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The Stress Distribution of Gear Tooth Due to Axial Misalignment Condition

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Abstract: Misalignment in gear dynamics mesh always lead to generate a vibration that causes of un-even dynamic load on transmitting torque to the gear tooth. This type of load is also considered as one of the main criteria on contributing the high stresses happen to the teeth that lead to the fatigue breakage after in some duration of cyclic loading time. This effort of study is made to analyze theoretical forces that create stresses with the effect of axial misalignment to a spur gear in meshing condition. A 3D CAD model of the pinion hobbing gear tooth was created with Autodesk Inventor 2010 and analyzes using ANSYS V13 FEA method. The transmitted dynamic load was calculated with change of misalignment angle proportionally to the theoretical contact area on the tooth face. As a result the stress distribution at the contact region and the tooth root is clearly seen variant with the misalignment angle and the equivalent stress is directly proportional with the misalignment deviation. The values of equivalent stresses and its distribution are change with the changing of deviation angle. The stress concentration is higher at the contact region and the tooth root with augmentation of misalignment angle. The face load factor in align and misalign shows the increasing of the load deviation will cause the factor to increase and probably the major contribution to the vibration of the gear mesh in dynamic condition.

Key words: Misalignment, gear mesh, axial misalignment, stress analysis, face load factor, FEA analysis

INTRODUCTION

Misalignment of meshing in pinion and gear is considered as a major contribution of increasing the stress concentration on specific regions of the gear tooth. This condition is related to the non uniform distribution of transmitted load along the tooth face. The force that creates a torque with misalignment causes high vibration with symptoms that sometimes cannot be explained (Al-Hussain and Redmond, 2002). The meshing tooth in this condition will influence the rigidity and the forces which is originated of complicated vibration and stress distribution in the gears (Li and Yu, 2001). The importance of stress analysis is focused on the determination of stress concentration regions that failure or fracture may initiate at these regions (Ali and Mohammad, 2008).

Several researchers had created a computerized FEM methodology of determining the stress concentration on the spur gear tooth. Sfakiotakis *et al.* (2001) had described a finite element procedure which simulates the conjugation action of spur gear drives. Baud and Velex (2002) then had created the experimental procedure to validate using a similar model of dynamic tooth load from the AGMA. Kawalec *et al.* (2006) then grouped these models in two main categories based on their methodologies:

- **Semi-analytical:** Where the empirical studies parametric data is considered
- **Numerical models:** A model of developed by using discretization methods such as the boundary element method (BEM) and the finite element method (FEM)

The most recent studies made by Hassan (2010) that study on transient stress of the spur gear using a super position mode calculation. A 2D finite element model of three segment gear teeth was tested and the natural frequency model proposed by Block Lanczos was used to determine the stress profile at the root.

Regardless all the gear studies, it is mostly considered a stress state at the root based on assuming the gear component meshing pinion to gear is in aligning with each other. In actual condition, most of the component fitting, e.g., shaft to gear, driven to slave gears

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etc., will affect to the misalignment error due to tolerance, transmission, machining, assembly errors and others parameter related.

In the gear contact sense, mesh misalignment implies the axial shifting of the position of the meshing surfaces due to either deflections or errors in the manufacture of the gears and their housing. The axial misalignment is essentially results in a change in center distance of the shafts depends upon the plane that it acts in. Axial misalignment parallel to the plane of action tends to shift the load to the sub side of the tooth by increasing the separation at one side of the tooth and reducing the separation at the other side of the tooth. In this case, the shape and area of the theoretical active contact plane remains the same as the ideal shape of the tooth (Houser *et al*., 2006).

In this study one effect of axial misalignments that creates a parabolic function at the surface of the gear flank in meshing condition and the stress distribution at the contact region and tooth root of the spur gear is investigated. The comparison of the load distribution factor to the face width of the gear mesh in misalignment will also be considered.

ACTIVE GEAR CONTACT REGION

When gear meshes with perfect align condition of shaft, maximum torque is transferred at the line of action in between pinion and the gear. The transmission force F_n, which is normal to the tooth surface, as in Fig. 1, can be resolved into a tangential component, F_t and a radial component, F_r as refer to equation:

$$F_t = F_n \cos \varphi' \quad (1)$$

$$F_r = F_n \sin \varphi' \quad (2)$$

There will be no axial force, F_x. and direction of the forces acting on the gears are shown in The load is considered uniform as refer to Fig. 2a but when the axial misalign happens the load will be offset from the contact area of the tooth flank as demonstrated in Fig. 2b.

