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A Telescoping Numerical Scheme for the Solution of Retarded Delay Differential Systems

S. Jahangiri, S. Karimi and J. Sedighi
Department of Mathemetics, Khomeini Shahr Branch, Islamic Azad University, Isfhan, Iran

Abstract: In this study, approximate and/or exact analytical solutions of the Retarded Delay Differential Systems (RDDSs) are obtained by Telescoping Decomposition Method (TDM). TDM is a modified form of the well-known Adomian Decomposition Method (ADM). The main features of the TDM are that it deforms a difficult problem into a set of problems which are easier to solve and avoids calculating the Adomian polynomials. The analytical approximations with high accuracy are obtained using the TDM which agree well with the numerical results. Some illustrative linear and nonlinear experiments are given to indicate the validity and great potential of the proposed method for solving RDDSs.

Key words: Adomian decomposition method, telescoping decomposition method, delay differential systems

INTRODUCTION

Retarded delay differential systems with proportional delays represent a particular class of delay differential systems. Such systems play an important role in the mathematical modeling of real world phenomena such as physical problems, circuit analysis, computer-aided design, power systems, simulation of mechanical systems and more general optimal control problems; thus, they have attracted the attention of numerical analysts (Hale and Lunel, 1993; Taiwo and Odetunde, 2010; Hafshejani *et al*., 2011; Shieh *et al*., 2011; Vanani *et al*., 2011a; Vanani and Aminataei, 2009, 2010). A RDDS is presented as follows (Bellen and Zennaro, 2003):

$$\dot{U}(x) = A(x)U(x) + B(x)U_\alpha(x) + F(x),\ a \le x \le b, \quad U(x) = \Psi(x),\ x \le a \tag{1}$$

Where:

$$U(x) = [u_0(x),\ u_1(x), \ldots, u_m(x)]^T,\ u_k(x) \in C,\ k = 0, 1, \ldots, m \tag{2}$$

is the state vector and:

$$U_\alpha(x) = [u_0(\alpha_0(x)),\ u_1(\alpha_2(x)), \ldots,\ u_m(\alpha_m(x))]^T \tag{3}$$

such that $\{\alpha_k(x) \le b\}^m_{k=0}$ are delay functions; A(x) and B(x) are (m+1)-dimensional matrices which their entries are complex functions of x. Also:

$$\Psi(x) = [\psi_0(x),\ \psi_1(x), \ldots, \psi_m(x)]^T,\ \psi_k(x) \in C,\ k = 0, 1, \ldots, m \tag{4}$$

$$F(x) = [f_0(x), f_1(x), \ldots, f_m(x)]^T, f_k(x) \in C,\ k = 0, 1, \ldots, m \tag{5}$$

represent the initial vector function and known vector function, respectively.

Obviously, most of these systems cannot be solved exactly. It is therefore necessary to design efficient numerical methods to approximate their solutions. TDM as a modification of ADM is considered as a efficient method for solving RDDSs. The ADM was first introduced by Adomian (1968, 1988) and has been used to integrate various systems of functional equations (Adomian, 1988). Recently, many literatures have been developed for the application of ADM (Adomian, 1988, 1994; Adomian and Rach, 1992; Chowdhury, 2011; Jaradat, 2008; Kooch and Abadyan, 2011, 2012). Several modifications of ADM have been presented various fields of applied mathematics and physics (Hosseini, 2006; Wazwaz, 1999a, b, 2000, 2002; Vanani *et al*., 2011b).

The difficult parts of ADM is to calculate the Adomian polynomials. There are large number of literature to present an efficient algorithm for computing Adomian polynomials (Wazwaz and El-Sayed, 2001). The most popular one is the formula obtained bu Adomian (1994, 1988) as:

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n} f\left(\sum_{i=0}^{\infty} \lambda^i u_j\right)_{|\lambda=0} \tag{6}$$

Corresponding Author: S. Jahangiri, Department of Mathemetics, Khomeini Shahr Branch, Islamic Azad University, Isfhan, Iran

where, A_n denotes the Adomian polynomial of degree n, $u=\sum_{i=0}^{\infty}u_i$ is the exact solution of the problem and f(u) is the nonlinear term in the equation. It should b noted that the calculation of the Adomian polynomials is too difficult for large n and Eq. 6 can not be applied if f is a function of more than one variable, such as f = f(u, u') In addition, the ADM may diverge for some problems with special conditions (Hosseini and Nasabzadeh, 2006). Hence, to remove this difficulty we proposed a new and efficient method (TDM) for solving desired RDDS.

