35,000	5	2,000
A_5	A_3	4	2	3,000	5,000	2	1,000
A_6	A_3	10	6	40,000	60,000	4	5,000
A_7	A_3	1	1	5,000	5,000	0	0
A_8	A_3	6	3	7,500	14,000	3	2,166.66
A_9	A_4	2	2	5,000	5,000	0	0
A_{10}	A_5	10	7	8,000	10,000	3	666.66
A_{11}	A_6	5	5	6,000	6,000	0	0
A_{12}	A_7	5	3	4,000	5,000	2	500
A_{13}	A_9	8	6	12,000	14,000	2	1,000
A_{14}	A_{10}	10	6	10,000	16,000	4	1,500
A_{15}	A_{11},A_{12}	3	3	6,500	6,500	0	0
A_{16}	A_8,A_{15}	5	4	7,000	8,000	1	1,000
A_{17}	A_{13},A_{14}	2	1	4,000	5,000	1	1,000
A_{18}	A_{16},A_{17}	1	1	3,000	3,000	0	0
A_{19}	A_{18}	10	8	20,000	28,000	2	4,000
A_{20}	A_{18}	3	2	8,000	10,000	1	2,000
A_{21}	A_{19}	5	3	10,000	15,000	2	2,500
A_{22}	A_{21}	5	3	8,500	12,000	2	1,750
A_{23}	A_{20}	4	3	4,000	5,000	1	1,000
A_{24}	A_{23}	10	8	14,000	19,000	2	2,500
A_{25}	A_{22}	10	8	14,000	16,000	2	1,000
A_{26}	A_{25}	5	3	5,000	6,000	2	500
A_{27}	A_{25}	5	3	9,000	11,000	2	1,000
A_{28}	A_{24}	3	3	2,500	2,500	0	0
A_{29}	A_{25}	3	3	2,500	2,500	0	0
A_{30}	A_{28}	5	4	13,000	15,000	1	2,000
A_{31}	A_{29}	5	3	12,000	15,000	2	1,500
A_{32}	A_{31}	5	3	10,000	12,000	2	1,000
A_{33}	A_{30}	5	2	8,000	14,000	3	2,000
A_{34}	A_{26},A_{27},A_{32}	10	7	15,000	18,000	3	1,000
A_{35}	A_{33}	10	6	13,000	19,000	4	1,500
A_{36}	A_{34},A_{35}	10	7	18,000	26,000	3	2,666.66
Total cost				$345,000	$462,000		

$$\theta_{Stretcht} = \sum_{x=1}^{z} D_{r,x} \tag{10}$$

RESULTS AND DISCUSSION

Finding normal duration and cost to complete the project: We can find the critical path and total normal cost for the five projects from Eq. 1 and 2, respectively. So the costs and times to complete the projects 1-5 under normal conditions are ($57,000, 27 weeks), ($186,500, 63 weeks), ($158,500, 35 weeks), ($5,120,000, 77 weeks) and ($345,000, 98 weeks) respectively. Note that the clients need their projects within T.

Reducing project duration by CCA mechanism: The objective of the project crashing in CPM is to find which activities should be crashed with the use of additional resources if the duration of the project must be shortened. The cost-slope concept can be used to determine the most efficient way of shortening the project. We begin by assuming that all activities are done under a normal pace, therefore the project is completed in 77 weeks at an expense of $5,120,000. Suppose that we want to shorten the project duration or the critical path (77 weeks). The way to shorten the project is to simply shorten any critical activities A, B, C,…,U, W. Reducing an activity's duration increases its cost, but because the reduction can be made anywhere on the critical path, the cost increase is minimized by selecting the activity with the smallest cost slope. Thus, Activity K is selected because it has the smallest cost slope. Reducing K by 1 week shortens the project duration to 76 weeks and adds $5,000 (the cost slope of K) to the project cost, thereby increasing the project cost to $5,120,000 + $5,000 = $5,125,000. This step does not change the critical path because the critical path still is the longest (76 weeks) the last step reduces K to 1 week, its crash time, so no further reductions can be made to K if need be, an additional week can be cut from another activity on the critical path that has a smallest cost slope. The processes are summarized in Table 10. We can compute the total extra cost ($T.E_C$) to reduce the project duration to 46 weeks and the total

Table 7: Mechanism of CCA for Project 1

Critical activity that has a smallest cost slope respectively	Max reduction in duration	Smallest cost slope respectively	Length of path A_1 A_2 A_4 A_7 A_8 A_{10}	 A_1 A_2 A_5 A_6 A_9 A_{10}	 A_1 A_2 A_3 A_{dum} A_6 A_9 A_{10}	 A_1 A_2 A_3 A_{DUM} A_{11}	 A_1 A_2 A_5 A_{11}	Total cost after adding extra cost
			23	27	26	20	21	57,000
A_2	1	500	22	26	25	19	20	57,500
A_2	1	500	21	25	24	18	19	58,000
A_{10}	1	750	20	24	23	18	19	58,750
A_{10}	1	750	19	23	22	18	19	59,500
A_9	1	1,000	19	22	21	18	19	60,500
A_9	1	1,000	19	21	20	18	19	61,500
A_9	1	1,000	19	20	19	18	19	62,500
A_5	1	1,500	19	19	19	18	18	64,000
A_1	1	2,000	18	18	18	17	17	66,000
A_6,	1,	2,000,						
A_7	1	750	17	17	17	17	17	68,750[1]