The load of the active contact in case of axial misalignment can be specified by coating the teeth by a thin layer of a soft material which explain the pressure zone on the meshing tooth with refer to the experiment by Ameen (2010). Result shows there are 4 major axial misalignment contact happen due to axial misalignment in shaft contributes the non uniform load happen to the gear teeth. The alteration of distribution load concentrated on one side (B') as refer to Fig. 3 can be described as a parabolic function based on the Eq. 3:

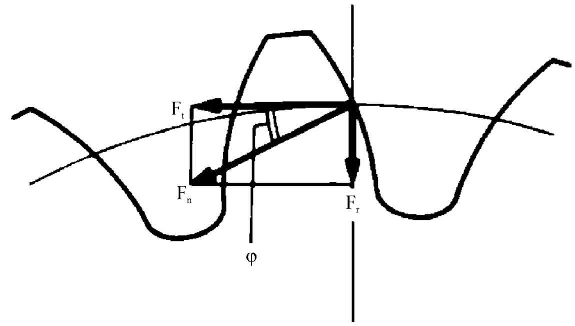


Fig. 1: Forces acting on a spur gear mesh

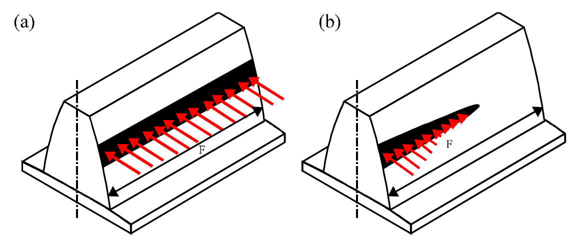


Fig. 2(a-b): Load distribution along tooth surface (a) Align and (b) Misalign

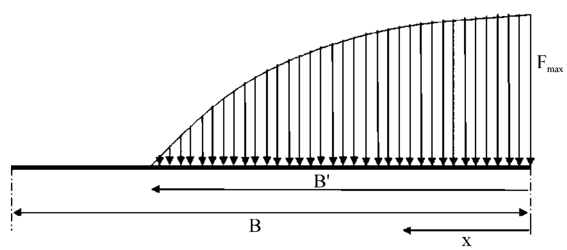


Fig. 3: Theoretical active Load distribution along tooth surface

$$F(x) = F_{max}\left(1-\left(\frac{x}{B'}\right)\right)^3 \quad (3)$$

Finally the equation:

$$B' = \sqrt{\frac{4F_t B}{3f_{RW}C_z}} \quad (4)$$

Hence, Eq. 4 use as represents the active gear width at tooth surface in misalignment condition with refer to the static condition. In actual, gear always operates in dynamic motion. From Buckingham's dynamic load equation that consider as a small machining error and deflection of teeth under load cause periods of acceleration, inertia forces and impact loads on the teeth similar to variable load superimposed on a steady load (Buckingham, 2011). Those the equation:

$$F_{dy} = F_t + \frac{21V(Bce + F_t)}{21V + \sqrt{Bce + F_t}} \quad (5)$$

and

$$F_t = F_i \cos \varphi$$

where, F_{dy} is dynamic load, F_t is transmitted load, V is the pitch line velocity, m/s, B is the face width actual length, mm, c is the dynamic factor depending on machining error, e is the error in tooth shape, 21 is the value of constant k. Hence:

$$c = a\left(\frac{E_p E_g}{E_p + E_g}\right) \quad (6)$$

where, a is the material contact constant depending on angle, $E_p E_g$ is the Modulus Young for pinion and gears.

To represent the theoretically active misalignment contact width (B'), it can be divided into several segments of interval where each segment dynamic forces is calculated as centre forces with the equation:

$$F_{total} = \int_{x_1}^{x_2} F_{dy}\left[1 - \left(\frac{x}{B'}\right)\right] dx \quad (7)$$

Thus, to determine the distribution forces on the gear contact during misalignment would be based on the integration of Eq. 7 with Simpson's rule method for numerical integration.

