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Estimation of R = P[Y<X] for Weighted Exponential Distribution

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Abstract: This study deals with the estimation of P = (Y<X) when X and Y are two independent weighted exponential distributions with different parameters. The MLE of the R based on one simple iteration procedure is obtained. Furthermore, a simulation study was conducted to compute Bayesian estimates of the model. Finally, we carried out its Bayesian estimations of parameters with a real data set.

Key words: Weighted exponential distribution, Bayesian estimation, reliability, Newton-Raphson algorithm

INTRODUCTION

In reliability contexts, inferences about R = (Y<X), where X and Y independent distribution, are a subject of interest. Note that the estimation of R is very common in the statistical literature. For example, in mechanical reliability of a system, if X is the strength of a component which is subject to stress Y, then R is a measure of system performance. The system fails, if at any time the applied stress is greater than its strength. The Maximum Likelihood Estimator (MLE) of R when X and Y have bivariate exponential distribution has been considered by Awad *et al.* (1981). Church and Harris (1970), Downtown (1973), Woodward and Kelley (1977) and Owen *et al.* (1964) considered the estimation of R when X and Y are normally distributed. Similar problem for multivariate normal distribution has been considered by Gupta and Gupta (1990). Kelley *et al.* (1976) and Sathe and Shah (1981) considered the problem of estimating R when X and Y are independent exponential random variable. Constantine *et al.* (1986) considered the estimation of R when X and Y are independent gamma random variable. Surles and Padgett (1998, 2001) considered the estimation of R when X and Y are Burr type X random variables. Kundu and Gupta (2005) considered this problem when X and Y are generalized exponential distribution. So Rezaei *et al.* (2010) obtained estimation of P[Y<X] when X and Y are two independent generalized Pareto distributions with different parameters. Gupta and Kundu (2009) introduced a new class of weighted exponential distribution by using of the idea of Azzalini (1985). In this study, we focus on estimation of R = P[Y<X], where X and Y follow the Weighted Exponential (WE) distribution with different parameters.

WEIGHTED EXPONENTIAL DISTRIBUTION

A random variable X is said to have weighted exponential distribution, if its Probability Density Function (PDF) is given by:

$$f_X(x;\alpha,\lambda)=\frac{\alpha+1}{\alpha}\lambda e^{-\lambda x}\left(1-e^{-\alpha\lambda x}\right);\ x>0 \qquad (1)$$

And it's 0 otherwise. We will denote it as WE(α,λ). Note that in the model (1) the location parameter can be easily incorporated. The cumulative distribution function is defined by:

$$F_X(x;\alpha,\lambda)=\frac{\alpha+1}{\alpha}\left[1-e^{-\lambda x}-\frac{1}{\alpha+1}\left(1-e^{-(\alpha+1)\lambda x}\right)\right] \qquad (2)$$

For λ>0 and α>0. Here, α and λ are the shape and scale parameters, respectively (Gupta and Kundu, 2009).

ESTIMATION OF MAXIMUM LIKELIHOOD OF R WITH COMMON SCALE PARAMETER

Here, we investigate the properties of R, when the common scale parameter λ, is the same. To investigate the properties of R, denote by WE(α,λ) the distribution of reparametrized WE. Let X~WE(α,λ) and Y~WE(β,λ), where X and Y are independent random variables. Therefore:

$$\begin{aligned}R&=P[Y<X]=\int_0^\infty P(Y<X|X=x)P(X=x)\,dx\\&=\int_0^\infty F_Y(x)f(x)\,dx\\&=\int_0^\infty\frac{\beta+1}{\beta}\left[1-e^{-\lambda x}-\frac{1}{\beta+1}\left(1-e^{-(\beta+1)\lambda x}\right)\right]\frac{\alpha+1}{\alpha}\lambda e^{-\lambda x}\left(1-e^{-\alpha\lambda x}\right)dx\\&=\frac{(\beta+1)(\alpha+1)}{2\alpha\beta}-\frac{(\beta+1)}{\alpha\beta}+\frac{(\alpha+1)(\beta+1)}{\alpha\beta(2+\alpha)}-\frac{(\alpha+1)}{\alpha\beta}+\frac{1}{\alpha\beta}+\frac{(\alpha+1)}{\alpha\beta(2+\beta)}-\frac{(\alpha+1)}{\alpha\beta(2+\alpha+\beta)}\end{aligned} \qquad (3)$$

The result follows after simplification.