APPLICATION OF TDM ON RDDSs

The structure of TDM is as follows. Let us the problem (1) is given. So, we can consider its solution in the following form:

$$U(x)=U_0(x)+U_1(x)+U_2(x)+\ldots+U_n(x) \tag{7}$$

Where:

$$U_i(x)=\left[u_{0i}(x),\, u_{1i}(x),\ldots,\, u_{ni}(x)\right]^T,\quad i=0,1,\ldots,n \tag{8}$$

have to be determined sequentially upon the following algorithm:

$$\begin{cases} U_0(x)=\Psi(a)+\int_0^x F(t)dt, \\ U_1(x)=\int_0^x\left[A(t)U_0(t)+B(t)U_{a0}(t)\right]dt, \\ U_2(x)=\int_0^x\left[A(t)\sum_{k=0}^{1}U_k(t)+B(t)\sum_{k=0}^{1}U_{ak}(t)\right]dt-\int_0^x\left[A(t)U_0(t)+B(t)U_{a0}(t)\right]dt, \\ U_3(x)=\int_0^x\left[A(t)\sum_{k=0}^{2}U_k(t)+B(t)\sum_{k=0}^{2}U_{ak}(t)\right]dt-\int_0^x\left[A(t)\sum_{k=0}^{1}U_k(t)+B(t)\sum_{k=0}^{1}U_{ak}(t)\right]dt, \\ \vdots \\ U_n(x)=\int_0^x\left[A(t)\sum_{k=0}^{n-1}U_k(t)+B(t)\sum_{k=0}^{n-1}U_{ak}(t)\right]dt-\int_0^x\left[A(t)\sum_{k=0}^{n-2}U_k(t)+B(t)\sum_{k=0}^{n-2}U_{ak}(t)\right]dt, \end{cases} \tag{9}$$

and so on.

Adding the above equations, we obtain:

$$U(x)=\sum_{i=0}^{n}U_i(x)=\Psi(a)+\int_0^x F(t)\,dt+\int_0^x\left[A(t)\sum_{k=0}^{n-1}U_k(t)+B(t)\sum_{k=0}^{n-1}U_{ak}(t)\right]dt \tag{10}$$

This method is called TDM which is useful for different problems in finite, infinite, regular and irregular domains. The convergency and more details of this method are given (Al-Refai *et al*., 2008).

ILLUSTRATIVE NUMERICAL EXPERIMENTS

Here, three experiments of RDDSs are given to illustrate the efficiency and validity of the method. All experiments are considered on the interval [0, 1]. To simplify the computations, we have used Taylor series expansion of each iteration. The computations associated with the experiments discussed below were performed in Maple 14 on a PC with a CPU of 2.4 GHz.

Experiment 1: Consider the RDDS (1) with the following conditions:

$$A=\begin{bmatrix}1 & x\\ x & 1\end{bmatrix},\quad B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix},\quad U_\alpha(x)=\begin{bmatrix}u_0\left(\frac{x}{2}\right)\\ u_1\left(\frac{x}{3}\right)\end{bmatrix},\quad F(x)=\begin{bmatrix}-xe^{-x}-e^{\frac{x}{2}}\\ -2e^{-x}-xe^{x}-e^{-\frac{x}{3}}\end{bmatrix},\quad U(0)=\begin{bmatrix}1\\ 1\end{bmatrix} \tag{11}$$

The exact solution is $U(x) = [e^x, e^{-x}]$.

We have solved this problem using TDM with n = 4. The sequence of approximate solution is obtained as follows:

$$\begin{cases} v_{00}(x)=1-x-\frac{3}{4}x^2+\frac{7}{24}x^3-\frac{25}{192}x^4+O(x^5), \\ v_{01}(x)=2x-\frac{1}{4}x^2-\frac{21}{16}x^3+\frac{191}{728}x^4+O(x^5), \\ v_{02}(x)=\frac{3}{2}x^2+\frac{9}{16}x^3-\frac{381}{512}x^4+O(x^5), \\ v_{03}(x)=\frac{5}{8}x^3+\frac{755}{1536}x^4+O(x^5), \\ v_{04}(x)=\frac{25}{256}x^4+O(x^5), \end{cases}$$
$$\begin{cases} v_{10}(x)=1-3x+\frac{2}{3}x^2-\frac{37}{54}x^3-\frac{13}{324}x^4+O(x^5), \\ v_{11}(x)=2x-\frac{3}{2}x^2-\frac{7}{81}x^3-\frac{4259}{11664}x^4+O(x^5), \\ v_{12}(x)=\frac{4}{3}x^2+\frac{1}{9}x^3-\frac{2971}{34992}x^4+O(x^4), \\ v_{13}(x)=\frac{40}{81}x^3+\frac{785}{1944}x^4+O(x^4), \\ v_{14}(x)=\frac{280}{2187}x^4+O(x^5). \end{cases} \tag{12}$$

Hence, we get:

$$\begin{cases} u_0(x)=1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+O(x^5), \\ u_1(x)=1-x+\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}+O(x^5). \end{cases} \tag{13}$$

Therefore, we conclude that:

$$\begin{cases} u_0(x)=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots+\frac{x^n}{n!}+O(x^{n+1}), \\ u_1(x)=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots+(-1)^n\frac{x^n}{n!}+O(x^{n+1}) \end{cases} \tag{14}$$

This has the closed form $U(x) = [e^x, e^{-x}]^T$ which is the exact solution of the problem. To show the fastness of the method, the runtime of the proposed algorithm is also computed. Table 1 shows the results including the maximum absolute error and runtime of the method for different n.

Table 1 illustrates that the solutions of TDM are in good agreement with the exact solution. Also, the runtime of the proposed algorithm illustrate the method as a fast and powerful tool.

Experiment 2: Consider the RDDS (1) with the following conditions:

$$A=\begin{bmatrix} x & e^x & x^2 \\ e^{-x} & 1 & 1 \\ x & x^2 & x^3 \end{bmatrix},\quad B=-\begin{bmatrix} x & x^2 & 1 \\ 1 & x & x^2 \\ x & x^2 & 1 \end{bmatrix},\quad U_\alpha(x)=\begin{bmatrix} u_0(x^2) \\ u_1(x^3) \\ u_2(x^4) \end{bmatrix},\quad U(0)=\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$F(x)=\begin{bmatrix} \cos(x)-x\sin(x)-e^{2x}-x^2\cos(x)+x\sin(x^2)+x^2e^{x^3}+\cos(x^4) \\ -e^{-x}\sin(x)-\cos(x)+\sin(x^2)+xe^{x^3}+x^2\cos(x^4) \\ -\sin(x)-x\sin(x)-x^2e^x-x^3\cos(x)+x\sin(x^2)+x^2e^{x^3}+\cos(x^4) \end{bmatrix} \tag{15}$$

The exact solution is $U(x) = [\sin(x), e^x, \cos(x)]^T$.

We have solved this problem using TDM with n = 6. The sequence of approximate solution is obtained as follows:

$$\begin{cases} v_{00}(x)=x-x^2-\dfrac{7x^3}{6}-\dfrac{x^4}{12}+\dfrac{x^5}{120}+\dfrac{11x^6}{90}+O(x^7), \\ v_{01}(x)=\dfrac{x^3}{6}-\dfrac{x^4}{24}-\dfrac{41x^5}{120}+\dfrac{269x^6}{720}+O(x^7), \\ v_{02}(x)=x^2+\dfrac{2x^3}{3}-\dfrac{3x^4}{8}+\dfrac{7x^5}{120}-\dfrac{199x^6}{720}+O(x^7), \\ v_{03}(x)=\dfrac{x^3}{6}+\dfrac{3x^4}{8}+\dfrac{2x^5}{15}-\dfrac{27x^6}{80}+O(x^7), \\ v_{04}(x)=\dfrac{x^4}{8}+\dfrac{7x^5}{60}+\dfrac{17x^6}{360}+O(x^7), \\ v_{05}(x)=\dfrac{x^5}{30}+\dfrac{11x^6}{180}+O(x^7), \\ v_{06}(x)=\dfrac{7x^6}{120}+O(x^7), \end{cases}$$

