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Decoupling and Soft Sensor Design for a Class of MIMO System

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Abstract: The aim of this research was to maintain the interacting liquid level and temperature parameter values at the desired level. The design of decoupling and linearization algorithm (Hirschorn's algorithm) and soft sensor techniques for an approximated model of an interacting thermal non-linear process is presented in this work. First the nonlinear interacting system was converted into linear non interacting system using decoupling and linearization algorithm. The soft sensor techniques such as the Kalman observer and the Unknown Input Observer (UIO) was then applied to estimate the system parameters namely level and temperature for a non interacting linear MIMO system. The obtained estimated error for the plant using UIO observer was less when compared to Kalman observer.

Key words: Multivariable process control, Hirschorn's algorithm, Kalman observer, unknown input observer

INTRODUCTION

The chemical processes possess non-linear dynamic characteristics and the design of controller for a non-linear chemical process involves linearizing the process model around its steady state operating point and applying the linear control theory. The decoupling and linearization control (Akkari *et al*., 2009) provides satisfactory response when the process model is available. However if there is a difference between the real process and the process model, application of this model may give unsatisfactory results. But the degree of the mismatch is generally not high in many chemical processes. In such cases, it is sufficient to add external controllers which compensates for the mismatch. In this work the state feedback law (Thosar *et al*., 2008) is applied to the nonlinear process. That is linearized as per the above process. The state feedback law is developed as a part of linearization of the nonlinear process. This yields us a control structure called Global Linearizing Control which responds like linear system. This type of control is tried in this work for a MIMO nonlinear system with equal number of inputs and outputs.

For a linear non interacting MIMO system, Kalman and Unknown Input Observer (UIO) is designed from control point of view. The system states of a dynamic system is estimated using the unknown input observers which may receive input excitation of any kind. Recently there have been many researchers aiming to simultaneously estimate the system state and the unknown input. The estimation of parameters is important in many engineering applications.

APPLICATION TO CHEMICAL PROCESS CONTROL

Application of global linearizing control for the Level and Temperature Control Process is studied here.

The non-linear system is defined as:

$$\dot{x} = f(x)+g(x)\,u \tag{1}$$

$$Y = h(x) \tag{2}$$

where, 'x' is the state vector of dimension 'n', 'u' is an input vector of dimension 'm', 'y' is an output vector of dimension of 'p', 'f(x)' is a smooth function, h(x) is a (p,1) vector with a row element $h_j(x)$ also a smooth function and g(x) is an (n, m) matrix with elements of each column being $g_j(x)$.

The model is chemical process with a liquid level and temperature as variables to be controlled (Kravaris and Chung, 1987) is shown in Fig. 1.

The mathematical equations formed for the above system is:

$$\frac{dx_1}{dt} = -\left(\frac{k}{s}\right)x_1^{\frac{1}{2}} + \frac{1}{s}u_1 \tag{3}$$

$$\frac{dx_2}{dt} = \left(\frac{T_o - x_2}{sx_1}\right)u_1 + \left(\frac{1}{c_p \varsigma s x_1}\right)u_2, \qquad y_1 = x_1, \qquad y_2 = x_2 \tag{4}$$

where , x_1 and x_2 are the liquid level and the temperature in the tank, respectively. u_1 and u_2 are the feed flow rate to the tank and the heat flow rate from the heater,

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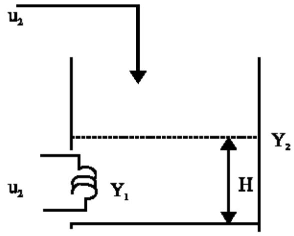

Fig. 1: Schematic diagram for the level and temperature control process, u_1: Feed flow rate of the liquid, u_2: Heater input, Y_1: Level sensor output, Y_2: Temperature sensor output, H: Height of the liquid

respectively. The feed flow rate and heat flow rate are constrained as $0=u_1=22$ cm^3 sec^{-1} and $0=u_2 = 2700$ J sec^{-1}. k is the constant coefficient, 1.8, S is the cross sectional area, 191 cm^2, x_1 is the liquid level in cm, x_2 is the liquid temperature, °C, T_0 is the temperature of the feed 18°C, C_p is the specific heat, = 4.2 J^{-1} K^{-1}, ς is the density of the liquid (water).

