Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 12 (15): 1596-1601, 2012
ISSN 1812-5654 / DOI: 10.3923/jas.2012.1596.1601

Combining Robust H_∞ Control Performance and Parametric Output Feedback Eigenstructure Assignment Using Enhanced LMI Characteristics

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Abstract: This study proposes a method for combining parametric Eigenstructure Assignment with H_∞ control performance. In practical cases most of the time, measurement of the states is not possible so we have to use the output feedback controller for satisfying control performances. In the proposed approach in this article, designing robust H_∞ controller is considered. The output feedback controller is designed such that the H_∞ control performance is satisfied. Based on Enhanced LMI characterizations and eigenstructure assignment parameterization, the problem is translated to LMI problem. The suggested LMI can be solved by LMI toolbox or YALMIP easily. Also, in the end of this manuscript, an example is given for demonstrating the effectiveness of proposed method.

Key words: Eigenstructure, robust, linear matrix inequality, output feedback

INTRODUCTION

Eigenstructure assignment one of the most powerful methods in multivariable control systems design. According to the results of Konstantopoulos and Antsaklis (1996) and degrees of freedom in eigenstructure assignment method by using output feedback (Duan, 2003), in recent two decades using eigenstructure assignment has been developed in all field of multivariable control systems. Reconfigurable control design (Konstantopoulos and Antsaklis, 1996), Helicopter control systems (Manness and Smiths, 1992), Control reconfiguration in second order dynamic systems (Wang *et al*., 2005), missile control systems (Sobel and Cloutier, 1992), robust control design in electrical induction machine (Duval *et al*., 2006) are examples of using eigenstructure assignment.

Most of the methods that have used eigenstructure assignment for designing robust control, only consider eigenstructure assignment sensitivity and they haven't combined eigenstructure assignment with other robust control performances. In recent years, some researches have been performed for such problems. Apkarian *et al*. (2001) a new method for combining eigenstructure assignment with H_2 constraint has been suggested. He *et al*. (2004) a new method for designing state feedback control to satisfy multi-objective controller design (specially H_∞ constraint) has been proposed. However, this method can be used when all states are available.

In this study, combining eigenstructure assignment problem and robust H_∞ control design is investigated while states aren't measurable. According to parametric eigenstructure assignment (Duan, 2003) characteristics of enhanced LMI (Shimomura *et al*., 2001), a method is proposed. Based on this method the output feedback controller will be designed such that robust H_∞ control performance is satisfied. Also, the method is developed for dynamical output feedback. By developing this method for dynamical output feedback, degrees of freedom in dynamical feedback can be used.

In this study M* is transpose conjugate of M.M>0 means M is positive definite matrix. H_∞ is used for Hermitian matrix. Diag(.) is used for diagonal matrix.

PARAMETRIC EIGENSTRUCTURE ASSIGNMENT VIA OUTPUT FEEDBACK

Consider the following LTI system:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (1)$$

where, $x \in R^n$, $u \in R^r$, $y \in R^m$ are, respectively, state vector, input vector, output vector. A, B and C are known matrices with appropriate dimensions. By applying the static output feedback of the form:

$$u = Ky \quad (2)$$

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In Eq. 1, the closed loop matrix is obtained as follow:

$$A_{cl} = A+BKC \quad (3)$$

Based on eigenstructure assignment definition, we have:

$$A_{cl}V = JV \quad (4)$$

where, V is right eigenvectors matrix and J represents the Jordan form of the closed loop matrix (3). We know that the eigenvalues of non-defective matrix are distinct and they have less sensitivity to parameter changes (Duan, 2003). Assume that the closed loop matrix A_{cl} is non-defective then, the Jordan form of A_{cl} is diagonal. The Jordan form of A_{cl} is:

$$F = \text{diag}(s_1, s_2,..., s_n) \quad (5)$$

where, s_i (i = 1,2,...,n) are desired closed loop eigenvalues. Also s_i (i = 1,2,...,n) are self conjugate complex numbers. Assume (A,B) is controllable. By applying SVD to the matrix $[B\ s_iI-A]$, we have:

$$\Psi^c_i, [B\ s_iI-A]\varphi^c_i = [\Sigma^c_i\ o] \quad (6)$$

where, Ψ^c_i, φ^c_i are orthogonal matrices with appropriate dimensions and Σ^c_i is non-singular diagonal matrix that diagonal elements are the singular values of $[B\ s_iI-A]$. Partition φ^c_i as:

$$\varphi^c_i = \begin{bmatrix} * & D_i \\ * & N_i \end{bmatrix} \quad N_i \in R^{n\times n}, D_i \in R^{r\times n} \quad (7)$$