$T.E_C$ = \$11,750, θ_{crashT}^{1} =11 steps to complete the project within T

Table 8: Mechanism of CCA for Project 2

Critical activity that has a smallest slope respectivelyin	Max reduction duration	Smallest cost slope respectively	Length of path A_1 A_3 A_4 A_7 A_{11} A_{13} A_{14} A_{16} A_{18}	 A_1 A_3 A_4 A_7 A_8 A_{dum} A_{14} A_{16} A_{18}	 A_1 A_3 A_4 A_7 A_9 A_{16} A_{18}	 A_2 A_5 A_6 A_{10} A_{12} A_{15} A_{17} A_{18}	Total cost after adding extra cost
			63	60	55	50	186,500
A_{16}	1	1,000	62	59	54	50	187,500
A_{16}	1	1,000	61	58	53	50	188,500
A_{16}	1	1,000	60	57	52	50	189,500
A_{14}	1	1,000	59	56	52	50	190,500
A_{14}	1	1,000	58	55	52	50	191,500
A_7	1	1,000	57	54	51	50	192,500
A_7	1	1,000	56	53	50	50	193,500
A_{13}	1	1,500	55	53	50	50	195,000
A_{13}	1	1,500	54	53	50	50	196,500
A_4	1	1,750	53	52	49	50	198,250
A_4	1	1,750	52	51	48	50	200,000
A_3	1	2,500	51	50	47	50	202,500
A_3	1	2,500	50	49	46	50	205,000
A_{18}	1	2,666.66	49	48	45	49	207,667
A_{18}	1	2,666.66	48	47	44	48	210,333
A_{18}	1	2,666.66	47	46	43	47	213,000
A_1,	1,	4,000,					
A_5	1	1,000	46	45	43	46	218,000
A_1,	1,	4,000,					
A_{17}	1	1,500	45	44	42	45	223,500[1]

$T.E_C$ = \$37,000, θ_{crashT}^{1} = 20 steps to complete the project within T

cost of the project by CCA ($T.C_C$) from Eq. 5 and 6, respectively.

$$T.E_c = \sum_{s=k}^{G} D_{r,s}.U_s = D_{r,k}.U_k + D_{r,A}.U_A + \ldots + D_{r,s}.U_s + D_{r,G}.U_G$$

$$= \$1,295,000$$

The total extra cost ($T.E_C$) to reduce the duration of the projects 1-5 to 17, 45, 24, 46 and 69 weeks are (\$11,750), (\$37,000), (\$40,500), (\$1,295,000) and (\$77,500), respectively:

$$T.C_c = \sum_{i=A}^{W} C_{N,i} + \sum_{s=k}^{G} D_{r,s}.U_S$$

$=(C_{N,A} + C_{N,B} + \cdots + C_{N,V} + C_{N,W}) + D._{r,k} \cdot U_K + D_{r,A} \cdot U_A + \cdots + D_{r,S} \cdot U_S + D_{r,G} \cdot U_G) = (\$5,120,000) + (\$1,295,000) = \$6,415,000$

Table 9: Mechanism of CCA for Project 3

Critical activity that has a smallest slope respectively	Max reduction in duration	Smallest cost slope respectively	Length of path A_1 A_2 A_3 A_4 A_9 A_{13} A_{17} A_{18}	 A_1 A_2 A_3 A_5 A_{10} A_{14} A_{17} A_{18}	 A_1 A_2 A_3 A_6 A_{11} A_{15} A_{16} A_{18}	 A_1 A_2 A_3 A_7 A_{12} A_{15} A_{16} A_{18}	 A_1 A_2 A_3 A_8 A_{16} A_{18}	Total cost after adding extra cost
			35	34	31	22	19	158,500
A_1	1	1,000	34	33	30	21	18	159,500
A_2	1	2,000	33	32	29	20	17	161,500
A_3	1	3,000	32	31	28	19	18	164,500
A_{13}	1	1,000	31	31	28	19	18	165,500
A_{17}	1	1,000	30	30	28	19	18	166,500
A_{13},	1,	1,000,						
A_{10}	1	667	29	29	28	19	18	168,167
A_4,	1,	2,000,						
A_{10}	1	667	28	28	28	19	18	170,833
A_4,	1,	2,000,						
A_{10},	1,	667,	27	27	27	18	17	174,500
A_{16}	1	1,000						
A_4,	1,	2,000,						
A_5,	1,	1,000,						
A_6	1	5,000	26	26	26	18	17	182,500
A_4,	1,	2,000,						
A_5,	1,	1,000,						
A_6	1	5,000	25	25	25	18	17	190,500
A_4,	1,	2,000,						
A_{14},	1,	1,500,						
A_6	1	5,000	24	24	24	18	17	199,000[1]

$T.E_C$ = \$40,500, θ_{crashT}[1] = 21 steps to complete the project within T

The total cost ($T.C_C$) of the projects 1-5 by CCA are (\$68,750), (\$223,500), (\$199,000), (\$6,415,000) and (\$422,500), respectively.