SPUR GEAR FEM MODEL

The physical gear model for FEM analysis in this study is a particular spur gear pinion with tooth hobbing process manufacturing for transporting high load of machinery transmission. The main characteristic parameter is defined as Table 1 below:

Table 1: Model characteristic parameter

Parameter	Symbol	Value use in model: Pinion	Gear
Geometry			
Module (mm)	m	4	4
Pressure angle (°)	φ	20	20
Number of teeth	z	50	30
Pitch circle diameter (mm)	PCD	200	120
Face width (mm)	B	100	100
Working depth (mm)	h	9	9
Dedendum (mm)	S_m	6.338	
Tooth thickness	S_t	8.891	
Torque (Nm)	T	765.5	
Speed (rpm)	N	5000	
Material(cast steel)			
Modulus Y(N/mm)	E	21000	
Poisson Ratio	V	0.3	
Tooth stiffness (N/mm)	K	239405.2	

In accordance with investigations given in paper Atanasovska and Nikolic (2000), expected maximum contact stress point on path of contact is point B where point of passing from period with two tooth pairs in contact to single meshed tooth pair period. The finite element types chosen for the gear 3D pair is Autodesk Inventor CAD model isoperimetric structural solid element defined by eight points gear modeling and surface contact element. The tooth contact modeling is analyzed at the pinion gear only with assumption, the load transferred in perfect form (the effect between pinion and gear approximately the same). A 3D FEM gear pair models are derived like sweeped (copied) 2D model in normal direction along the length equal to gears face width (Nikolic and Atanasovska, 2004). Singular element, spider-web pattern and sweep-mesh scheme is implemented continuously along with several original strategies on free mesh size control (Ariatedja and Mamat, 2011). The face width of the models is then divided into 20 segments (each segments as 5 mm incrementally) which give a possibility for accuracy determination of stress state and load distribution along gear face width.

The boundary conditions on any 3D FEM model are defined by displacement constraints at the direction normal to the surfaces which separate the modeled gear segment from the rest of gear body (Atanasovska and Nikolic, 2000). Model loading consist of the applied mechanical load which is modeled as the load control and displacement control (Rahman *et al*., 2008). To achieve statically stable models, the elements-teeth have displacement constraints at the direction normal to teeth transverse plane at z-axis as Fig. 6. The external load is defined on the elements-teeth by few concentrated forces at the path of contact direction at y-axis of Global Cartesian coordinate system.

Two symmetrical models have been developed for the studied (align- refer to Fig. 4 and misalign condition-Fig. 5) with constant total forces F_{total} acting to the nominal surface segment divided. Then, simulation in ANSYS-FEM software in absolute align condition with active gear contact (B) applied to the first model with nominal forces. On second model, the theoretical active contact length B' (calculated as Eq. 4) with theoretical misalignment angle of 0.2°, 0.3° is applied gradually to the discrete segment on the gear tooth face. The data for determining deformation and principle stress state and load factor will be discussed as comparisons between both results and compared to the previous studies from Atanasovska and Nikolic-Stanojevic (2007).

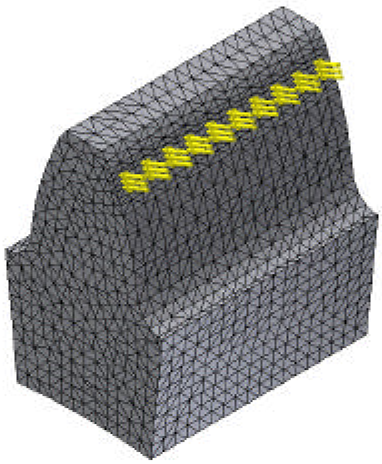


Fig. 4: Active gear mesh contact load at full length (B)

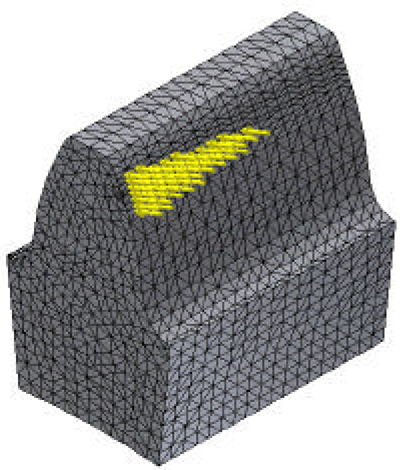


Fig. 5: Active gear mesh contact at misalignment (B')