According to the above preliminaries and the results of Duan (2003), the following theorem for parametric eigenstructure assignment by using output feedback is described (Duan, 2003).

Theorem 1: Consider (A,B) is controllable and rank (C) = m then here exist matrices $V\epsilon C^{n\times n}$, det(V) ≠ 0 and $K\epsilon R^{r\times m}$ satisfying (4), iff there exists parameter vectors $f_i\epsilon C^r$ which is satisfying the following constraints:

$$s_i = \bar{s}_j \text{ Then } f_i = \bar{f}_j$$

$$\det[N_1.f_1\ N_2.f_2 \cdots N_n.f_n] \neq 0$$

$$\text{rank}\begin{bmatrix} CN_1f_1 & CN_2f_2 & \ldots & CN_nf_n \\ D_1f_1 & D_2f_2 & \ldots & D_nf_n \end{bmatrix} = m$$

If the above conditions are satisfied, all matrices $K\epsilon R^{r\times m}$ and $V\epsilon C^{n\times n}$ (det(V) ≠ 0 are obtained as follow:

$$K = W(CV)^{-1}[(CV)(CV)^T]^{-1},\ W = KV \quad (8)$$

$$\begin{cases} V = [N_1f_1 \quad N_2f_2 \ \ldots\ N_nf_n] \\ W = [D_1f_1 \quad D_2f_2 \ \ldots\ D_nf_2] \end{cases} \quad (9)$$

where, N_i, D_i are determined by Eq. 7. In special case rank(C) = n the output feedback is given by:

$$K = W(CV)^{-1} \quad (10)$$

PROBLEM FORMULATION

Consider the following LTI systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1w(t) + B_2u(t) \\ z(t) = C_1x(t) + D_{12}u(t) \\ y(t) = C_2x(t) \end{cases} \quad (11)$$

where, $x\epsilon R^n$, $w\epsilon L^q_2[0,\infty)$, $u\epsilon R^r$, $z\epsilon R^1$, $y\epsilon R^m$, respectively are state vector, exogenous disturbance vector, input vector, state combination (objective functional signal) and output vector. A, B_1, B_2, C_1, C_2 and D_{12} are known matrices with appropriate dimensions. Assume that (A,B_2) is controllable and $D^*_{12} D_{12}>0$. By applying the static output feedback of the following form to Eq. 11:

$$u(t) = Ky(t) \quad (12)$$

The closed loop system is obtained as:

$$\begin{cases} \dot{x}(t) = A_{cl}x(t) + B_1w(t) + B_2u(t) \\ z(t) = (C_1 + D_{12}K) \\ y(t) = C_2x(t) \end{cases} \quad (13)$$

Based on Enhanced LMI characterizations that have been proposed (Shimomura *et al*., 2001), we combine eigenstructure assignment problem with H_∞ control design. Consider the closed loop system Eq. 13. There exist positive definite $Y\epsilon H_n$ and $S\epsilon H_n$ such that:

$$\begin{bmatrix} A_{cl}S+S^*A_{cl} & * & * & * \\ C_{cl}S & -I_1 & * & * \\ B_1^* & 0 & -\gamma_\infty^2 I_q & * \\ Y-S+\varepsilon S^*A_{cl}^* & \varepsilon S^*C_{cl}^* & 0 & -\varepsilon(S+S^*) \end{bmatrix} < 0, \quad (14)$$
$$C_{cl} = (C_1 + D_{12}KC_2),\ A_{cl} = (A + B_2KC_2)$$

For some ε>0, iff $\|T_{zw}\|_\infty < \gamma_\infty$. Note that S is variable which is represented by the Enhanced LMI characterizations.