The number of steps (θ_{CrashT}) to complete the Project 4 within T by CCA can be computed from 'Eq (9)' as follows (Table 10):

$$\theta_{Crasht4} = \sum_{s=A}^{w} D_{r,s} = D_{r,k} + D_{r,A} + \ldots + D_{r,s} + D_{r,G} = 31 \text{ (Weeks)}$$

In the same way we can compute the number of steps (θ_{CrashT}) to complete the projects 1-5 within T by CCA. So, θ_{Crash1}= 11, θ_{Crash2}= 20, θ_{Crash3}= 21, θ_{Crash4}= 31 and θ_{Crash5}= 41 (Tables 7-11).

Project completion by SNA mechanism: As a result of crashing all activities simultaneously yields the project duration of 46 weeks and the expense of crashing all the activities is \$6,830,000 an artificially high amount because, as would shown, it is not necessary to crash every activity to finish the projec t in the shortest time.

From Eq. 3 and 4, we can find the total cost and the projects completion time (C.P) by considering crashing all activities.

So, the total cost and time to complete projects 1-5 under crashing conditions are (\$78,500, 17 weeks), (\$246,000, 45 weeks), (\$216,000, 24 weeks), (\$6,830,000, 46 weeks) and (\$462,000, 69 weeks), respectively.

We can also find the total saving cost by SNA ($T.S.C_n$) without extending the project duration from Eq. 7.

So, the total saving cost by SNA ($T.S.C_n$) for the projects 1-5 are (\$9,750), (\$22,500), (\$17,000), (\$415,000) and (\$39,500), respectively.

From Eq. 8, we can compute the total cost to complete the project within T by SNA ($T.C.S_n$). So, the total cost to complete the projects 1-5 are (\$68,750), (\$223,500), (\$199,000), (\$6,415,000) and (\$422,500), respectively.

We can compute the number of steps ($\theta_{StretchT}$) to complete the Project 4 within T by *SNA* from 'Eq (10)' (Table 15, Column 2).

$$\theta_{Stretcht} = \sum_{x=T}^{M} D_{r,x} + D_{r,p} + \ldots + D_{r,M} = 8 \text{ (steps)}$$

In the same way we can compute the number of steps to complete the projects 1-5 within T by *SNA* ($\theta_{StretchT}$). So, $\theta_{StretchT1}$= 5, $\theta_{StretchT2}$= 7, $\theta_{StretchT3}$= 4, $\theta_{StretchT4}$= 8 and $\theta_{StretchT5}$= 11 respectively (Tables 12-16).

Comparison between CCA and SNA: CCA as discussed thus far determines, step-by-step, which activities to speed up so as to reduce the project completion

