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Determination of SIF for a Crack Emanating From a Rivet Hole in a Plate using Displacement Extrapolation Method

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Abstract: Modern aircraft structures are designed using a damage tolerance philosophy. This design philosophy envisions sufficient strength and structural integrity to sustain major damage and to avoid catastrophic failure. The most likely places for crack initiating and development are the rivet holes, due to high stress concentration in this area. Such cracks may grow in time and reduces the lifetime of the sheet. The Stress Intensity Factor (SIF) is one the most important parameters in fracture mechanics analysis. The objective of this work is to determine SIF (plane stress) for a crack emanating from a rivet hole in a plate Finite Element Method (FEM). From this study it was observed that the value of SIF rises suddenly when the crack tip is near to the hole and drops down as the crack tip move far from the hole. The SIF values evaluated for different crack length is compared with the analytical values obtained from Bowie's equation. This provides important information for subsequent studies such as the crack growth rate determination and prediction of residual strength.

Key words: Fracture mechanics, stress intensity factor, rivet hole

INTRODUCTION

Modern aircraft structures are designed using a damage tolerance philosophy. This design philosophy envisions sufficient strength and structural integrity of the aircraft to sustain major damage and to avoid catastrophic failure. However, structural aging of the aircraft may significantly reduce the strength below an acceptable level. This raises many important safety issues (Chen *et al*., 1999).

The most likely places for crack initiating and development are the rivet holes due to the high stress concentration in this area. Such cracks may grow in time, leading to a loss of strength and the reduction of the lifetime of the sheet as shown in Fig. 1. If the structure is concerned with different loading, the crack behavior must be assessed in order to avoid catastrophic failures. For this, the knowledge of the crack size, service stress, material properties and Stress Intensity Factor (SIF) is required (Karlsson and Backlund, 1978).

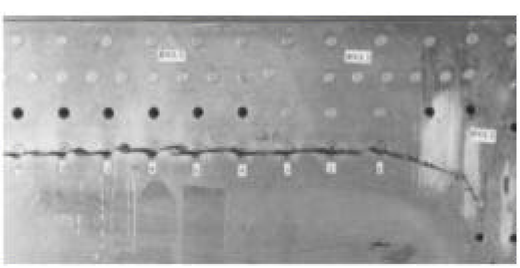

Fig. 1: Larger crack formed by the link-up of fatigue cracks at adjacent rivets

FRACTURE MECHANICS

Fracture mechanics involves a study of the presence of the cracks on overall properties and behavior of the engineering component. The process of fracture may be initiated at defect locations like micro-cracks, voids and the cavities at the grain boundaries. These defects can lead to the formation of a crack due to the rupture and disentanglement of molecules, rupture of atomic bonds or dislocation slip (Broek, 2002).

Cracked body can be subjected to one of the three modes of loads as shown in Fig. 2. In some cases, body may experience combination of the three modes:

- **Opening mode:** The principal load is applied normal to the crack surfaces which tends to open the crack. This is also referred as Mode I loading (Fig. 2a)

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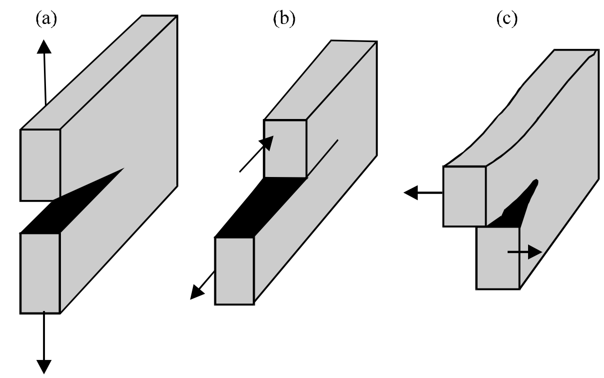

Fig. 2(a-c): Three modes of loading that can be applied to a crack

- **In-plane shear mode:** This mode corresponds to in-plane shear loading which tends to slide one crack surface with respect to the other. This is also referred as Mode II loading (Fig. 2b)
- **Out-of-plane shear mode:** This is the tearing and anti-plane shear mode where the crack surfaces move relative to one another and parallel to the leading edge of the crack (Fig. 2c)

The Stress Intensity Factor (SIF) is one the most important parameters in fracture mechanics analysis. It defines the stress field close to the crack tip and provides fundamental information of how the crack is going to propagate. In this study, the displacement extrapolation method is employed to calculate the SIF.