According the previous results and theorem (1) we state the main problem of this study.

1. **Main problem:** Consider the system (13) and a set of desired closed loop eigenvalues $M = \{S_i, i = 1,2,...,n\}$. We want to determine the output feedback of the from Eq. 12 such that H_∞ control performance $\varepsilon>0$, $\|T_{zw}\|_\infty <\gamma_\infty$ or based on theorem 1 the LMI (14) is satisfied.

According to theorem 1 and Enhanced LMI (14) the following theorem is represented to solve the main problem.

Theorem 2: Consider the system (1) that (A,B_2) is controllable. There exist the output feedback of the form Eq. 12 for the main problem, if there exist $0<\varepsilon<1$ and matrices $Y>0$, M such that:

$$\begin{bmatrix} NMJ+(NMJ)^* & * & * & * \\ C_1NJ+D_{12}DJ & -I_1 & * & * \\ B_1^* & 0 & -\gamma_\infty^2 I_q & * \\ Y-NM+(NMJ)^* & \varepsilon(C_1NM+D_{12}DM)^* & 0 & -\varepsilon(NM+(NM)^*) \end{bmatrix} < 0 \tag{15}$$

where, $J = \text{diag}(s_1, s_2,...,s_n)$, also the following conditions are satisfied:

$$s_i = \bar{s}_j \text{ Then } f_i = \bar{f}_j$$

$$\det[N_1.f_1\ N_2.f_2 \cdots N_n.f_n] \neq 0$$

$$\text{rank}\begin{bmatrix} CN_1f_1 & CN_2f_2 & \dots & CN_nf_n \\ D_1f_1 & D_2f_2 & \dots & D_nf_n \end{bmatrix} = m$$

If the above conditions are met, the output feedback can be calculated as:

$$\begin{gathered} K = DM(C_2DM)^T[(C_2DM)(C_2DM)^T]^{-1} \\ M = \text{diag}(f_1, f_2, ..., f_n) \in R^{nr\times n} \end{gathered} \tag{16}$$

$$\begin{gathered} N = [N_1\ \ N_2\ \dots\ N_n] \in R^{n\times nr} \\ D = [D_1\ \ D_2\ \dots\ D_n] \in R^{n\times nr} \end{gathered}$$

Proof: Based on the above definition we can rewrite (16) as follows:

$$\begin{gathered} V = [N_1\ \ N_2\ \dots\ N_n]\begin{bmatrix} f_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & f_n \end{bmatrix} = NM \\ W = [D_1\ \ D_2\ \dots\ D_n]\begin{bmatrix} f_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & f_n \end{bmatrix} = DM \end{gathered} \tag{17}$$

The output feedback is obtained as:

$$K = DM(C_2DM)^T[(C_2DM)(C_2DM)^T]^{-1} \tag{18}$$

By substituting V instead of S in Eq. 14, the closed loop system is satisfied the H_∞ control performance if the following LMI is feasible:

$$\begin{bmatrix} A_{cl}V+V^*A_{cl} & * & * & * \\ C_{cl}V & -I_1 & * & * \\ B_1^* & 0 & -\gamma_\infty^2 I_q & * \\ Y-V+\varepsilon V^*A_{cl}^* & \varepsilon V^*C_{cl}^* & 0 & -\varepsilon(V+V^*) \end{bmatrix} < 0, \tag{19}$$
$$C_{cl} = (C_1 + D_{12}KC_2),\ A_{cl} = (A + B_2KC_2)$$

According to Eq. 4 and $C_{cl} = C_1+D_{12}KC_2$ the following LMI is obtained:

$$\begin{bmatrix} VJ+(VJ)^* & * & * & * \\ C_1V+D_{12}KC_2V & -I_1 & * & * \\ B_1^* & 0 & -\gamma_\infty^2 I_q & * \\ Y-V+\varepsilon(VJ)^* & \varepsilon V^*(C_1+D_{12}KC_2)^* & 0 & -\varepsilon(V+V^*) \end{bmatrix} < 0, \tag{20}$$
$$C_{cl} = (C_1 + D_{12}KC_2),\ A_{cl} = (A + B_2KC_2)$$