Table 10: Mechanism of CCA for project 4

Critical activity that has a smallest slope respectively	Max reduction in duration	Smallest cost slope respectively	Length of path: ABCDEGHOSUW	ABCDEGHNRSUW	ABCDEGHMQRSUW	ACDEGHIPTVW	ABCDEGHIKLQRSUW	ABCDEGHJLQRSUW	ABCDEGHJTVW	ABFGHOSUW	ABFGHNRSUW	ABFGHMQRSUW	ABFGHIPTVW	ABFGHIKLQRSUW	ABFGHJLQRSUW	ABFGHJTVW	Total cost after adding extra cost
			43	50	53	60	77	67	45	38	45	48	55	72	62	40	5,120,000
K	1	5,000	43	50	53	60	76	67	45	38	45	48	55	71	62	40	5,125,000
A	1	10,000	42	49	52	60	75	67	44	37	44	47	54	70	61	39	5,135,000
C	1	10,000	41	48	51	59	74	66	43	37	44	47	54	70	61	39	5,145,000
D	1	10,000	40	47	50	58	73	65	42	37	44	47	54	70	61	39	5,155,000
B	1	20,000	39	46	49	57	72	64	41	36	43	46	53	69	60	29	5,175,000
B	1	20,000	38	45	48	56	71	63	40	35	42	45	52	68	59	28	5,195,000
E	1	25,000	37	44	47	55	70	62	39	35	42	45	52	68	59	28	5,220,000
E	1	25,000	36	43	46	54	69	61	38	35	42	45	52	68	59	28	5,245,000
W	1	25,000	35	42	45	53	68	60	37	34	41	44	51	67	58	27	5,270,000
W	1	25,000	34	41	44	52	67	59	36	33	40	43	50	66	57	26	5,295,000
U	1	35,000	33	40	43	52	66	58	36	32	39	42	50	65	56	26	5,330,000
U	1	35,000	32	39	42	52	65	57	36	31	38	41	50	64	55	26	5,365,000
I	1	40,000	32	39	42	51	64	57	36	31	38	41	49	63	55	26	5,405,000
I	1	40,000	32	39	42	50	63	57	36	31	38	41	48	62	55	26	5,445,000
I	1	40,000	32	39	42	49	62	57	36	31	38	41	47	61	55	26	5,485,000
I	1	40,000	32	39	42	48	61	57	36	31	38	41	46	60	55	26	5,525,000
I	1	40,000	32	39	42	47	60	57	36	31	38	41	45	59	55	26	5,565,000
L	1	40,000	32	39	42	47	59	56	36	31	38	41	45	58	54	26	5,605,000
L	1	40,000	32	39	42	47	58	55	36	31	38	41	45	57	53	26	5,645,000
L	1	40,000	32	39	42	47	57	54	36	31	38	41	45	56	52	26	5,685,000
H	1	40,000	31	38	41	46	56	53	35	30	37	40	44	55	51	25	5,725,000
H	1	40,000	30	37	40	45	55	52	34	29	36	39	43	54	50	24	5,765,000
H	1	40,000	29	36	39	44	54	51	33	28	35	38	42	53	49	23	5,805,000
R	1	70,000	29	35	38	44	53	50	33	28	34	37	42	52	48	23	5,875,000
R	1	70,000	29	34	37	44	52	49	33	28	33	36	42	51	47	23	5,945,000
R	1	70,000	29	33	36	44	51	48	33	28	32	35	42	50	46	23	6,015,000
Q	1	75,000	29	33	35	44	50	47	33	28	32	34	42	49	45	23	6,090,000
Q	1	75,000	29	33	34	44	49	46	33	28	32	33	42	48	44	23	6,165,000
S	1	75,000	28	32	33	44	48	45	33	27	31	32	42	47	43	23	6,240,000
S	1	75,000	27	31	32	44	47	44	33	26	30	31	42	46	42	23	6,315,000
G	1	100,000	26	30	31	43	46	43	32	25	29	30	41	45	41	22	6,415,000[1]

$T.E_C$ = \$1,295,000, $\theta_{crashT}{}^{1}$ = 31 steps to complete the project within T

time. This stepwise reduction of the project duration eventually leads to the shortest possible project duration and its associated cost. However, if we want to directly find the shortest possible project duration and avoid the intermediate steps, a simpler procedure represent with SNA which supposed to simultaneously crash all activities at once. This also yields the shortest project duration. However, the expense of crashing all activities is an artificially high amount because, it is not necessary to crash every activity to finish the project in the shortest time. So we must stretch noncritical activities to reduce the cost of the project completion.

Similarity between the two approaches:

- These approaches aim to reduce the project duration at the least total cost
- These approaches need to determine critical and noncritical activities