Various numerical methods have been used to derive SIF such as Finite Difference Method (FDM), Finite Element Method (FEM) and Boundary Element Method (BEM). Among them, FEM has been widely employed for the solution of both fracture problems linear elastic and elasto-plastic. A typical and practical point matching technique, called Displacement Extrapolation Method (DEM) is chosen for the numerical analysis method (Guinea *et al*., 2000).

Solving a fracture mechanics problem involves performing a linear elastic or elastic-plastic static analysis and then using specialized postprocessing commands or macros to calculate desired fracture parameters. The two main aspects of in this procedure are modeling the crack region and calculating fracture parameters (Souiyah *et al*., 2009).

EVALUATING STRESS INTENSITY FACTOR (K_I)

The stress or strain state is always there in three dimensional analysis. But in most cases, they can be simplified to either plane strain or plane stress by ignoring either the out of plane strain or plane stress. In a thin body generally, the stress through the thickness (σ_z) cannot vary appreciably due to the thin section. Because there can be no stresses normal to a free surface, $\sigma_z = 0$ throughout the section and a biaxial state of stress results. This is termed as plane stress condition.

Plane stress assumption is valid for very thin-walled structures, while plane strain is predominant condition in structures with large thickness.

The evaluation of SIF (K_I) by Displacement Extrapolation Method (DEM) is as discussed bellow for plane stress condition.

The stress intensity factors at a crack for a linear elastic fracture mechanics analysis may be computed using the KCALC command. The analysis uses a fit of the nodal displacements in the vicinity of the crack. The actual displacements at and near a crack for linear elastic materials are:

$$u = +\frac{K_{II}}{2\pi}\sqrt{\frac{r}{2\pi}}(1+k) \quad (1)$$

$$v = +\frac{K_I}{2G}\sqrt{\frac{r}{2\pi}}(1+k) \quad (2)$$

$$w = +\frac{2K_{III}}{G}\sqrt{\frac{r}{2\pi}} \quad (3)$$

Where:

u, v, w	=	Displacements in a local Cartesian coordinate system as shown in Fig. 3
r, θ	=	Coordinates in a local cylindrical coordinate system as shown in Fig. 3
G	=	Shear modulus

In plane stress:

$$k = \frac{3v}{1+v} \quad (4)$$

v = Poisson's ratio

For Mode 1, SIF at crack tip is expressed as:

$$K_I = \sqrt{2\pi}\frac{G}{1+k}\frac{|\Delta V|}{\sqrt{r}} \quad (5)$$

where, Δv, are the motions of one crack face with respect to the other.

Then A and B are determined so that:

$$\frac{|V|}{\sqrt{r}} = A + Br \quad (6)$$

At points J and K.

Next, let r approach 0:

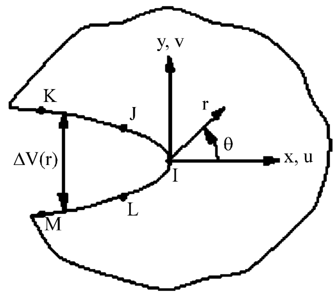

Fig. 3: Nodes used for the approximate crack-tip displacements for full crack model

$$\lim_{r \to 0} \frac{|V|}{\sqrt{r}} = A \tag{7}$$

Thus, Eq. 5 becomes:

$$K_I = \sqrt{2\pi} \frac{2GA}{1+k} \frac{N}{mm^2} \sqrt{mm} \tag{8}$$

FINITE ELEMENT MODEL DEVELOPMENT

"A through crack emanating from holes" is one among the practical problems in chapter 14 from the text book "elementary engineering fracture mechanics" by David Broek Published by Kluwer Academic Publishers reprinted in year 2002 (Broek, 2002).

Bowie has presented the K solution for radial through cracks emanating for unloaded open holes. For the case where the crack is not small compared to the hole, one might assume as a first engineering approach that the combination behave as if the hole were part of the crack is as shown in Fig. 4. The effective crack size is then equal to the physical crack plus the diameter of the hole. The stress intensity factor for the asymmetric case with 2aeff = D+a:

$$K = \sigma\sqrt{\pi a_{eff}} = \sigma\sqrt{\pi a}\ \sigma\sqrt{\left(\frac{D}{2a} + \frac{1}{2}\right)} = \sigma\sqrt{\pi a}.\, f_1\left(\frac{a}{D}\right) \tag{9}$$

Where:

D = Diameter of the hole (10 mm)
a = Crack length (1 to 30 mm)
σ = Tensile load (10 MPa)

The objective of this study is to determine SIF for a crack emanating from a rivet hole in a plate as shown in Fig. 4. The objective is achieved by developing a 2D finite element model of a plate with rivet holes and a through crack subjected to a tensile load. The SIF is calculated at crack tip for various crack length by generating mesh using crack tip elements.