To compute the MLE of R, suppose $X_1, X_2, \ldots X_N$ is a random sample from WE(α,λ) and WE(β,λ) is a random sample from WE(β,λ). Therefore, the log-likelihood function of the observed sample is:

$$\begin{aligned} L(\alpha,\beta,\lambda) &= n\ln\left(\frac{\alpha+1}{\alpha}\right) + n\ln(\lambda) - \lambda\sum_{i=1}^{n} x_i + \sum_{i=1}^{n}\ln\left(1-e^{-\alpha\lambda x_i}\right) \\ &+ m\ln\left(\frac{\beta+1}{\beta}\right) + m\ln(\lambda) - \lambda\sum_{j=1}^{m} y_j + \sum_{j=1}^{m}\ln\left(1-e^{-\beta\lambda y_j}\right) \end{aligned} \quad (4)$$

The MLEs of α, β and λ say $\hat{\alpha}, \hat{\beta}$ and $\hat{\lambda}$, respectively, can be obtained as the solutions of:

$$\frac{\partial L}{\partial \alpha} = \frac{-n}{\alpha(\alpha+1)} + \lambda\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}} = 0 \quad (5)$$

$$\frac{\partial L}{\partial \beta} = \frac{-m}{\beta(\beta+1)} + \lambda\sum_{j=1}^{m}\frac{y_j e^{-\beta\lambda y_j}}{1-e^{-\beta\lambda y_j}} = 0 \quad (6)$$

$$\frac{\partial L}{\partial \lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i + \alpha\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}} + \frac{m}{\lambda} - \sum_{j=1}^{m} y_j + \beta\sum_{j=1}^{m}\frac{y_j e^{-\beta\lambda y_j}}{1-e^{-\beta\lambda y_j}} = 0 \quad (7)$$

There is no closed-form solution to this system of equations, so we will solve for $\hat{\alpha}, \hat{\beta}$ and $\hat{\lambda}$ iteratively, using the Newton-Raphson method, a tangent method for root finding. In our case we will estimate $\theta = (\alpha,\beta,\lambda)$ iteratively:

$$\hat{\theta}_{i+1} = \hat{\theta}_i - G^{-1}g$$

where, g is the vector of normal equations for which we want:

$$g = [g_1\ g_2\ g_3]$$

with:

$$g_1 = \frac{-n}{\alpha(\alpha+1)} + \lambda\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}},$$

$$g_2 = \frac{-m}{\beta(\beta+1)} + \lambda\sum_{j=1}^{m}\frac{y_j e^{-\beta\lambda y_j}}{1-e^{-\beta\lambda y_j}},$$

$$g_3 = \frac{n}{\lambda} - \sum_{i=1}^{n} x_i + \alpha\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}} + \frac{m}{\lambda} - \sum_{j=1}^{m} y_j + \beta\sum_{j=1}^{m}\frac{y_j e^{-\beta\lambda y_j}}{1-e^{-\beta\lambda y_j}}$$

and G is the matrix of second derivatives:

$$G = \begin{bmatrix} \frac{dg_1}{d\alpha} & \frac{dg_1}{d\beta} & \frac{dg_1}{d\lambda} \\ \frac{dg_2}{d\alpha} & \frac{dg_2}{d\beta} & \frac{dg_2}{d\lambda} \\ \frac{dg_3}{d\alpha} & \frac{dg_3}{d\beta} & \frac{dg_3}{d\lambda} \end{bmatrix},$$