Table 11: Mechanism of *CCA* for Project 5

Critical activity that has smallest a slope respectively	Max reduction in duration	Smallest cost slope respectively	Length of path																				Total cost after adding extra cost
			A_{1}	A_{1}			A_{1}	A_{1}			A_{1}	A_{1}				A_{1}							
			A_{2}	A_{2}		A_{1}	A_{2}	A_{2}		A_{1}	A_{2}	A_{2}		A_{1}	A_{1}	A_{2}		A_{1}					
			A_{3}	A_{3}	A_{1}	A_{2}	A_{3}	A_{3}	A_{1}	A_{2}	A_{3}	A_{3}	A_{1}	A_{2}	A_{3}	A_{3}	A_{1}	A_{2}	A_{1}	A_{1}			
			A_{4}	A_{4}	A_{2}	A_{3}	A_{5}	A_{5}	A_{2}	A_{3}	A_{6}	A_{6}	A_{2}	A_{3}	A_{7}	A_{7}	A_{2}	A_{3}	A_{2}	A_{2}		A_{1}	
			A_{9}	A_{9}	A_{3}	A_{4}	A_{10}	A_{10}	A_{3}	A_{5}	A_{11}	A_{11}	A_{3}	A_{6}	A_{12}	A_{12}	A_{3}	A_{7}	A_{3}	A_{3}	A_{1}	A_{2}	
			A_{13}	A_{13}	A_{4}	A_{9}	A_{14}	A_{14}	A_{5}	A_{10}	A_{15}	A_{15}	A_{6}	A_{11}	A_{15}	A_{15}	A_{7}	A_{12}	A_{8}	A_{8}	A_{2}	A_{3}	
			A_{17}	A_{17}	A_{9}	A_{13}	A_{17}	A_{17}	A_{10}	A_{14}	A_{16}	A_{16}	A_{11}	A_{15}	A_{16}	A_{16}	A_{12}	A_{15}	A_{16}	A_{16}	A_{3}	A_{8}	
			A_{18}	A_{18}	A_{13}	A_{17}	A_{18}	A_{18}	A_{14}	A_{17}	A_{18}	A_{18}	A_{15}	A_{16}	A_{18}	A_{18}	A_{15}	A_{16}	A_{18}	A_{18}	A_{8}	A_{16}	
			A_{19}	A_{19}	A_{17}	A_{18}	A_{19}	A_{19}	A_{17}	A_{18}	A_{19}	A_{19}	A_{16}	A_{18}	A_{19}	A_{19}	A_{16}	A_{18}	A_{19}	A_{19}	A_{16}	A_{18}	
			A_{21}	A_{21}	A_{18}	A_{20}	A_{21}	A_{21}	A_{18}	A_{20}	A_{21}	A_{21}	A_{18}	A_{20}	A_{21}	A_{21}	A_{18}	A_{20}	A_{21}	A_{21}	A_{18}	A_{20}	
			A_{22}	A_{22}	A_{19}	A_{23}	A_{22}	A_{22}	A_{19}	A_{23}	A_{22}	A_{22}	A_{19}	A_{23}	A_{22}	A_{22}	A_{19}	A_{23}	A_{22}	A_{22}	A_{19}	A_{23}	
			A_{25}	A_{25}	A_{21}	A_{24}	A_{25}	A_{25}	A_{21}	A_{24}	A_{25}	A_{25}	A_{21}	A_{24}	A_{25}	A_{25}	A_{21}	A_{24}	A_{25}	A_{25}	A_{21}	A_{24}	
			A_{29}	A_{26}	A_{22}	A_{28}	A_{29}	A_{26}	A_{22}	A_{28}	A_{29}	A_{26}	A_{22}	A_{28}	A_{29}	A_{26}	A_{22}	A_{28}	A_{29}	A_{26}	A_{22}	A_{28}	
			A_{31}	A_{dum}	A_{25}	A_{30}	A_{31}	A_{dum}	A_{25}	A_{30}	A_{31}	A_{dum}	A_{25}	A_{30}	A_{31}	A_{dum}	A_{25}	A_{30}	A_{31}	A_{dum}	A_{25}	A_{30}	
			A_{32}	A_{32}	A_{27}	A_{33}	A_{32}	A_{32}	A_{27}	A_{33}	A_{32}	A_{32}	A_{27}	A_{33}	A_{32}	A_{32}	A_{27}	A_{33}	A_{32}	A_{32}	A_{27}	A_{33}	
			A_{34}	A_{34}	A_{34}	A_{35}	A_{34}	A_{34}	A_{34}	A_{35}	A_{34}	A_{34}	A_{34}	A_{35}	A_{34}	A_{34}	A_{34}	A_{35}	A_{34}	A_{34}	A_{34}	A_{35}	
			A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	A_{36}	
			98	95	90	85	97	94	89	84	94	91	86	81	85	82	77	72	82	79	74	69	345,000
A_{1}	1	1,000	97	94	89	84	96	93	88	83	93	90	85	80	84	81	76	71	81	78	73	68	346,000
A_{17}	1	1,000	96	93	88	83	95	92	87	82	93	90	85	80	84	81	76	71	81	78	73	68	347,000
A_{34}	1	1,000	95	92	87	83	94	91	86	82	92	89	84	80	83	80	75	71	80	77	72	68	348,000
A_{34}	1	1,000	94	91	86	83	93	90	85	82	91	88	83	80	82	79	74	71	79	76	71	68	349,000
A_{34}	1	1,000	93	90	85	83	92	89	84	82	90	87	82	80	81	78	73	71	78	75	70	68	350,000
A_{32}	1	1,000	92	89	85	83	91	88	84	82	89	86	82	80	80	77	73	71	77	74	70	68	351,000
A_{32}	1	1,000	91	88	85	83	90	87	84	82	88	85	82	80	79	76	73	71	76	73	70	68	352,000
A_{25}	1	1,000	90	87	84	83	89	86	83	82	87	84	81	80	78	75	72	71	75	72	69	68	353,000
A_{25}	1	1,000	89	86	83	83	88	85	82	82	86	83	80	80	77	74	71	71	74	71	68	68	354,000
A_{13}	1	1,000	88	85	82	82	88	85	82	82	86	83	80	80	77	74	71	71	74	71	68	68	355,000
A_{31}	1	1,500	87	85	82	82	87	85	82	82	85	83	80	80	76	74	71	71	73	71	68	68	356,500
A_{31}	1	1,500	86	85	82	82	86	85	82	82	84	83	80	80	75	74	71	71	72	71	68	68	358,000
A_{13}	1	1,000,	85	84	81	81	85	84	81	81	84	83	80	80	75	74	71	71	72	71	68	68	359,667
A_{10}	1	666.7																					
A_{22}	1	1,750	84	83	80	81	84	83	80	81	83	82	79	80	74	73	70	71	72	70	67	68	361,417
A_{22}	1	1,750	83	82	79	81	83	82	79	81	82	81	78	80	73	72	69	71	71	69	66	68	363,166
A_{2}	1	2,000	82	81	78	80	82	81	78	80	81	80	77	79	72	71	68	70	70	68	65	67	365,166
A_{21}	1	2,500	81	80	77	80	81	80	77	80	80	79	76	79	71	70	67	70	68	67	64	67	367,666
A_{21}	1	2,500	80	79	76	80	80	79	76	80	79	78	75	79	70	69	66	70	67	66	63	67	370,166
A_{4},	1	2,000,	79	78	75	79	79	78	75	79	79	78	75	79	70	69	66	70	67	66	63	67	372,833
A_{10}	1	666.7																					
A_{36}	1	2,667	78	77	74	78	78	77	74	78	78	77	74	78	69	68	65	69	66	65	62	66	375,500
A_{36}	1	2,667	77	76	73	77	77	76	73	77	77	76	73	77	68	67	64	68	65	64	61	65	378,167
A_{36}	1	2,667	76	75	72	76	76	75	72	76	76	75	72	76	67	66	63	67	64	63	60	64	380,834
A_{3}	1	3,000	75	74	71	75	75	74	71	75	75	74	71	75	66	65	62	66	63	64	59	63	383,834
A_{10}	1	666.7,	74	73	70	74	74	73	70	74	74	73	70	74	65	64	61	65	62	63	58	62	387,500
A_{4}	1	2,000,																					
A_{16}	1	1,000																					
A_{19}	1	4,000,	73	72	69	73	73	72	69	73	73	72	69	73	64	63	60	64	61	62	57	61	392,500
A_{23}	1	1,000																					
A_{19}	1	4,000,	72	71	68	72	72	71	68	72	72	71	68	72	63	62	59	63	60	61	56	60	398 ,000
A_{35}	1	1,500																					
A_{4}	1	2,000,	71	70	69	71	71	70	69	71	71	70	69	71	63	62	59	63	60	61	56	60	406,000
A_{5}	1	1,000,																					
A_{6}	1	5,000																					
A_{4},	1	2,000,	70	69	68	70	70	69	68	70	70	69	68	70	63	62	59	63	60	61	56	60	414, 000
A_{5},	1	1,000,																					
A_{6}	1	5,000																					
A_{4}	1	2,000,	69	68	67	69	69	68	67	69	69	68	67	69	63	62	59	63	60	61	56	60	422,500[1]
A_{14}	1	1,500,																					
A_{6}	1	5,000																					