Extensive numerical simulations are carried out on the proposed algorithm as detailed below. A standard Runge-Kutta Gill algorithm is used for the numerical integration of the set of ordinary differential equations. Prior to decoupling, liquid level Y_1 depended on u_1 (flow rate) and liquid temperature Y_2 depended on both u_1 and u_2 (heater input). After the application of decoupling algorithm Y_1 depends only on u_1 and Y_2 developed here depends only on u_2 as can be observed from Eq. 9 and 10.

DEVELOPMENT OF HIRSCHORN'S CONTROL LAW

In order to calculate a control law that induces linear input/ output behavior of a MIMO system, a Decoupling and Linearization (Hirschorn's) algorithm (Kravaris and Soroush, 1990) was developed. It helps to find a differential operator such that, when applied to the outputs, it will provide a set of algebraic expressions in x and u that gives solution to u.

The use of this control law does not require any structured constraints to be imposed on the closed loop system dynamics. Therefore, the control designer has the flexibility to adjust the parameters β_{ik} for fast closed-loop response and desirable level of coupling.

From Kravaris and Soroush (1990) if:

$$\varsigma^{(k^*)} = m \quad (5)$$

$F_1(x)$ = constant, 1 = 0,..., k*-1, then the system Eq. 1 is input/output linearizable. [mx1] matrice β_{ik}, i = 0,.., m, k = 0,..., r_i-1. mx(m-$\varsigma^{(0)}$), mx(m-$\varsigma^{(1)}$),..., mx(m-$\varsigma^{(k^*-1)}$) matrices γ_0, γ_1,..., γ_{k^*-1} and an m×m invertible matrix Γ.

Applying the linearising algorithm, the decoupling and linearization control law obtained as given by (Kravaris and Soroush, 1990) is reproduced below:

$$U = \left[\Gamma L_g H^{(k')}(x)\right]^{-1}\left\{V - \sum_{i=1}^{m}\sum_{k=0}^{r_i-1}\beta_{ik}L_f{}^k h_i(x) - \sum_{l=0}^{k'-1}\gamma_l[F_l \vdots I_m - \varsigma^{(l)}]E_l L_f H^{(l)}(x) - \Gamma L_f H^{(k')}(x)\right\} \quad (6)$$

$$u_1 = s\left[V_1 - d_{10}x_1 + \frac{k}{s}x_1^{1/2} - \frac{k^2}{2s^2}\right] \quad (7)$$

$$u_2 = -\left(\frac{To - x_2}{x_1}\right)C_p ? s x_1\left[V_1 - d_{10}x_1 + \frac{k}{s}x_1^{1/2} - \frac{k^2}{2s^2}\right] C_p \varsigma s x_1 [V_2 - \delta_{20}x_2] \quad (8)$$

Applying the procedure and data as given in (Akkari *et al.*, 2009) and substituting u_1 and u_2 in the Eq. 3 and 4 the state equation obtained is both decoupled and linearised forms. The resulting Eq. 9 and 10 are in decoupled form:

$$\frac{dx_1}{dt} = V_1 - ?_{10}\,x_1 - \frac{k^2}{2s^2} \quad (9)$$

$$\frac{dx_2}{dt} = V_2 - ?_{20}x_2 \quad (10)$$

The advantage of using Hirschorn's algorithm is that the control law is less complex. In addition to that it also offers more dynamic feed flow rate of liquid u_1 and heat input rate u_2. The simulation results show that Hirschorn's algorithm has better effect.

IMPLEMENTATION OF KALMAN OBSERVER

Kalman observer is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms, i.e., only the estimated state from the previous time step and the current measurement are needed to compute the estimate of the current state. The Kalman filter operates by propagating the mean and covariance of the state through time. The notation $\hat{x}_{n|m}$ represents the estimate of the state vector X at time 'n' given observations till 'm'.