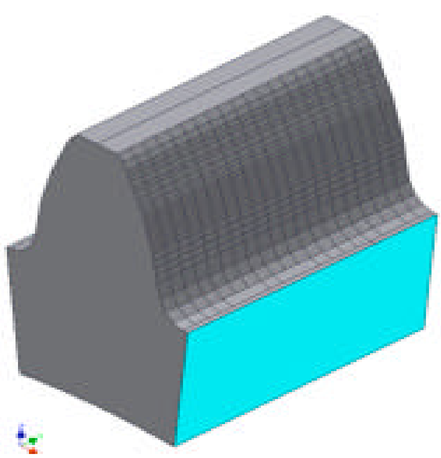
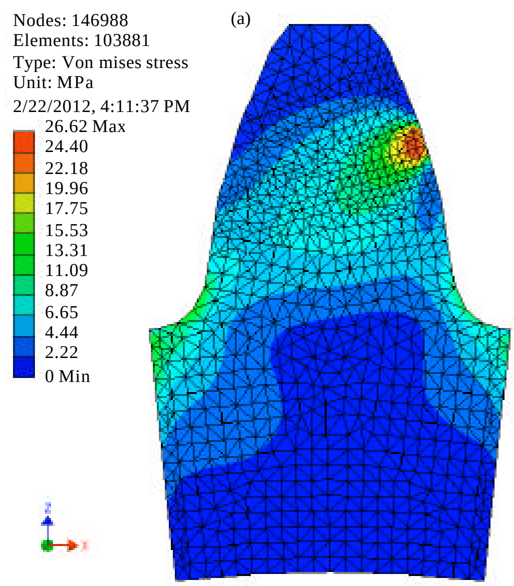


Fig. 6: Displacement constraint of gear mesh

RESULTS AND DISCUSSION

The numerical ANSYS FEA result in Fig. 7a and b represent the equivalent stress state of Von misses criteria at active contact region and the tooth root of gear surface is in absolute align condition. This criteria is based on 3 dimension complex system of stress develop at any node at the contact path within the surface body which the stress is acting in different direction proportional to the change of magnitude through each node and point. This



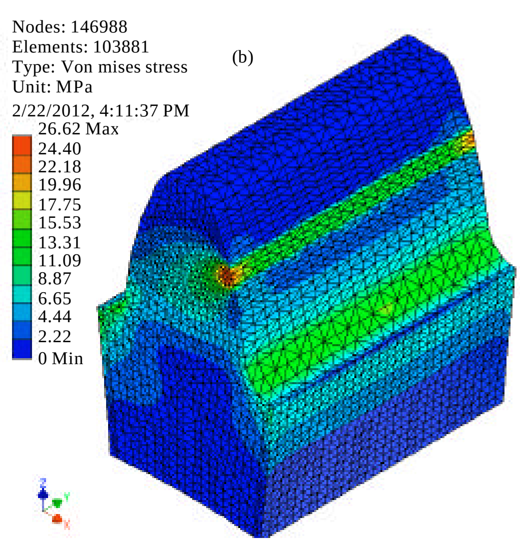


Fig. 7(a-b): Equivalent stress of von- misses criteria at contact region and the tooth root

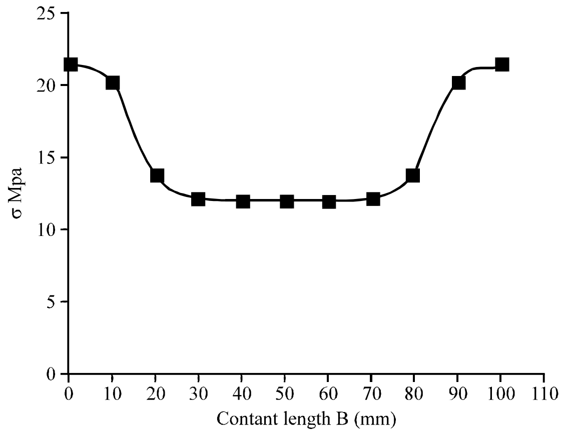


Fig. 8: Monitoring of tooth stiffness and load variation over face width in align condition