$T.E_{C}$ = \$77,500, θ_{crashT}^{1} = 41 steps to complete the project within T

Table 12: Mechanism of SNA for Project 1

Noncritical activity that has a greatest slope respectively	Max increasing in time	Greatest cost slope respectively	Length of paths A_2 A_2 A_5 A_{11}	 A_1 A_2 A_3 A_{dum} A_{11}	 A_1 A_2 A_5 A_6 A_9 A_{10}	 A_1 A_3 A_4 A_7 A_8 A_{10}	 A_1 A_2 A_3 A_{dum} A_6 A_9 A_{10}	Total cost after subtracting saving cost
			13	15	15	12	17	78,500
A_5	2	1,500	15	15	17	12	17	75,500
A_8	2	1,500	15	15	17	14	17	72,500
A_4	2	1,000	15	15	17	16	17	70,500
A_7	1	750	15	15	17	17	17	69,750
A_{11}	2	500	17	17	17	17	17	68,750[1]

$T.S.C_n$= \$9,750, $\theta_{StretchT}{}^1$= 5 steps to complete the project within T

Table 13: Mechanism of SNA for Project 2

Noncritical activity that has a greatest slope respectively	Max increasing in time	Greatest cost slope respectively	Length of paths A_1 A_3 A_4 A_7 A_{11} A_{13} A_{14} A_{16} A_{18}	 A_1 A_3 A_4 A_7 A_8 A_{dum} A_{14} A_{16} A_{18}	 A_1 A_3 A_5 A_4 A_7 A_9 A_{16} A_{18}	 A_2 A_5 A_6 A_{10} A_{12} A_{15} A_{17} A_{18}	Total cost after subtracting saving cost
			45	42	39	35	246,000
A_6	2	2,500	45	42	39	37	241,000
A_{15}	3	2,000	45	42	39	40	235,000
A_2	1	2,000	45	42	39	41	233,000
A_{12}	1	2,000	45	42	39	42	231,000
A_{17}	3	1,500	45	42	39	45	226,500
A_9	2	1,000	45	42	41	45	224,500
A_8	2	500	45	44	41	45	223,500[1]

$T.S.C_n$= \$22,500, $\theta_{StretchT}{}^1$ = 7 steps to complete the project within T

Table 14: Mechanism of SNA for Project 3

Noncritical activity that has a greatest slope respectively	Max increasing in time	Greatest cost slope respectively	Length of paths A_1 A_2 A_3 A_4 A_9 A_{13} A_{17} A_{18}	 A_1 A_2 A_3 A_5 A_{10} A_{14} A_{17} A_{18}	 A_1 A_2 A_3 A_6 A_{11} A_{15} A_{16} A_{18}	 A_1 A_2 A_3 A_7 A_{12} A_{15} A_{16} A_{18}	 A_1 A_2 A_3 A_8 A_{16} A_{18}	Total cost after subtracting saving cost
			24	21	23	16	12	216,000
A_6	1	5,000	24	21	24	16	12	211,000
A_8	3	2,167	24	21	24	16	15	204,499
A_{14}	3	1,500	24	24	24	16	15	199,999
A_{12}	2	500	24	24	24	18	15	198,999[1]