The state of the filter is represented by two variables (Tiano *et al.*, 2007):

- $\hat{x}_{n|m}$, a posteriori state estimate at time k. The given observation is up to and including at time k
- $P_{k|k}$, a posteriori error covariance matrix which measure the estimated accuracy of the state
- The Kalman filter has two distinct phases, prediction and correction

In a typical situation, first prediction phase provides an estimate of the current state which holds until the present scheduled observation. In the correction phase this observed information is used to update the estimate produced by the prediction phase. These two phases alternatively produce new estimate of the states. However, if the observation is not possible for some cases, the estimate can made by multiple prediction phases, skipping observation phase. Consider a linear time invariant discrete system given by the following equation:

$$X_{k+1} = FX_k+Bu_k+W_k \quad (11)$$

$$Z_{k+1} = HX_{k+1}+V_{k+1} \quad (12)$$

where, F is the state transition matrix, B is the control input matrix, W_k is the process noise with zero mean multivariate normal distribution having covariance Q_k. H is the observation matrix, V_{k+1} is the observation noise which is zero mean Gaussian white noise having covariance R_k. U_k is the control input.

Prediction (time update) equations: Predicted state estimate:

$$\hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1}+Bu_k \quad (13)$$

Predicted estimate covariance:

$$P_{k|k-1} = FP_{k-1|k-1}F_k^T+Q \quad (14)$$

Correction (measurement update) equations: Innovation or measurement residual:

$$\hat{y}_k = Z_k-H\hat{X}_{k|k-1} \quad (15)$$

Innovation (or residual) covariance:

$$S_k = HP_{k|k-1}H^T+R_k \quad (16)$$

Optimal Kalman gain:

$$K_k = P_{k|k-1}H^TS_k^{-1} \quad (17)$$

Updated (a posteriori) state estimate:

$$\hat{X}_{k|k} = \hat{X}_{k|k-1}+K_k\hat{y}_k \quad (18)$$

Updated (a posteriori) estimate covariance:

$$P_{k|k} = (I-K_kH)P_{k|k-1} \quad (19)$$

The results that are obtained using the Kalman observer technique are explained in the simulation results section.

IMPLEMENTATION OF UNKNOWN INPUT OBSERVER

Observer is capable of estimating the states with unknown inputs. The unknown inputs generally could be a combination or any of the unmeasurable or unmeasured disturbances, unknown control action or unmodelled system dynamics. This observer is very useful when we are dealing with problem of instrument fault detection. It can be implemented as reduced order observer or full order observer (Wang and Gao, 2003).

Consider a continuous linear time invariant steady space model of the system:

$$\dot{x}(t) = Ax(t)+Bu(t)+Ed(t)$$
$$y(t) = Cx(t) \quad (20)$$

where, $x\epsilon R_{n\times 1}$ is the state vector, u is the input vector, y is the sensor output, A is the system coefficient matrix, B is the input coefficient matrix, C is the output coefficient matrix, $d\epsilon R^{q\times 1}$ is the unknown input vector and $E\epsilon R^{n\times q}$ is the unknown input distribution matrix.

The structure of the UIO is described as:

$$\dot{z}(t) = Fx(t)+TBu(t)+Ky(t)$$
$$\hat{x}(t) = z(t)+Hy(t) \quad (21)$$

where, $\hat{x}\epsilon R^{n\times 1}$ is the estimated state vector and $T\epsilon R^{n\times n}$; $K\epsilon R^{n\times n}$ and $H\epsilon R^{n\times n}$ are matrices satisfying requirements.

The error vector is given by:

$$e(t) = x(t)-\hat{x}(t) \quad (22)$$

Using Eq. 21, error vector is obtained:

$$e(t) = x(t)-\hat{x}(t) = x(t)-z(t)-Hy(t) = x(t)-z(t)-HCx(t) = (I-HC)x(t)-z(t) \quad (23)$$

Using Eq. 23, derivative of the vector is:

$$\dot{e}(t) = (A-HCA-K_1C)\,e(t)+(A-HCA-K_1C)z(t)+(A-HCA-K_1C)Hy(t)+(I-HC)Bu(t)+(I-HC)Ed(t)-Fz(t)-TBu(t)-K_2y(t) = (A-HCA-K_1C)e(t)+[F-(A-HCA-K_1C)]z(t)-[K_2-(A-HCA-K_1C)]Hy(t)-[T-(1-HC)]Bu(t)-(1-HC)Ed(t) \quad (24)$$

The following relations also hold true:

$$(HC-I)\,E = 0 \quad (25)$$

$$T = (I\text{-}HC) \quad (26)$$

$$F = A\text{-}HCA\text{-}K_1+K_2 \quad (27)$$

$K1 = (A^T, C^T, p)$; The observer gain matrix:

$$K_2 = FH \quad (28)$$

$$K = K_1+K_2 \quad (29)$$

$H = L*inverse\ C*L)^T*(C*L))*(C*L)^T$; F, G, H, K_1, K_2 are the coefficients matrices with appropriate dimensions. I is the identity matrix. p is the desired closed loop poles. The desired observer response can be achieved by assigning suitable poles through the design of K_1 and K_2. The results that are obtained when the UIO observer technique is implemented, is explained in the simulation results section.