If we substituted C_2 instead of C in (8), where $W = KC_2V$ and also by substituting V,W from (17) the following equation is obtained:

$$\begin{bmatrix} NMJ+(NMJ)^* & * & * & * \\ C_1NJ+D_{12}DJ & -I_1 & * & * \\ B_1^* & 0 & -\gamma_\infty^2 I_q & * \\ Y-NM+(NMJ)^* & \varepsilon(C_1NM+D_{12}DM)^* & 0 & -\varepsilon(NM+(NM)^*) \end{bmatrix} < 0,$$
$$C_1V + D_{12}KV = C_1NM + D_{12}W \tag{21}$$

And the proof is completed. According to the above theorem, we can combine the parametric eigenstructure assignment problem with H_∞ control problem. If the LMI (21) is feasible, then the main problem has a solution. Based on the method that is proposed in this note, we design the parametric eigenstructure assignment elements f_i such that, in addition to satisfy H_∞ control performance, other performances are met too. Note that for solving main problem, satisfying all of the following conditions are important:

$$s_i = \bar{s}_j \text{ Then } f_i = \bar{f}_j$$

$$\det[N_1.f_1\ N_2.f_2 \cdots N_n.f_n] \neq 0$$

$$\text{rank}\begin{bmatrix} CN_1f_1 & CN_2f_2 & \dots & CN_nf_n \\ D_1f_1 & D_2f_2 & \dots & D_nf_n \end{bmatrix} = m$$

DYNAMICAL CONTROLLER DESIGN

For controllable system (8) if suppose that B,C are full rank then static output feedback can assign max (m,r) eigenvalues and corresponding eigenvectors (Konstantopoulos and Antsaklis, 1996). In general, we may desire to exercise some control over more than max(m,r) closed loop eigenvalues. So we generalize this method by using dynamical feedback of the form:

$$\begin{cases} \dot{x}_c = k_{11}x_c + k_{12}y \\ u = k_{21}x_c + k_{22}y \end{cases} \tag{22}$$

where, $x_c \in R^{n_c}$. If combine the above controller with (1), then closed loop system is obtained:

$$\begin{aligned} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ x_c \end{bmatrix} + \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}\tilde{u} \\ \tilde{y} &= \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}\begin{bmatrix} x \\ x_c \end{bmatrix} \\ \tilde{u} &= \begin{bmatrix} k_{22} & k_{21} \\ k_{12} & k_{11} \end{bmatrix}\tilde{y} \end{aligned} \tag{23}$$

Assume that:

$$\begin{aligned} &\bar{x} = \begin{bmatrix} x \\ x_c \end{bmatrix}, \ \bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \\ &\bar{B} = \begin{bmatrix} B & 0 \\ 0 & I \end{bmatrix}, \ \bar{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \end{aligned} \tag{24}$$

Based on (24), Eq. 23 changes as follows:

$$\begin{aligned} &\dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u} \\ &\bar{y} = \bar{C}\bar{x}, \qquad \bar{K} = \begin{bmatrix} k_{22} & k_{21} \\ k_{12} & k_{11} \end{bmatrix} \\ &\bar{u} = \bar{K}\bar{y} \end{aligned} \tag{25}$$

According to the above equations is clear that dynamical output feedback controller can be translated as static output feedback. A number of eigenvalues of this system (25) is $n+n_c$. By choosing appropriate n_c, all of the eigenvalues can be assigned. We can use theses degrees of freedom in dynamical feedback, for satisfying other control objectives.

For solving the main problem in dynamical output feedback case, By combining Eq. 11 and 22 the following equations are obtained:

$$\begin{aligned} &\begin{cases} \bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \bar{B}_2 = \begin{bmatrix} B_2 & 0 \\ 0 & L_{n_c} \end{bmatrix}, \bar{K} = \begin{bmatrix} k_{22} & k_{21} \\ k_{12} & k_{11} \end{bmatrix} \\ \tilde{C}_2 = \begin{bmatrix} C_2 & 0 \\ 0 & I_{n_c} \end{bmatrix}, \tilde{D}_{12} = \begin{bmatrix} D_{12} & 0 \\ 0 & 0 \end{bmatrix}, \tilde{C}_1 = \begin{bmatrix} C_0 & 0 \end{bmatrix} \end{cases} \\ &\begin{cases} \dot{\tilde{x}} + \tilde{A}\tilde{x} + \tilde{B}_1 w + \tilde{B}_2 u \\ \tilde{z} + \tilde{C}_1\tilde{x} + \tilde{D}_{12}u, \qquad \tilde{x} = \begin{bmatrix} x & x_c \end{bmatrix}^T \\ \tilde{y} = \tilde{C}_2\tilde{x} \end{cases} \end{aligned} \tag{26}$$

By substituting $\tilde{A}, \tilde{B}_1, \tilde{B}....$ instead of $A, B_1, ...$, respectively, the LMI (21) can be written for new augmented system (26). Based on the new augmented system (26) and using dynamical system, Eq. 21 can be represented as:

$$\begin{bmatrix} \tilde{N}\tilde{M}\tilde{J} + (\tilde{N}\tilde{M}\tilde{J})^* & * & * & * \\ \tilde{C}_1\tilde{N}\tilde{M} + \tilde{D}_{12}\tilde{D}\tilde{M} & -I_{1+n_c} & * & * \\ \tilde{B}_1^* & 0 & -\gamma_\infty^2 I_{q+n_c} & * \\ \tilde{Y} - \tilde{N}\tilde{M} + \varepsilon(\tilde{N}\tilde{M}\tilde{J})^* & \varepsilon(\tilde{C}_1\tilde{N}\tilde{M} + \tilde{D}_{12}\tilde{D}\tilde{M})^* C_{c1}^* & 0 & -\varepsilon(\tilde{N}\tilde{M} + (\tilde{N}\tilde{M})^*) \end{bmatrix} < 0 \tag{27}$$

where, $\tilde{M}, \tilde{D}, \tilde{N}$ are obtained from the similar Eq. 17, for augmented system (26).

According to degrees of freedom in dynamical controller (by selection n_c), the controller can be determined such that in addition to satisfy the main problem, satisfying other control objective.

NUMERICAL EXAMPLE

In order to investigate the proposed method the following example is stated. Consider the simplified third order linearized dynamics of a small aircraft witch is given by (Satoh and Sugimoto, 2004). We assume that the system has 40% uncertainty at A(1,1), A(2,2)and A(3,3). According to (Satoh and Sugimoto, 2004) this structured uncertainty can be described as follows:

$$\begin{aligned} &\dot{x} = (A + \Delta A(t))x(t) + Bu(t) \\ &y(t) = Cx(t) \end{aligned} \tag{28}$$

and

$$x = \begin{bmatrix} \beta \\ p \\ r \end{bmatrix}, \ u = \begin{bmatrix} s_a \\ s_r \end{bmatrix}$$

where, p and r are roll rate (rad/s) and yaw rate, respectively. β is the side slip angle (rad) and s_a, s_r are the aileron deflection and rudder deflection angle(rad). Our objective is to design the output feedback controller K such that the H_∞ performance $\|T_{zw}\|_\infty < 1$ is satisfied. The matrices in Eq. 11 are defined as:

$$A = \begin{bmatrix} -0.228 & 0.059 & -0.998 \\ -34.9 & -1.516 & 0.872 \\ 18.69 & 0.0379 & -0.564 \end{bmatrix}, B = \begin{bmatrix} -0.0037 & 0.0395 \\ 21.3 & 21.3 \\ 0.504 & -8.29 \end{bmatrix}$$

$$\hat{H} = [0.4A(1,1), 0, 0]^T, \ E_a = [1, 0, 0], F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\|\Delta(t)\| < 1, \Delta A(t)\hat{H}\Delta(t)E_a$$