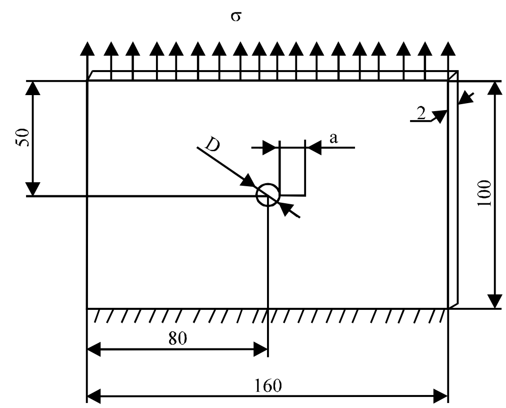

Fig. 4: Geometry of model

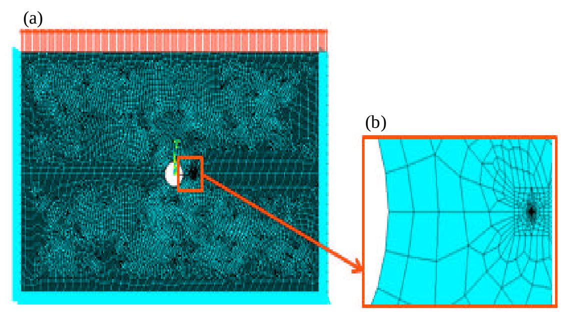

Fig. 5(a-b): (a) Finite element model and boundary condition and (b) zoomed view of crack tip

To achieve the required objective, 2D finite element model is developed and boundary conditions are applied in preprocessor of the ANSYS software. To mesh the model with crack, plane 82 with singular elements is used. In ANSYS, KSCON command is used to generate the singular elements around the crack tip. The model is then solved (Static Analysis) in solution menu. Then the SIF is evaluated in general postprocessor by using KCALC command.

The geometry of the test model created in ANSYS is as shown in the Fig. 4. It contains a through crack emanating from a hole. The meshing of the model is as shown in the Fig. 5a, b. The element used to mesh the model is 8-node plane 82 quadrilateral element. The symmetry boundary condition is applied at the both sides of the plate to make it as infinite length. The load is applied to the top edge and the bottom edge is fixed in all degree of freedom. The material considered is 2024-T3 Aluminium Alloy (ASME). The material is assumed to be linear elastic with young's modulus of 73.1 X 103 MPa and Poisson's ratio 0.33.

ANSYS preprocessor's (PREP7) KSCON command is used to generate the singular elements around the crack tip. For this model there are 36 singular elements around the crack tip and the radius of the first row elements is Δa.

Where:

$$\Delta a = \frac{a}{100} \tag{10}$$

RESULTS AND DISCUSSIONS

The geometry was imposed by plane stress condition and edge load (σ) applied under mode-I loading condition. The variation of normalized Stress Intensity Factor (K_I/K_O) (by plane stress method) with respect to a/D ratio [actual crack length (a) to the Diameter of the rivet hole (D)] is as shown in Table 1.

The normalized SIF (K_I/K_O) is used to obtain the characteristic curve of SIF which depends only on the geometrical factor and its variation within the given domain (a/D).

Where:

$$f\left(\frac{a}{D}\right) = \frac{K_I}{K_0} \tag{11}$$

$$K_O = \sigma\sqrt{\pi a}\,\frac{N}{mm^2}\sqrt{mm} \tag{12}$$

$$K_I\,(\text{Theo}) = \sigma\sqrt{\pi a}\,\sigma\sqrt{\left(\frac{D}{2a}+\frac{1}{2}\right)}\,\frac{N}{mm^2}\sqrt{mm} \tag{13}$$

The variation of normalized SIF (K_I/K_O) (by plane stress method) with respect to a/D ratio is as shown in Fig. 8. As the crack is near to the hole the stress concentration around holes has a strong influence on the SIF value. For a/D ratio 0.1 there is a steep rise in SIF K_I, this is due to crack is small and the crack tip is near to stress concentration at the hole from which crack in emanating. As the crack grow further (for a/D ranging from 0.1 to 3) the crack tip move far from the stressed areas hence the value of SIF drops down and become almost stable.

K_O is the stress intensity factor for a crack of length 2a in a large sheet subject to a remote uniform tensile stress perpendicular to crack direction and is given by $K_O = \sigma\sqrt{(\pi * a)}$ in a large sheet.