RESULTS AND DISCUSSION

Utilizing the model given by Kravaris and Chung (1987), decoupling and linearization algorithm was designed. At first the simulation was carried out without decoupling. Figure 2 shows the output response of the level and temperature when the set point of the level and temperature were changed from 1 to 60 cm and 1 to 30°C, respectively. When sudden disturbance was introduced at 200 sec in level, it affected the temperature process due to interaction.

Simulation was carried out after applying Hirschorn's algorithm with external PI controller (Nejati *et al*., 2012) as shown in Fig. 3. The sudden disturbance introduced at 200 sec in level does not affect the temperature process. It can be seen from the Fig. 2 that under the influence of the controller, ISE is improved.

Figure 4 illustrates the level tracking error between plant and Kalman observer. Kalman Observer was designed for the level process of the state space model. Actual level output of y and observer level output ŷ were obtained directly from simulation model of the plant and state estimation error was calculated. The level error varied between -2 to +2 cm when the setpoint of the level is changed from 1 to 60 cm. So, 6.66% error occurred as mentioned in Table 1.

Figure 5 illustrates the temperature tracking errors for plant and Kalman observer. Kalman observer was designed for the temperature process of the state space model. Actual temperature output of y and observer temperature output ŷ were obtained directly from the simulation model of the plant and state estimation error

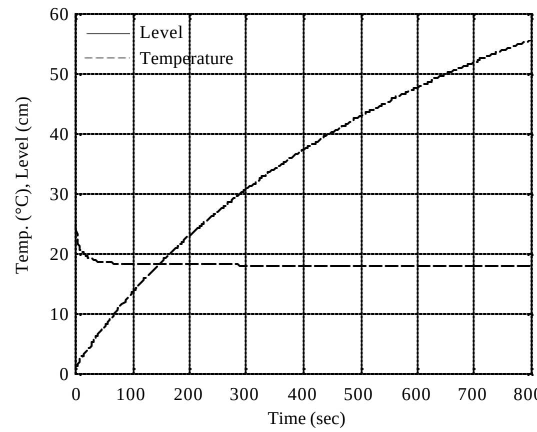

Fig. 2: Out put response for the step change in the level without decoupling with PI controllers

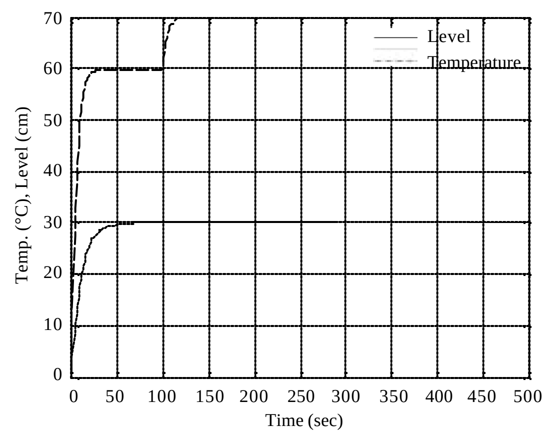

Fig. 3: Out put response for the step change in the level with decoupling with PI-SPW controllers