Where:

$$\frac{dg_1}{d\alpha} = \frac{2n(2\alpha+1)}{(\alpha^2+\alpha)} - \lambda^2\sum_{i=1}^{n}\frac{x_i^2 e^{-\alpha\lambda x_i}}{\left(1-e^{-\alpha\lambda x_i}\right)^2}$$

$$\frac{dg_1}{d\beta} = 0$$

$$\frac{dg_1}{d\lambda} = \sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}} - \alpha\lambda\sum_{i=1}^{n}\frac{x_i^2 e^{-\alpha\lambda x_i}}{\left(1-e^{-\alpha\lambda x_i}\right)^2}$$

$$\frac{dg_2}{d\alpha} = 0$$

$$\frac{dg_2}{d\beta} = \frac{2m(2\beta+1)}{(\beta^2+\beta)} - \lambda^2\sum_{j=1}^{m}\frac{y_j^2 e^{-\beta\lambda y_j}}{\left(1-e^{-\beta\lambda y_j}\right)^2}$$

$$\frac{dg_2}{d\lambda} = \sum_{j=1}^{m}\frac{y_j e^{-\beta\lambda y_j}}{1-e^{-\beta\lambda y_j}} - \beta\lambda\sum_{j=1}^{m}\frac{y_j^2 e^{-\beta\lambda y_j}}{\left(1-e^{-\beta\lambda y_j}\right)^2}$$

$$\frac{dg_3}{d\alpha} = \sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i} - x_i e^{-2\alpha\lambda x_i} - \alpha\lambda x_i^2 e^{-\alpha\lambda x_i}}{\left(1-e^{-\alpha\lambda x_i}\right)^2}$$

$$\frac{dg_3}{d\beta} = \frac{dg_2}{d\lambda}$$

$$\frac{dg_3}{d\lambda} = \frac{-n}{\lambda^2} - \alpha^2\sum_{i=1}^{n}\frac{x_i^2 e^{-\alpha\lambda x_i}}{\left(1-e^{-\alpha\lambda x_i}\right)^2} - \frac{m}{\lambda^2} - \beta^2\sum_{j=1}^{m}\frac{y_j^2 e^{-\beta\lambda y_j}}{\left(1-e^{-\beta\lambda y_j}\right)^2}$$

The Newton-Raphson algorithm converges, as our estimates of α,β and λ change by less than a tolerated amount with each successive iteration, to $\hat{\alpha}, \hat{\beta}$ and $\hat{\lambda}$. Since, ML estimators are invariant, so the MLE of R becomes:

$$\begin{aligned} \hat{R} &= \frac{(\hat{\beta}+1)(\hat{\alpha}+1)}{2\hat{\alpha}\hat{\beta}} - \frac{(\hat{\beta}+1)}{\hat{\alpha}\hat{\beta}} + \frac{(\hat{\alpha}+1)(\hat{\beta}+1)}{\hat{\alpha}\hat{\beta}(2+\hat{\alpha})} - \frac{(\hat{\alpha}+1)}{\hat{\alpha}\hat{\beta}} \\ &+ \frac{1}{\hat{\alpha}\hat{\beta}} + \frac{(\hat{\alpha}+1)}{\hat{\alpha}\hat{\beta}(2+\hat{\beta})} - \frac{(\hat{\alpha}+1)}{\hat{\alpha}\hat{\beta}(2+\hat{\alpha}+\hat{\beta})} \end{aligned} \quad (8)$$

BAYESIAN ESTIMATIONS

Here, we obtain the estimation of parameters weighted exponential distribution with using of posterior mode method.

Bayes notation and prior elicitation: Let $\bar{X}$ be the weighted Exponential random variables corresponding to the observed data $\underline{x}$ having WE distribution as its pdf $f(.|\lambda,\alpha)$, with the shape and scale parameters as $\alpha>0$ and $\lambda>0$. To obtain Bayesian estimates the posterior pdf of parameters should be built as a first step. Thus:

$$\prod(\alpha,\lambda\,|\,\text{data}) \propto L(\text{data},\alpha,\lambda)\pi_1(\lambda)\pi_2(\alpha) \quad (9)$$