A comparison of the results of Bowie equation and experiment for a through crack emanating from a hole with the finite element analysis result by using ANSYS software for a given rage of crack length is as shown in the Fig. 6-8. The present result which was obtained by using the finite element method are in good agreement with Bowie equation and experiment for a through crack emanating from a hole. The percentage deviation calculated for FEA and theoretical results is as show in column % error in the Table 1.

The Deformed geometries for crack length (a) of 1, 8 and 25 mm is as shown in Fig. 9-11. The maximum Von Mises is found to be at crack tip. The value of maximum

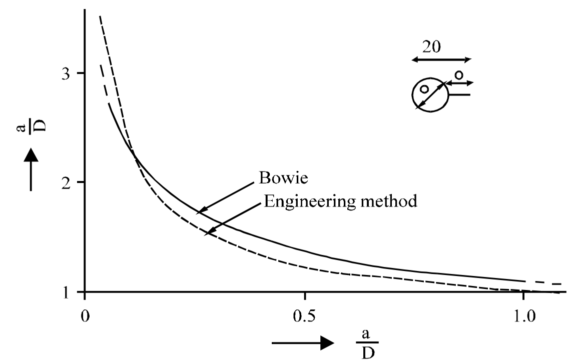

Fig. 6: Bowie's analysis as compared to the engineering method

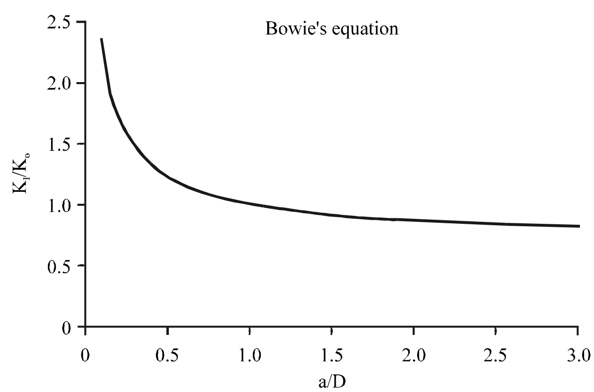

Fig. 7: Stress intensity factors for crack emanating from circular hole by using Bowie equation

Table 1: Variation of Stress Intensity Factor (K_I) using plane stress method for different crack lengths (a)

a	a/D	K_I (plane stress)	K_I(Theo)	K_O	(K_I/K_O)	K_I(Theo)/K_O	Error (%)
1.0	0.1	36.947	41.557	17.720	2.085	2.345	11.087
1.5	0.15	40.496	42.491	21.703	1.866	1.958	4.699
2.0	0.2	42.634	43.405	25.060	1.701	1.732	1.790
4.0	0.4	47.072	46.883	35.440	1.328	1.323	0.377
8.0	0.8	52.926	53.160	50.120	1.056	1.061	0.471
15.0	1.5	62.425	62.650	68.629	0.910	0.913	0.329
20.0	2.0	68.957	68.629	79.246	0.870	0.866	0.462
25.0	2.5	75.379	74.128	88.600	0.851	0.837	1.673
30.0	3.0	81.803	79.246	97.057	0.843	0.816	3.309

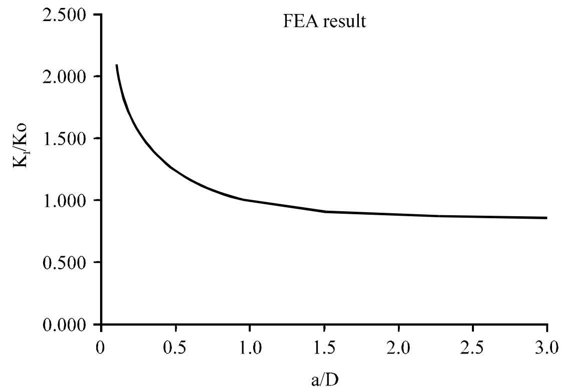

Fig. 8: Stress intensity factors for crack emanating from circular hole using finite element method