$$\begin{cases} v_{10}(x)=1-x+\dfrac{7x^3}{6}-\dfrac{x^4}{12}+\dfrac{23x^5}{120}+\dfrac{x^6}{180}+O(x^7), \\ v_{11}(x)=2x-\dfrac{3x^3}{2}+\dfrac{7x^4}{24}+\dfrac{17x^5}{40}-\dfrac{19x^6}{720}+O(x^7), \\ v_{12}(x)=\dfrac{x^2}{2}-\dfrac{x^4}{4}-\dfrac{29x^5}{60}-\dfrac{x^6}{240}+O(x^7), \\ v_{13}(x)=\dfrac{x^3}{2}-\dfrac{x^4}{12}-\dfrac{31x^5}{120}+\dfrac{x^6}{40}+O(x^7), \\ v_{14}(x)=\dfrac{x^4}{6}+\dfrac{3x^5}{40}-\dfrac{11x^6}{360}+O(x^7), \\ v_{15}(x)=\dfrac{7x^5}{120}+\dfrac{x^6}{60}+O(x^7), \\ v_{16}(x)=\dfrac{11x^6}{720}+O(x^7). \end{cases}$$

$$\begin{cases} v_{20}(x)=1+x-\dfrac{x^2}{2}-\dfrac{x^3}{3}-\dfrac{5x^4}{24}-\dfrac{x^5}{15}+\dfrac{53x^6}{240}+O(x^7), \\ v_{21}(x)=-x+\dfrac{x^3}{3}-\dfrac{x^4}{2}-\dfrac{7x^5}{30}+\dfrac{31x^6}{72}+O(x^7), \\ v_{22}(x)=\dfrac{x^4}{2}+\dfrac{x^5}{30}-\dfrac{85x^6}{144}+O(x^7), \\ v_{23}(x)=\dfrac{x^4}{4}+\dfrac{3x^5}{30}-\dfrac{11x^6}{48}+O(x^7), \\ v_{24}(x)=\dfrac{x^5}{30}+\dfrac{7x^6}{48}+O(x^7), \\ v_{25}(x)=\dfrac{x^6}{48}+O(x^7), \\ v_{26}(x)=0+O(x^7), \end{cases} \tag{16}$$

Thus, we obtain:

$$\begin{cases} u_0(x)=x-\dfrac{1}{6}x^3+\dfrac{1}{120}x^5+O(x^7), \\ u_1(x)=1+x+\dfrac{x^2}{2}+\ldots+\dfrac{x^6}{720}+O(x^7), \\ u_2(x)=1-\dfrac{1}{2}x^2+\dfrac{1}{24}x^4-\dfrac{1}{720}x^6+O(x^7) \end{cases} \tag{17}$$

Therefore, we conclude that:

$$\begin{cases} u_0(x)=x-\dfrac{1}{3!}x^3+\dfrac{1}{5!}x^5+\ldots+\dfrac{(-1)^n}{(2n+1)!}x^{2n+1}+O(x^{2n+3}) \\ u_1(x)=1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\dfrac{x^4}{4!}+\ldots+\dfrac{x^n}{n!}+O(x^{n+1}), \\ u_0(x)=1-\dfrac{1}{2!}x^2+\dfrac{1}{4!}x^4+\ldots+\dfrac{(-1)^n}{(2n)!}x^{2n}+O(x^{2n+2}), \end{cases} \tag{18}$$

This has the closed form $U(x) = [\sin(x), e^x, \cos(x)]^T$, which is the exact solution of the problem. The runtime of the method is also computed for different n to obtain a suitable approximation. Table 2 shows the results including the maximum absolute error and runtime of the method for different n.

The results also confirm the method as a fast method. Therefore, using the proposed method is preferred to facilitate the computations as shown in Table 2.

Table 1: Maximum absolute error and runtime of the method for different n of experiment 1

n	Max absolute error $u_0(x)$	$u_1(x)$	Run time (sec)
6	2.26×10^{-4}	1.71×10^{-4}	0.094
10	2.73×10^{-8}	2.31×10^{-8}	0.109
14	8.15×10^{-13}	7.19×10^{-13}	0.218
18	1.00×10^{-15}	1.00×10^{-16}	0.390

Table 2: Maximum absolute error and runtime of the method for different n of experiment 2

n	Max absolute error $u_0(x)$	$u_1(x)$	$u_2(x)$	Run time (sec)
6	1.95×10^{-4}	2.26×10^{-4}	2.45×10^{-5}	0.889
10	2.48×10^{-8}	2.73×10^{-8}	2.07×10^{-9}	2.589
14	7.61×10^{-13}	8.15×10^{-13}	4.7610^{-14}	4.680
18	1.00×10^{-16}	1.00×10^{-15}	1.00×10^{-16}	7.613