Now it is assumed that $x_1,\ldots,x_n$ is a random sample from $f(.|\lambda,\alpha)$ as given in Eq. 1. We assume that λ has a prior $\pi_1(.)$ with Gamma (a,b) distribution. Also the prior on α is $\pi_2(.)$ with Gamma (c,d) density and it is independent of $\pi_1(.)$:

$$\pi_1(\lambda) \propto \lambda^{b-1}\, e^{-\alpha\lambda},\ \lambda>0 \quad (10)$$

$$\pi_2(\lambda) \propto \lambda^{d-1}\, e^{-c\alpha},\ \alpha>0 \quad (11)$$

The likelihood function of the observed data is:

$$L(x_1,\ldots,x_n \mid \alpha,\lambda) = \left(\frac{\alpha+1}{\alpha}\right)^n \lambda^n e^{-\lambda\sum x_i} \prod_{i=1}^{n}\left(1-e^{-\alpha\lambda x_i}\right),\ x>0 \quad (12)$$

Therefore, the joint density function of the observed data, α and λ is:

$$\begin{aligned} l(\text{data},\alpha,\lambda) &\propto l(x_1,\ldots,x_n \mid \alpha,\lambda)\pi_1(\lambda)\pi_2(\alpha), \\ &\propto (\alpha+1)^n \alpha^{d-n-1}\lambda^{b+n-1} e^{-\lambda(\sum x_i+\alpha)-c\alpha} \prod_{i=1}^{n}\left(1-e^{-\alpha\lambda x_i}\right) \end{aligned} \quad (13)$$

The posterior density function of $\{\alpha,\lambda\}$, given the data is:

$$\prod(\alpha,\lambda \mid \text{data}) = \frac{(\alpha+1)^n \alpha^{d-n-1}\lambda^{b+n-1} e^{-\lambda(\sum x_i+\alpha)-c\alpha} \prod_{i=1}^{n}\left(1-e^{-\alpha\lambda x_i}\right)}{\int_0^\infty\int_0^\infty (\alpha+1)^n \alpha^{d-n-1}\lambda^{b+n-1} e^{-\lambda(\sum x_i+\alpha)-c\alpha} \prod_{i=1}^{n}(1-e^{-\alpha\lambda x_i})\, d\alpha\, d\lambda} \quad (14)$$

The following are different algorithms to generate WE(α,λ).

- **Interpretation 1:**
  - Step 1: $U \sim \text{Beta}\left(\frac{1}{\alpha},2\right)$
  - Step 2: $X = \frac{\ln U}{-\alpha\lambda} \sim WE(\alpha,\lambda)$
- **Interpretation 2:**
  - **Step 1:** U~exp(λ) and V~exp($\lambda(1+\alpha)$) (U and V are independent)
  - **Step 2:** X = U+V~WE(α,λ)

Throughout this article, we assume that all parameters follow independent gamma prior distribution.

BAYESIAN ESTIMATION OF PARAMETERS BY POSTERIOR MODE METHOD

Let X_j (i = 1,2,..., n) be a random sample from WE distribution. Using the corresponding pdf (1) and expression (9), the log posterior distribution for a sample of size n with priors Gamma (a,b) and Gamma(c,d) is given by:

$$\begin{aligned} &= n\ln(\alpha+1) + (d-n-1)\ln(\alpha) + (b+n-1)\ln(\lambda) - \\ &\lambda\left(\sum_{i=1}^{n} x_i + \alpha\right) - c\alpha + \sum_{i=1}^{n}\ln(1-e^{-\alpha\lambda x_i}) + \log(K) \end{aligned} \quad (15)$$

where, K does not depend on parameters α and λ the hyper parameters a, b, c and d are fixed. Thus to obtain Bayesian estimation of parameters via posterior mode method, we require to maximize (13). First, we obtain the likelihood equation:

$$\frac{\partial l_{WE}}{\partial\lambda} = \frac{b+n-1}{\lambda} - \left(\sum_{i=1}^{n} x_i + \alpha\right) + \alpha\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}} \quad (16)$$

$$\frac{\partial l_{WE}}{\partial\alpha} = \frac{n}{1+\alpha} + \frac{d-n-1}{\alpha} - \lambda - c + \lambda\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}} \quad (17)$$

For implementation of the work, we need to use either the scoring algorithm or the Newton-Raphson algorithm to solve the two non linear Eq. 14 and 15 simultaneously we prefer to use Newton-Raphson procedure to obtain the approximate roots of the equations. So we will solve for $\hat{\alpha}$ and $\hat{\lambda}$ iteratively, using the Newton-Raphson method, a tangent method for root finding.