$T.S.C_n$= \$17,000, $\theta_{StretchT}{}^1$= 4 steps to complete the project within T

- Normal time and cost as well as crash time and cost must be determined in order to find the cost slope for these approaches
- It is possible for these approaches to find T

Difference between the two approaches:

- CCA is the expression of gradually crashing the critical path activities to reduce a completion of project duration gradually too, while SNA depends to crash all activities simultaneously to yield the shortest possible duration of project completion then stretching noncritical activities to reduce the total cost
- CCA begins to crash the critical activity that has the least cost slope to reduce the project duration at least cost, while SNA depends on the stretching noncritical activity that has the greatest cost slope to obtain the greatest saving

Table 15: Mechanism of SNA for Project 4

Noncritical activity that has a greatest slope respectively	Max increasing in time	Greatest cost slope respectively	Length of path														Total cost after subtracting saving cost
			ABCDEGHOSUW	ABCDEGHNRSUW	ABCDEGHMQRSUW	ABCDEGHIPTVW	ABCDEGHIKLQRSUW	ABCDEGHJLQRSUW	AB C DE G H J T V W	AB F GH O SU W	AB F GH NR SU W	AB FG H M QR SU W	AB FG H I P TV W	AB FG H I KL QR SU W	AB FG H J LQ RS U W	AB FG H J TV W	
			28	29	30	35	46	40	26	26	27	28	33	44	38	24	6,830,000
T	2	50,000	28	29	30	37	46	40	28	26	27	28	35	44	38	26	6,730,000
P	3	40,000	28	29	30	40	46	40	28	26	27	28	38	44	38	26	6,610,000
V	2	35,000	28	29	30	42	46	40	30	26	27	28	40	44	38	28	6,540,000
F	1	30,000	28	29	30	42	46	40	30	27	28	29	41	45	39	29	6,510,000
J	2	25,000	28	29	30	42	46	42	32	27	28	29	41	45	41	31	6,460,000
O	2	15,000	30	29	30	42	46	42	32	29	28	29	41	45	41	31	6,430,000
N	1	10,000	30	30	30	42	46	42	32	29	29	29	41	45	41	31	6,420,000
M	1	5,000	30	30	31	42	46	42	32	29	29	30	41	45	41	31	6,415,000[1]

T.S.C_n= $415,000, $\theta_{StretchT}{}^{1}$ = 8 steps to complete the project within T