Experiment 3: Consider the RDDS (1) with the following conditions:

$$A = B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}, \quad U_\alpha(x) = \begin{bmatrix} u_0\left(\frac{x}{2}\right) \\ u_1\left(\frac{x}{3}\right) \\ u_2\left(\frac{x}{4}\right) \\ u_3\left(\frac{x}{5}\right) \end{bmatrix}, \quad U(0) = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$F(x) = \begin{bmatrix} -\sinh(x) - \sin(x) - e^{x^2} - \sinh\left(\frac{x}{2}\right) - \cosh\left(\frac{x}{3}\right) - \sin\left(\frac{x}{4}\right) - e^{\frac{x^2}{25}} \\ -\cosh(x) - \sin(x) - e^{x^2} - \sinh\left(\frac{x}{2}\right) - \cosh\left(\frac{x}{3}\right) - \sin\left(\frac{x}{4}\right) - e^{\frac{x^2}{25}} \\ \cos(x) - \sinh(x) - \cosh(x) - \sin(x) - e^{x^2} - \sinh\left(\frac{x}{2}\right) - \cosh\left(\frac{x}{3}\right) - \sin\left(\frac{x}{4}\right) - e^{\frac{x^2}{25}} \\ 2xe^{x^2} - \sinh(x) - \cosh(x) - \sin(x) - e^{x^2} - \sinh\left(\frac{x}{2}\right) - \cosh\left(\frac{x}{3}\right) - \sin\left(\frac{x}{4}\right) - e^{\frac{x^2}{25}} \end{bmatrix} \quad (19)$$

The exact solution is $U(x) = [\sinh(x), \cosh(x), \sin(x), e^{x^2}]^T$.

We have solved this problem using TDM with n = 7. The s following results are obtained:

$$\begin{cases} u_0(x) = x + \frac{1}{6}x^3 + \frac{1}{120}x^5 + \frac{1}{5040}x^7 + O(x^9), \\ u_1(x) = 1 + \frac{1}{2}x^2 + \frac{1}{24}x^4 + \frac{1}{720}x^6 + O(x^8), \\ u_2(x) = x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + O(x^9), \\ u_3(x) = 1 + x^2 + \frac{1}{2}x^4 + \frac{1}{6}x^6 + O(x^8) \end{cases} \quad (20)$$

Therefore, we conclude that:

$$\begin{cases} u_0(x) = x + \frac{1}{3!}x^3 + \ldots + \frac{1}{(2n+1)!}x^{2n+1} + O(x^{2n+3}), \\ u_1(x) = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \ldots + \frac{1}{(2n)!}x^{2n} + O(x^{2n+2}), \\ u_2(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \ldots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + O(x^{2n+3}), \\ u_3(x) = 1 + x^2 + \frac{1}{2!}x^4 + \frac{1}{3!}x^6 + \ldots + \frac{1}{n!}x^{2n} + O(x^{2n+2}) \end{cases} \quad (21)$$

This has the closed form $U(x) = [\sinh(x), \cosh(x), \sin(x)\ e^{x^2}]^T$ which is the exact solution of the problem. Table 3 shows the results including the maximum absolute error and runtime of the method for different n.

The high accuracy of the method can be observed again. Therefore, we prefer the proposed method for solving RDDSs.

Table 3: Maximum absolute error and runtime of the method for different n of experiment 3

	Max absolute error				
n	$u_0(x)$	$u_1(x)$	$u_2(x)$	$u_3(x)$	Run time (sec)
10	2.52×10^{-8}	2.09×10^{-9}	2.48×10^{-8}	1.61×10^{-3}	0.250
20	1.00×10^{-16}	1.00×10^{-16}	1.00×10^{-16}	2.73×10^{-8}	2.543
30	1.00×10^{-16}	1.00×10^{-16}	1.00×10^{-16}	5.10×10^{-14}	16.786

CONCLUSION

This study has presented a reliable algorithm based on the TDM to solve RDDSs. Some experiments are given to illustrate the validity and accuracy of the proposed method. The main feature of the TDM is to avoid calculating the Adomian polynomials. Furthermore, this method yields the desired accuracy only in a few terms in a series form of the exact solution. The method is also quite straightforward to write computer code. The reliability of TDM and the reduction in computations give TDM a wider applicability.

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