In our case we will estimate $\beta = (\alpha,\lambda)$ iteratively:

$$\hat{\beta}_{i+1} = -\hat{\beta}_i G^{-1} g$$

where, g is the vector of normal equations for which we want:

$$g = [g_1\ g_2]$$

With:

$$g_1 = \frac{n}{1+\alpha} + \frac{d-n-1}{\alpha} - \lambda - c + \lambda\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}}$$

$$g_2 = \frac{b+n-1}{\lambda} - \left(\sum_{i=1}^{n} x_i + \alpha\right) + \alpha\sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}}$$

Therefore, the second derivatives are indispensable. Following, are the elements of symmetric observed information matrix or symmetric Hessian matrix:

$$\frac{\partial g_2}{\partial\lambda} = \frac{\partial^2 l_{WE}}{\partial\lambda^2} = \frac{-(b+n-1)}{\lambda^2} - \alpha^2\sum_{i=1}^{n}\frac{x_i^2 e^{-\alpha\lambda x_i}}{(1-e^{-\alpha\lambda x_i})^2};$$

$$\frac{\partial g_1}{\partial\alpha} = \frac{\partial^2 l_{WE}}{\partial\alpha^2} = \frac{-n}{(1+\alpha)^2} + \frac{-(d-n-1)}{\alpha^2} - \lambda^2\sum_{i=1}^{n}\frac{x_i^2 e^{-\alpha\lambda x_i}}{(1-e^{-\alpha\lambda x_i})^2};$$

$$\frac{\partial g_2}{\partial\alpha} = \frac{\partial^2 l_{WE}}{\partial\lambda\partial\alpha} = -1 + \sum_{i=1}^{n}\frac{x_i e^{-\alpha\lambda x_i}}{1-e^{-\alpha\lambda x_i}} - \alpha\lambda\sum_{i=1}^{n}\frac{x_i^2 e^{-\alpha\lambda x_i}}{(1-e^{-\alpha\lambda x_i})^2};$$

$$\frac{\partial g_1}{\partial\lambda} = \frac{\partial^2 l_{WE}}{\partial\alpha\partial\lambda} = \frac{\partial^2 l_{WE}}{\partial\lambda\partial\alpha}$$

and G is the matrix of second derivatives:

$$G = \begin{bmatrix} \frac{dg_1}{d\alpha} & \frac{dg_1}{d\lambda} \\ \frac{dg_2}{d\alpha} & \frac{dg_2}{d\lambda} \end{bmatrix}$$

The Newton-Raphson algorithm converges as our estimates of α and λ change by less than a tolerated amount with each successive iteration, to $\hat{\alpha}$ and $\hat{\lambda}$.

Estimation of hyper parameters of prior distribution via moment of we model: Let $x_1, x_2, ..., x_n$ is a random sample from $f(.|\lambda,\alpha)$ and denote the prior distribution as Eq. 10 and 11. We obtain the estimate of a,b,c and d from the past data. To estimate of the hyper parameters, first we obtain the moment estimators of parameters of WE distribution, then their corresponding mean and variance of each estimator are obtained by using simulation consequently the estimated value of a,b,c and d are obtained by solving two simple equation:

$$E(\lambda) = \frac{a}{b},\ E(\alpha) = \frac{c}{d},\ Var(\lambda) = \frac{a}{b^2} \text{ and } Var(\alpha) = \frac{c}{d^2}$$

We can estimate the hyper parameters of prior distribution by moment estimators which is well-known in the literature as empirical Bayes Procedure. For implementation, in a simulation study we can use one set of data to estimate hyper parameters of prior distribution and use another set of data to obtain posterior mode. Empirical Bayes approach is used when we do not have prior knowledge about hyper parameters of prior distribution. In later section we carry out the procedure of computing the moment for WE distribution.