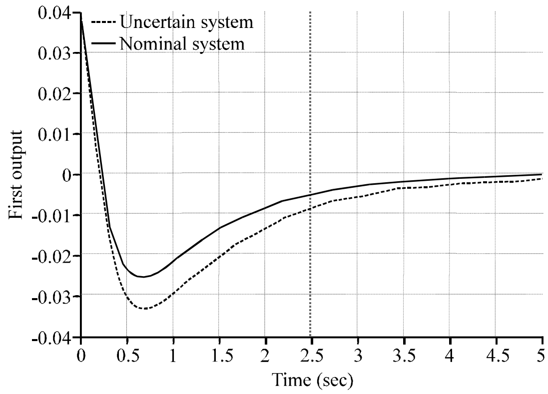

Fig. 1: First initial output response

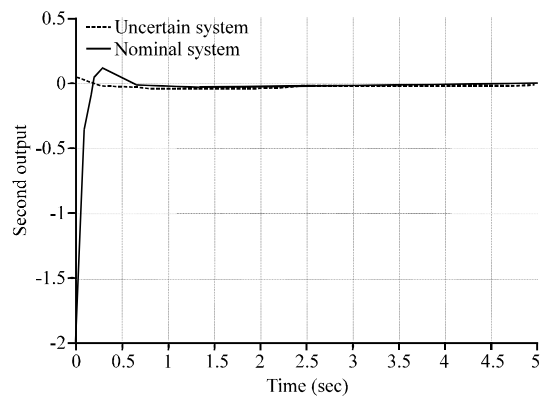

Fig. 2: Second initial output response

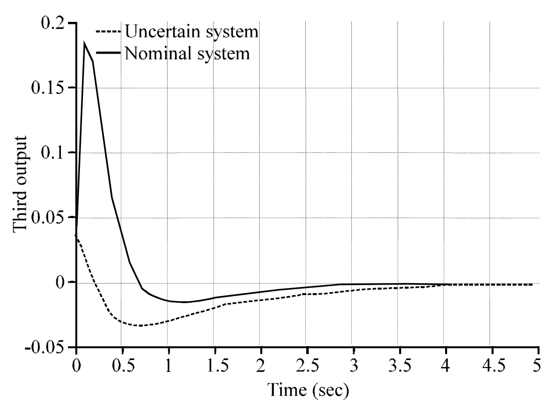

Fig. 3: Third initial output response

Note that all of the calculation have been performed by MATLAB, Sedumi 1.1 and YALMIP 2.4. (Lofberg, 2004) Also the desired eigenvalues are selected as:

$$\Gamma = \{-1, -5, 10\}$$

By applying theorem 2 for $\varepsilon = 0.01$ and $\gamma = 1$ the output feedback and parameter matrix are obtained as:

$$\Lambda = \begin{bmatrix} 0.4189 & 0 & 0 \\ 2.2537 & 0 & 0 \\ 0 & 0.1947 & 0 \\ 0 & 0.2504 & 0 \\ 0 & 0 & 0.1779 \\ 0 & 0 & 0.6173 \end{bmatrix}$$

$$K = \begin{bmatrix} 1.0294 & -0.3369 & 0.2182 \\ 1.7119 & 0.2092 & 0.2126 \end{bmatrix}$$

Based on the above controller the eigenvalues are given by:

$\lambda_1 = 1.0009$
$\lambda_2 = -5.0002$
$\lambda_3 = -10.0011$

The responses of the system (28) for initial condition [0.04 -1 0] are shown in Fig. 1-3. Figure 1 demonstrates the initial response of the first output, Fig. 2 shows the second initial response and Fig. 3 demonstrates the third initial response.

In Fig. 1-3, the H_∞ control design of uncertain and nominal system are shown. The uncertain system with robust H_∞ output feedback controller is calculated from theorem 2. It can be seen that the proposed method is more effective for designing robust H_∞ output feedback controller. It has been seen that the proposed controller can tolerate in the presence of uncertainties.

CONCLUSION

In this study robust H_∞ control design via eigenstructure assignment is considered. According to enhanced LMI and parametric eigenstructure a method is proposed such that H_∞ control performance is satisfied. Also based on degrees of freedom in dynamical output feedback the proposed method is developed. In order to demonstrate the effectiveness of proposed method the example is considered and from this example we saw that the proposed robust controller can deal with the uncertainty. Also, when we want to satisfy other control objective in addition to robust performance we can use the dynamical controller. By using dynamical controller, degrees of freedom in dynamical controller can be used.

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