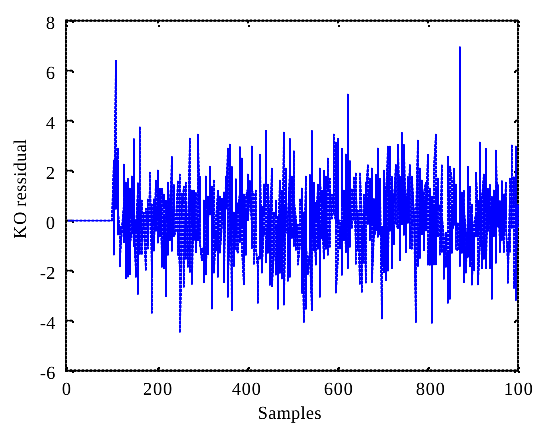

Fig. 4: Estimated error of the plant using Kalman observer (KO) for level

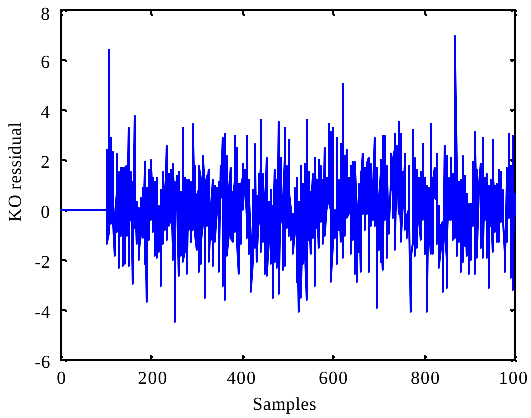

Fig. 5: Estimated error of the plant using Kalman observer (KO) for temperature

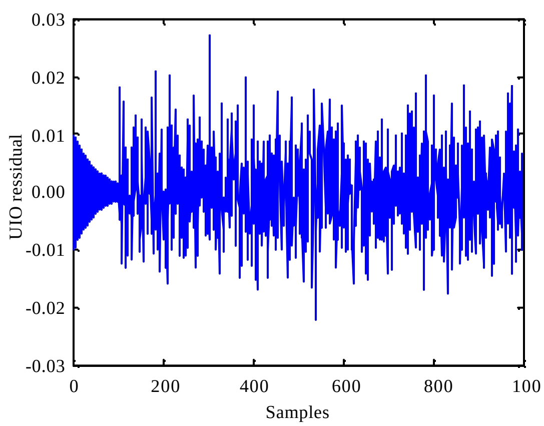

Fig. 6: Estimated error of the plant using UI observer (UIO) for level

Table 1: Comparative performance of observers

Observer type	Level error (%)	Temp. error (%)
Kalman observer	6.66	13.30
Unknown input observer	0.06	0.13

was calculated. The temperature error varied between -2 to +2°C when the setpoint of the level is changed from 1 to 30°C. So 13.3% error occurred as mentioned in Table 1.

Figure 6 illustrates the level tracking errors for plant and Unknown Input Observer. Unknown Input Observer was designed for the level process of the state space model. Actual level output of y and observer level output ŷ were obtained directly from the simulation model of the plant and state estimation error was calculated. The level error varied between +0.02 to -0.02 cm when the setpoint of the level is changed from 1 to 60 cm. So the observer is designed in such a way that the observer output follows the system output and only 0.06% error occurred.

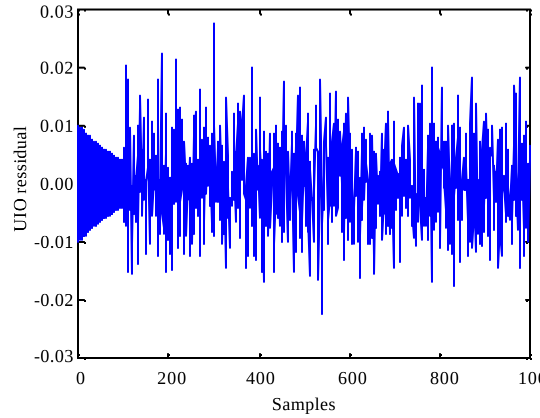

Fig. 7: Estimated error of the plant using UI observer (UIO) for temperature

Figure 7 illustrates the temperature tracking errors for plant and Unknown Input Observer. Unknown Input Observer was designed for the temperature parameter of the state space model. Actual temperature output of y and observer temperature output ŷ were obtained directly from simulation model of the plant and the state estimation error was calculated. The temperature error varied between +0.02 to -0.02 cm when the setpoint of the level is changed from 1 to 30°C. so the UI observer is designed in such a way that the observer output follows the system output and only 0.13% error occurred as mentioned in Table 1. Table 1 show that UIO gives less error.

CONCLUSION

The decoupling linearization algorithm was applied to a nonlinear MIMO interacting thermal process. The simulation results had shown that even if the processes are non-linear and interactive a satisfactory control performance could be obtained. The Kalman and Unknown Input Observer (UIO) was designed to estimate the system parameters like level and temperature for a non interacting linear MIMO system. Results of these simulations are presented in Table 1. Performance of a UIO was found to be better. The obtained outputs for UIO give less error when compared to Kalman observer.

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