Algorithm of computing for hyper parameters by moment method: If X follows, $WE(\alpha,\lambda)$ then the MGF of X for $-1<t<1$ can be obtained as:

$$M_X(t) = \frac{\lambda(\alpha+1)}{\alpha}\left[\frac{1}{\lambda-1} - \frac{1}{\lambda - t + \alpha\lambda}\right]$$

Therefore, differentiating $M_X(t)$and having t=0, we obtain:

$$M'_X(0) = E(X) = \frac{\alpha+2}{\lambda(\alpha+1)}$$

and:

$$M''_X(0) = E(X^2) = \frac{2(\alpha^2+3\alpha+3)}{\lambda^2(\alpha+1)^2}$$

So:

$$Var(X) = M''_X(0) - (M'_X(0))^2 = \frac{1}{\lambda^2}\left(1 + \frac{1}{(\alpha+1)^2}\right)$$

For $\lambda = 1$ it is given by Gupta and Kundu (2009).

Now, we can to obtain the values of α and λ for replication samples and so estimate the hyper parameters of the prior distribution by Mont Carlo simulation by following equations:

$$E(\hat{\alpha}) = \frac{c}{d} \Rightarrow \bar{\alpha} = \frac{\hat{c}}{\hat{d}}$$

$$Var(\hat{\alpha}) = \frac{c}{d^2} \Rightarrow S^2_{\alpha} = \frac{\hat{c}}{\hat{d}^2}$$

Also to computation of estimation of the hyper parameters of a and b we apply this method. In this paper we prefer to consider the hyper parameters as fix but instead we estimate Bayesian parameters of the model in the cases Non Informative priors, informative priors and most informative priors by method namely Posterior mode.

NUMERICAL SIMULATION STUDY FOR THE POSTERIOR MODE METHOD

For WE model, 10000 samples, each of size 10, 30, 50 were generated as past data. In this simulation that has carried out with a SAS code, we follow up priors distribution as non informative , i.e., "a = b = c = d = 0", less informative prior that in the case we take a = 8, b = 16, c = 2, d = 1; for the informative prior we take a = 80, b = 160, c = 20, d = 10 and finally for the most informative prior we get a = 400, b = 800, c = 100, d = 50. The results derived due to the simulation have sited in Table 1 and 2.

Table 1: Estimations for $\alpha = 0.5$ and $\lambda = 2$

Parameters	n	Non informative (a = b = c = d = 0)	Less informative (a = 8, b = 16, c = 2, d = 1)	Informative (a = 80, b = 160, c = 20, d = 10)	Most informative (a = 400, b = 800, c = 100, d = 50)
$\hat{\alpha}$	10	0.0031194	0.0082553	0.3946383	0.4782735
$Sd(\hat{\alpha})$		0.1980014	0.0387253	0.0329586	0.0066745
$\hat{\lambda}$		3351005.98	10275.410	1.9495332	1.9885215
$Sd(\hat{\lambda})$		16581669.4	978964.86	0.0110933	0.0026188
$\hat{\alpha}$	30	0.5504746	0.4015584	0.4198821	0.4767851
$Sd(\hat{\alpha})$		0.2293231	0.1598698	0.0447861	0.0109774
$\hat{\lambda}$		1.4929561	1.6199636	1.8690043	1.9659110
$Sd(\hat{\lambda})$		0.0725162	0.0266724	0.0149338	0.0042806
$\hat{\alpha}$	50	0.7690850	0.6094339	0.4520162	0.4769940
$Sd(\hat{\alpha})$		0.1067031	0.1025889	0.0466176	0.0133157
$\hat{\lambda}$		1.4485841	1.5505762	1.8119080	1.9454336
$Sd(\hat{\lambda})$		0.0363946	0.0294196	0.0156702	0.0051537