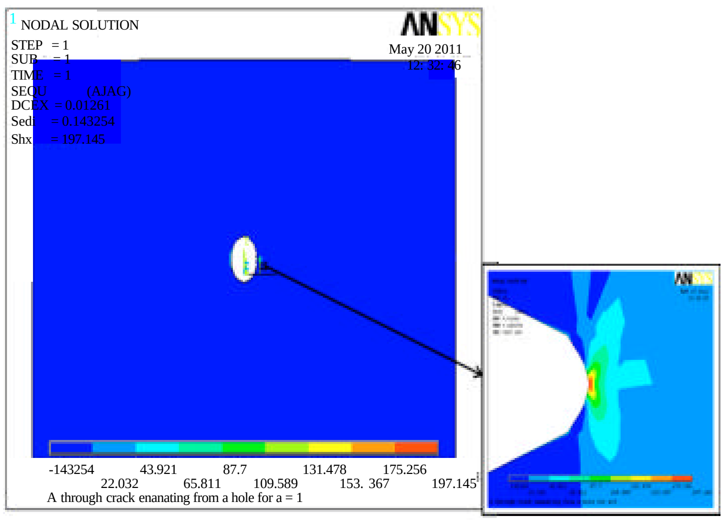

Fig. 9: Von mises stress distribution for a = 1 mm

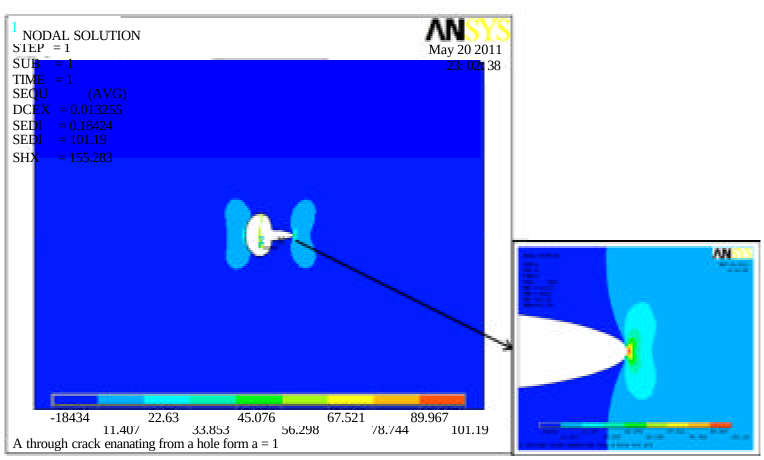

Fig. 10: Von mises stress distribution for a = 8 mm

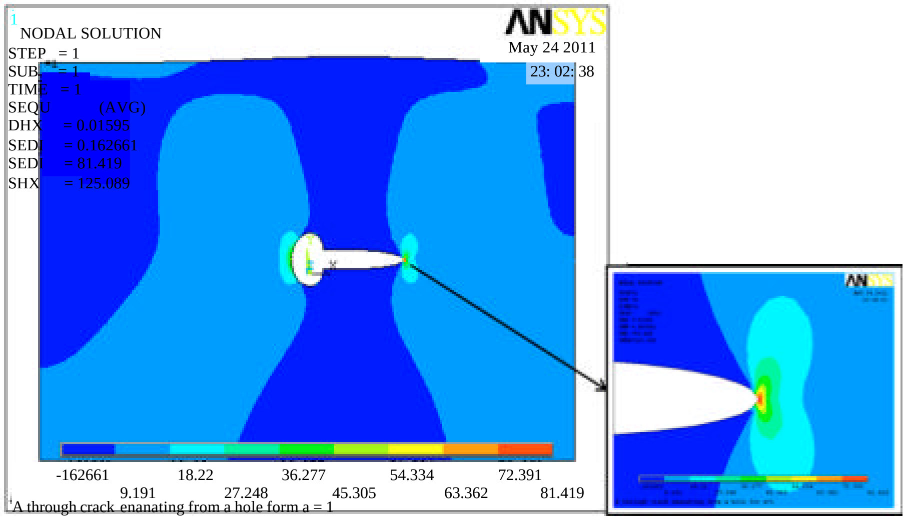

Fig. 11: Von mises stress distribution for a = 25 mm

Table 2: Y-Directional stresses, Von Mises stresses and SIF at crack tip for a = 1, 8 and 25 mm

Crack length (a)	a/D	SIF-mode plane stress	Y-direction stress (σy) N mm^{-2}	Von mises stress (σVon) N mm^{-2}
1.0	0.10	36.947	225.86	197.145
8.0	0.80	52.925	116.27	101.130
25.0	2.50	75.379	93.73	81.400

Y-directional stresses, Von Mises stresses and Stress Intensity Factor (K_I) at crack tip is as shown in Table 2.

CONCLUSION

The problem of determining stress intensity factors for a crack emanating from a rivet hole in a tensile loaded infinite plate is of prime importance in damage tolerance analysis. The method used in this report can be utilized for calculating the stress intensity factor for many other loading cases and many values of the crack length. This provides important information for subsequent studies, especially for fatigue loads, where stress intensity factor is necessary for the crack growth rate determination.

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