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Three Terminal Quantum Dot System

Narra Sunil Kumar, N. Chandrasekar and G. Pavan
School of Electrical and Electronics Engineering, SASTRA University, Thanjavur, Tamil Nadu-613401, India

Abstract: In this study, the transmission rate for the three terminal quantum dot system is determined using Keldysh nonequilibrium Green's function technique for interacting and non-interacting cases. The three terminal quantum dot systems consist of three leads and three quantum dots that are arranged in a triangular form. Each led is coupled with each dot. The lesser and retarded Green's functions are used for the calculations of transmission rates and how the transmission rates vary for interacting and non-interacting system are studied is investigated.

Key words: Green's function, quantum dot system, transmission rates, non-interacting system

INTRODUCTION

Quantum dots are mesoscopic particles whose dimensions are of the order of tens of nanometers. The Quantum Dot (QD) is also termed as artificial atom because of resemblance of its energy levels to that of an atom. QDs are semiconductor structures whose electronic energy levels depend upon the shape and the size of the QD. The electronic energy levels of the dot can be varied by varying the lead voltage which is coupled to the Quantum dot. Initially the calculation of conductance for a two terminal single QD was done through the equivalent RC circuit model by neglecting the coherence between the dots (Wang and Chou, 1994). The theoretical and experimental approach for the two terminal single quantum dot system is discussed by Yuan and Gu (1993) and Kastner (2000). Coulomb blockade effect plays a main role for a two terminal single dot system (Korotkov *et al*., 1995). Calculation of conductance in terms of probability approach was done using Gibbs distribution formalism (Beenakker, 1991). The conductance for a system of two terminal array of QDs is analyzed as the extension of two terminal single quantum dot and it was found that the conduction characteristics depend number of dots in the quantum dot system (Chen *et al*., 1994). There will be conduction peaks for the even number of dots and the peaks will not be present for odd number of dots (Chen *et al*., 1994). The transmission for a QD array with quantum wires as leads for a non-interacting system can be studied by using Anderson Hamiltonian (Rostami *et al*., 2011). The conductance for different configurations (like 1D array of circular order) for a two terminal system can be done by using Green function (Li *et al*., 2005). The calculation of transmission probability for a two terminal array of circular dots can be calculated from the Green's function. In this article the three terminal QD system can be analyzed by using Keldysh nonequilibrium Green's function technique.

THREE TERMINAL QD SYSTEM

This QD system consists of three terminals named as lead M, lead L and lead R and three QDs named as QD1, QD2 and QD3 in Fig. 1. Dots QD1, QD2 and QD3 are coupled to leads M, L and R respectively. All the QDs are similar to each other and are arranged in a triangular form as shown in Fig. 1. In this QD system each QD consists of a single pair of electrons accommodated in a single energy level. The QDs are placed such that there is no interaction between dots. The lead M is given a positive potential and leads L and R are given a negative potential. The leads and QDs are considered as non-interacting system. To this system we will apply retarded and lesser Green's function to derive the conduction equation. The total Hamiltonian of this system can be given as:

$$H = H_L + H_C + H_{dot}$$

$$H_L = \sum_{k\sigma,\alpha} \varepsilon^0_{k\sigma,\alpha} C^+_{k\sigma,\alpha} C_{k\sigma,\alpha}, \alpha \in L, R, M.$$

$$H_C = \sum_k (V_L C^+_{k\sigma,L} d_{k\sigma,\alpha} + V_R C^+_{k\sigma,R} d_{k\sigma,\alpha} + V_M C^+_{k\sigma,M} d_{k\sigma,\alpha})$$

$$H_{dot} = \sum_{i=1}^{3} \varepsilon^0_i d^+_{i\sigma} d_{i\sigma} + V_{1,2} d^+_{2\sigma} d_{1\sigma} + V_{1.3} d^+_{3\sigma} d_{1\sigma}$$

H_L, H_C and H_{dot} are the Hamiltonians of the coupling between the dots and the leads. c^+ and d^+ are the creation

Corresponding Author: N. Chandrasekar, School of Electrical and Electronics Engineering, SASTRA University, Thanjavur, Tamil Nadu-613401, India

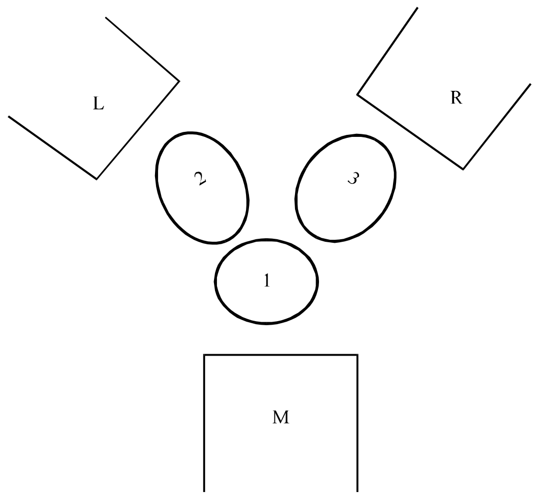

Fig. 1: Three terminal quantum dot system

operators in the lead and the dot respectively. $\varepsilon^0{}_1$ is the Fermi energy level of the ith QD. The current equation can be written as:

$$J = \frac{ie}{h}\langle[H, N_L]\rangle \tag{1}$$

N_L is the number operator and Eq. 1 denote the commutator relation HN_L-N_LH. The total expression for the Hamiltonian is given:

$$\begin{aligned} H = {} & V_m C_L{}^+ d_{2\sigma} C_m C_m^+ + V_m C_R{}^+ d_{3\sigma} C_m C_m^+ + V_m C_M{}^+ d_\sigma C_m C_m^+ \\ & - V_m C_L{}^+ d_\sigma C_m C_m^+ - V_m C_R{}^+ d_\sigma C_m C_m^+ - V_m C_M{}^+ d_\sigma C_m C_m^+ \end{aligned} \tag{2}$$

The net flow of electrons arises when there is difference in electron tunneling from positive-potential lead to QD system and from QD system to negative-potential lead or from QD system to positive lead and QD system to negative leads. If we apply creation and annihilation operator for one electron $C^+C = 1$. The first three terms in Eq. 2 refer to the tunneling of electron from leads to QD system and remaining terms refer to tunneling of electron from QD system to lead. From the QD system electrons tunnel from lead in to the QD system through only one path and electrons tunnel from QD system to the leads through two paths, thus Hamiltonian system can be modified by subtracting the term for one incoming path and two outgoing paths and the Eq. 3 can be written as:

$$[H, N_L] = V_m C_L^+ d_{2\sigma} + V_m C_R^+ d_{3\sigma} - V_m C_M^+ d_{1\sigma} \tag{3}$$

The above expression gives the Hamiltonian H of the total system.

GREEN'S FUNCTION OF QD SYSTEM

$$G^<_{2\sigma,L} = C_L^+ d_{2\sigma}, G^<_{3\sigma,R} = C_R^+ d_{3\sigma}, G^<_{1\sigma,M} = C_M^+ d_{1\sigma} \tag{4}$$

$$G^<_{2\sigma,L} = V[G^r_{2,2} g^<_L - G^<_{1,1} g^a_l], G^<_{1\sigma,M} = V[G^r_{1,1} g^<_m - G^<_{1,1} g^a_m], G^<_{3\sigma,R} = V[G^r_{3,3} g^<_L - G^<_{1,1} g^a_r] \tag{5}$$

$$\begin{aligned} G^r_{1,1} &= g^r_{1,1} + g^r_{1,1}\Sigma_{1,2}G^r_{1,2} + g^r_{1,1}\Sigma_{1,3}G^r_{1,3} + g^r_{1,1}\Sigma_M G^r_{1,1} \\ G^<_{1,1} &= G^r_{1,1}\Sigma^<_M G^a_{1,1} \end{aligned} \tag{6}$$

If we express in terms of the green function in Fourier transform by transforming time domain to frequency domain it can be written as in Eq. 4. By using Dyson equation the above Eq. 5 can be written as mentioned in Eq. 6.

The retarded and lesser Green's function for QD1, QD2 and QD3 can be written from the quantum dot system. By applying the retarded and lesser Green's function to the QD1.The retarded function is calculated based up on the number of dots and lead interacting with the QD1. Similarly the retarded and lesser Green's functions can be written for QD2 and QD3.

$$\begin{aligned} G^r_{1,1} &= g^r_{1,1} + g^r_{1,1}\Sigma_{1,2}G^r_{1,2} + gr_{1,1}\Sigma_{1,3}G^r_{1,3} + g^r_{1,1}\Sigma_M G^r_{1,1} \\ G^r_{1,1} &= g^r_{1,1} + g^r_{1,1}\Sigma_{1,2}G^r_{1,2} + gr_{1,1}\Sigma_{1,3}G^r_{1,3} + g^r_{1,1}\Sigma_M G^r_{1,1} \\ G^r_{1,1} &= g^r_{1,1} + g^r_{1,1}\Sigma_{1,2}G^r_{1,2} + gr_{1,1}\Sigma_{1,3}G^r_{1,3} + g^r_{1,1}\Sigma_M G^r_{1,1} \\ G^<_{1,1} &= G^r_{1,1}\Sigma^<_M G^a_{1,1} \\ G^<_{1,1} &= G^r_{1,1}\Sigma^<_M G^a_{1,1} \end{aligned}$$

$$J_n = [\Gamma^M(\varepsilon) f_M(\varepsilon) + \Gamma^R(\varepsilon) f_R(\varepsilon) - \Gamma^L(\varepsilon) f_L(\varepsilon)] T(\omega) \tag{7}$$

From Eq. 1 to 4 we will arrive at the final expression for Jn as mentioned in Eq. 7. This expression gives the current density of three terminal QD system.