Table 2: Estimations for $\alpha = 0.5$ and $\lambda = 1$

Parameters	n	Non informative	Less informative	Informative	Most informative (a = 400, b = 800, c = 100, d = 50)
$\hat{\alpha}$	10	2010752.67	0.05235480	0.4197867	0.4833332
Sd($\hat{\alpha}$)		105712544	0.10931270	0.0329939	0.0067713
$\hat{\lambda}$		1269573.97	0.90800240	0.9736889	0.9940988
Sd($\hat{\lambda}$)		9416949.04	0.02588556	0.0056889	0.0013327
$\hat{\alpha}$	30	0.7154153	0.49416460	0.4411414	0.4816058
Sd($\hat{\alpha}$)		0.2012635	0.15324040	0.0441687	0.0108099
$\hat{\lambda}$		0.7280340	0.80146350	0.9334672	0.9827815
Sd($\hat{\lambda}$)		0.0189144	0.01522060	0.0075075	0.0021165
$\hat{\alpha}$	50	0.8355492	0.66283080	0.4689475	0.4815055
Sd($\hat{\alpha}$)		0.0997305	0.09804500	0.0463818	0.0135064
$\hat{\lambda}$		0.7173897	0.77058510	0.9047638	0.9725123
Sd($\hat{\lambda}$)		0.0196441	0.01536740	0.0079447	0.0026258

Table 3: Estimates for α and λ

Methods	$\hat{\alpha}$	Sd($\hat{\alpha}$)	$\hat{\lambda}$	Sd($\hat{\lambda}$)
Maximum likelihood	1.62485870	1.2696140	0.01383409	0.002408713
Non informative	1.61137510	4.1820054	0.01385510	0.000012400
Informative	1.59656870	0.2014449	0.01390260	7.7756E-7
Most informative	1.59995260	0.0485232	0.01391440	2.099E-7

Table 4: Survival time of Guinea pigs

Sample No.	Survival time (day)	Sample No.	Survival time (day)	Sample No.	Survival time (say)
1	12	25	60	49	96
2	15	26	60	50	98
3	22	27	60	51	99
4	24	28	61	52	109
5	24	29	62	53	110
6	32	30	63	54	121
7	32	31	65	55	127
8	33	32	65	56	129
9	34	33	67	57	131
10	38	34	68	58	143
11	38	35	70	59	146
12	43	36	70	60	146
13	44	37	72	61	175
14	48	38	73	62	175
15	52	39	75	63	211
16	53	40	76	64	233
17	54	41	76	65	258
18	54	42	81	66	258
19	56	43	83	67	263
20	57	44	84	68	297
21	58	45	85	69	341
22	58	46	87	70	341
23	59	47	91	71	376
24	60	48	95		

CASE STUDY

The data set consists of survival times of guinea pigs injected with different amount of tubercle bacilli and was studied by Bjerkedal (1960). Guinea pigs are known to have high susceptibility of human tuberculosis which is one reason for choosing this species. We consider only the study in which animals in a single cage are under the same regimen. Table 4 represents the survival times of Guinea pigs. The data are given below:

In this case n = 71, the sample mean $\bar{x} = 99.82$ and the sample standard deviation s = 80.55. That gives the moment estimates of α and λ as $\tilde{\alpha} = 2.6124$ and $\tilde{\lambda} = 0.0128$.

The maximum likelihood estimate of the parameters are $\hat{\alpha} = 1.62485870$ and $\hat{\lambda} = 0.01383409$ with the corresponding standard deviation as 1.269614 and 0.0022408713. The Kolmogorov-Smirnov (K-S) distance between the empirical and fitted distribution functions is 0.1173 and the corresponding p-value is 0.2748. It is clearly indicates that the WE distribution provides a good fit to the data. We further calculate the Bayes estimates of the unknown parameters by using the posterior mode approach. We consider non-informative, informative and most-informative priors for both the parameters α and λ. The results are in Table 3.

CONCLUSION

In this article, we have addressed the problem of estimating P(Y<X) for the weighted exponential distribution. Also a Bayesian approach on estimation of the parameters of this model was performed via a simulation study. All results are sited in Table 1 and 2 and they show in case of "Most Informative priors", the estimations are better than the rest.

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