Table 16: Mechanism of SNA for project 5

Non critical activity that has a Greatest cost slope respectively	Max Increasing in duration	Greatest cost slope respectively	Length of path																				Total cost after subtracting saving cost
			$A_1 A_2 A_3 A_4 A_9 A_{13} A_{17} A_{18} A_{19} A_{21} A_{22} A_{25} A_{29} A_{31} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_4 A_9 A_{13} A_{17} A_{18} A_{19} A_{21} A_{22} A_{25} A_{26} A_{dum} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_4 A_9 A_{13} A_{17} A_{18} A_{19} A_{21} A_{22} A_{25} A_{27} A_{34} A_{36}$	$A_1 A_2 A_3 A_4 A_9 A_{13} A_{17} A_{18} A_{20} A_{23} A_{24} A_{28} A_{30} A_{33} A_{35} A_{36}$	$A_1 A_2 A_3 A_5 A_{10} A_{14} A_{17} A_{18} A_{19} A_{21} A_{22} A_{25} A_{29} A_{31} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_5 A_{10} A_{14} A_{17} A_{18} A_{19} A_{21} A_{22} A_{25} A_{26} A_{dum} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_5 A_{10} A_{14} A_{17} A_{18} A_{19} A_{21} A_{22} A_{25} A_{27} A_{34} A_{36}$	$A_1 A_2 A_3 A_5 A_{10} A_{14} A_{17} A_{18} A_{20} A_{23} A_{24} A_{28} A_{30} A_{33} A_{35} A_{36}$	$A_1 A_2 A_3 A_6 A_{11} A_{15} A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{29} A_{31} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_6 A_{11} A_{15} A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{26} A_{dum} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_6 A_{11} A_{15} A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{27} A_{34} A_{36}$	$A_1 A_2 A_3 A_6 A_{11} A_{15} A_{16} A_{18} A_{20} A_{23} A_{24} A_{28} A_{30} A_{33} A_{35} A_{36}$	$A_1 A_2 A_3 A_7 A_{12} A_{15} A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{29} A_{31} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_7 A_{12} A_{15} A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{26} A_{dum} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_7 A_{12} A_{15} A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{27} A_{34} A_{36}$	$A_1 A_2 A_3 A_7 A_{12} A_{15} A_{16} A_{18} A_{20} A_{23} A_{24} A_{28} A_{30} A_{33} A_{35} A_{36}$	$A_1 A_2 A_3 A_8 A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{29} A_{31} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_8 A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{26} A_{dum} A_{32} A_{34} A_{36}$	$A_1 A_2 A_3 A_8 A_{16} A_{18} A_{19} A_{21} A_{22} A_{25} A_{27} A_{34} A_{36}$	$A_1 A_2 A_3 A_8 A_{16} A_{18} A_{20} A_{23} A_{24} A_{28} A_{30} A_{33} A_{35} A_{36}$	
			69	66	63	59	66	63	60	56	68	65	62	58	61	58	55	51	57	54	51	47	462,000
A_6	1	5,000	69	66	63	59	66	63	60	56	69	66	63	59	61	58	55	51	57	54	51	47	457,000
A_{24}	2	2,500	69	66	63	61	66	63	60	58	69	66	63	61	61	58	55	53	57	54	51	49	452,000
A_8	3	2,167	69	66	63	61	66	63	60	58	69	66	63	61	61	58	55	53	60	57	54	52	445,500
A_{33}	3	2,000	69	66	63	64	66	63	60	61	69	66	63	64	61	58	55	56	60	57	54	55	439,500
A_{30}	1	2,000	69	66	63	65	66	63	60	62	69	66	63	65	61	58	55	57	60	57	54	56	437,500
A_{20}	1	2,000	69	66	63	66	66	63	60	63	69	66	63	66	61	58	55	58	60	57	54	57	435,500
A_{35}	3	1,500	69	66	63	69	66	63	60	66	69	66	63	69	61	58	55	61	60	57	54	60	431,000
A_{14}	3	1,500	69	66	63	69	69	66	63	69	69	66	63	69	61	58	55	61	60	57	54	60	426,500
A_{27}	2	1,000	69	66	65	69	69	66	65	69	69	66	65	69	61	58	57	61	60	57	56	60	424,500
A_{12}	2	500	69	66	65	69	69	66	65	69	69	66	65	69	63	60	59	63	60	57	56	60	423,500
A_{26}	2	500	69	68	65	69	69	68	65	69	69	68	65	69	63	62	59	63	60	59	56	60	422,500[1]

T.S.C_n= $39, 500, $\theta_{StretchT}{}^{1}$ = 11 steps to complete the project within T

Table 17: Comparison between θ_{CrashT} and $\theta_{StretchT}$

Projects	θ_{CrashT}	$\theta_{StretchT}$
1	11	5
2	20	7
3	21	4
4	31	8
5	41	11

- It is possible for CCA obtains F as well as T, while SNA obtains just the T
- We can illustrate the number of steps to complete the projects 1-5 within T by CCA (θ_{CrashT}) and the number of steps to complete the project within T by SNA ($\theta_{StretchT}$) from Table 17

DISCUSSION

CCA requires more computation then SNA specially when $\theta_{CrashT} > \theta_{StretchT}$, because CCA will be by virtue of crashing one time unit from critical activity that has smallest cost slope then we must be sure that this step does not change the nature of the problem (critical path is still the longest path in the network project), i.e., if all of the slack on another paths has been used up, the network two or more critical paths, any further reduction in has project duration must be made by shortening all of the paths. So if need to be, an additional time unit can be cut from the same activity unless it reaches to its crash time or transition to another critical activity has second smallest cost slope (if the previous activity reach to its crash time) to crash another time unit from the new activity etc. As we mentioned earlier, this procedure thus far determines, step-by-step, which activities to speed up so as to reduce the project completion time while SNA starts with all activities crashed, then stretch the noncritical activities with the greatest cost slope to use up available slack and obtain the greatest cost savings. An activity can be stretched up to its normal time, which is assumed to be its least-costly time. Therefore, we can consider this approach is faster and easier to complete the project in shortest possible duration at least cost within the maximum available budget specially when $\theta_{StretchT} < \theta_{CrashT}$.

CONCLUSION

The philosophy of time-cost trade-off via CCA and SNA has been presented in this paper. Based on the five projects of varying levels and difficulties and from Table 17, which shows the number of steps (θ_{CrashT}) required to complete each project within T by CCA and the number of steps ($\theta_{StretchT}$) required to complete each project within T by SNA we can observe the obvious difference between $\theta_{StretchT}$ and θ_{CrashT}, i.e., SNA could complete the project in the least number of steps than CCA. Therefore, we can conclude that SNA is a faster and easier to complete the project within T. Based on the five projects, we conclude the following points in SNA:

- Obtain the greatest cost saving by stretching the noncritical activities which have the greatest cost slope
- It is not necessary to crash every activity to finish the project within T
- In SNA the activity can be stretched up to its normal time, which is assumed to be its least-costly time, extending the activity beyond the normal pace will not produce any additional savings and might well increase the cost
- It is not necessary to use up all the slack in the different noncritical paths network when the noncritical activities are stretched

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