NUMERICAL RESULTS AND DISCUSSION

The graphs in Fig. 2 and 3 give the variation in transmission rate $T(\omega)$ with respect to ω. The accelerated Green's function can be converted to retarded function by using the identity given as:

$$G^r G^a = \frac{\Lambda(\omega)}{\Gamma(\omega)} = \frac{1}{[\omega - \varepsilon^0 - \Lambda(\omega)]^2 + [\frac{\Gamma(\omega)}{2}]^2}$$

where, $\Lambda(\omega)$ the interaction is term and $\Gamma(\omega)$ is the level width function. By substituting the identity in the transmission rate the graphs can be plotted. The transmission rate will be different for interacting and non-interacting cases.

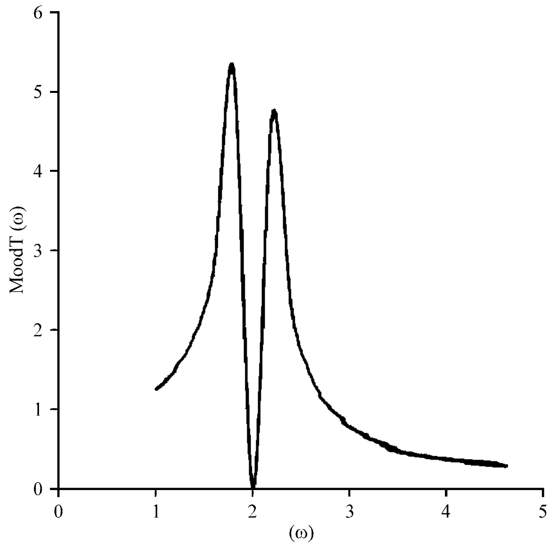

Fig. 2: Transmission rate for non interacting system

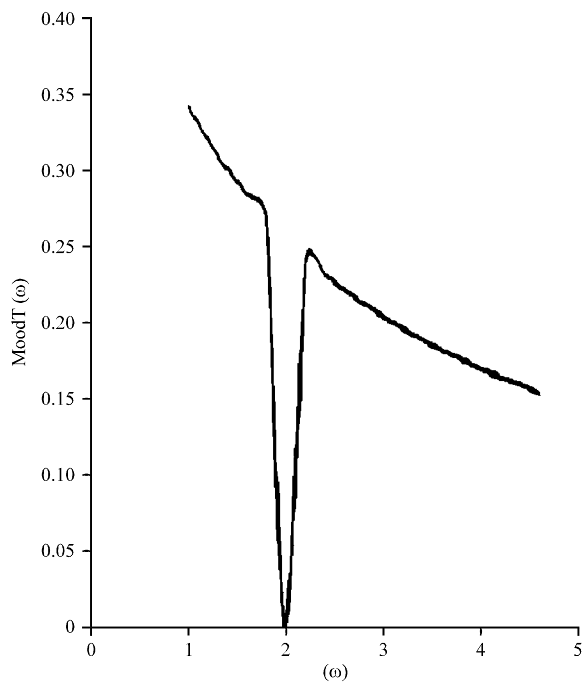

Fig. 3: Transmission rate for interacting system

The two cases are depicted in the Fig. 2 and 3. The graphs are plotted for transmission rates given by the expression:

$$T(\omega) = G^{r}_{1,1}\left(\frac{1}{V^2} - G^{a}_{1,1}\right)$$

This gives the transmission rate for the non-interacting system depicted in Fig. 2 and interacting system depicted in Fig 3.

In non-interacting case the applied potential will not have any effect on the levels of the quantum dot. The increase in excitation gives rise to the increase in conduction because of the increase of energy of electrons. The transmission ceases at a particular value of frequency as the increased electrons cannot accommodate on the other side of the barrier with similar energy level. At this stage the electron cannot tunnel and cannot generate a photon to reduce its energy level. If we increase the excitation beyond this value the electron with more energy value decreases its energy by generating a photon and participates in the tunneling process. After the ceasing point (point where transmission rate tends to zero) the conduction tend to decrease because of the increase in number of electrons with higher energy value and lesser number of electrons with lower energy value that can tunnel through barriers.

For interacting case the external potential can influence the energy levels of the QDs. The increase in external excitation leads to the decrease in similar energy levels in two QD for which tunneling is considered. Thus the increase in excitation leads to the decrease in the transmission rate. At a particular value of frequency (ceasing point) the transmission ceases and no tunneling of electrons takes place. If the excitation increases beyond this limit the electron tunnels through the barrier through a process of inelastic scattering.

CONCLUSIONS

Before the ceasing point, the tunneling in both interacting and non-interacting cases occur due to resonance tunneling. After the ceasing point, in interacting case the conduction process takes place through inelastic tunneling. For non-interacting cases the transmission occurs through emission of photon followed by tunneling between similar energy levels in QDs.

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