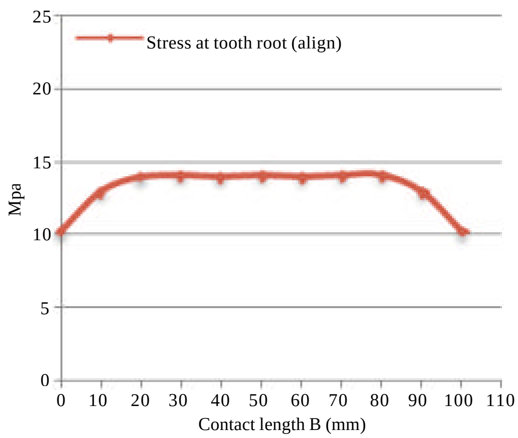


Fig. 9: Monitoring of tooth stiffness and load variation over face width in align condition

result had validated the gear model with precise result because of the fact that node with maximum direct stress value lie on the contact surface while node with maximum equivalent stress lie under the contact of small deep grain size. The formula given for the Von Misses criteria is:

$$\sigma_{vm} = \max[|\sigma_1\text{-}\sigma_2|, |\sigma_2\text{-}\sigma_3|, |\sigma_3\text{-}\sigma_1|] \quad (8)$$

where, σ_1, σ_2 and σ_3 is the direct stresses for a pinion tooth in different cross sections along gear face width.

From the contour, clearly we can see that the region in the curve section tangential between the root radius to the tooth path contact showed the smooth pattern of equivalent stress distribution. This result is expected as correspond to the result of FEM analysis by Nikolic and Atanasovska (2004).

As refer to the graph plotted at Fig. 8 and 9, the contact stress at the gear mesh active contact region and

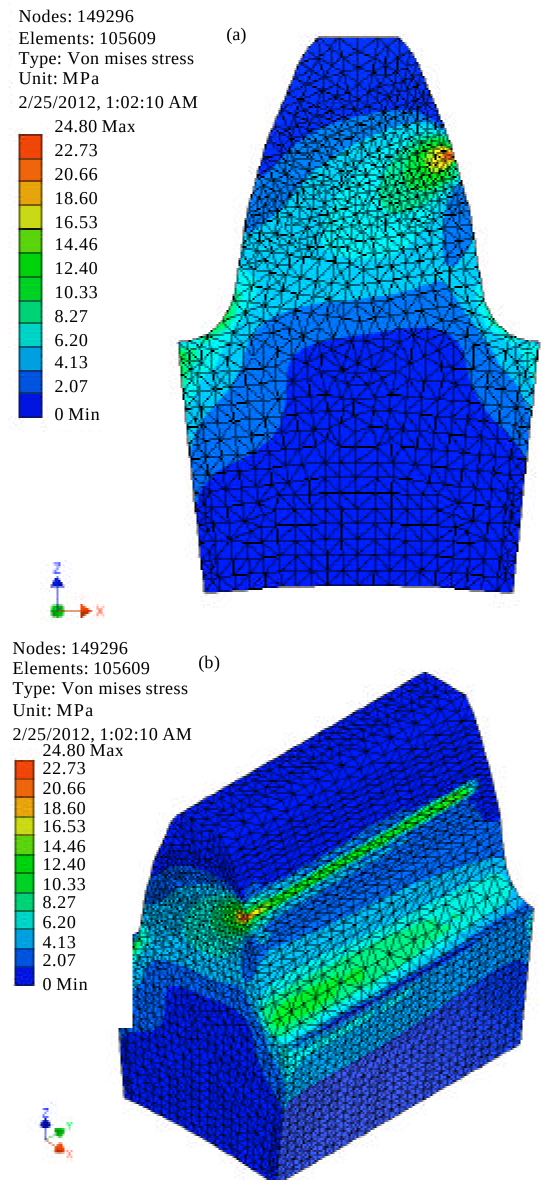


Fig. 10: Equivalent Stress at Gear Tooth root in Misalign condition

the tooth root showed a symmetrical pattern of curve with equivalent stress along contact length of B. For the stress at contact region, the plotted clearly showed that the maximum stress happens at the end of both side of the tooth flank. This result is opposite to the value of the stress happen at the root of the gear. Thus, according to the definitions based on common gear theory (Atanasovska and Nikolic-Stanojevic, 2007), the face load factor can be calculated as ratio between maximum value

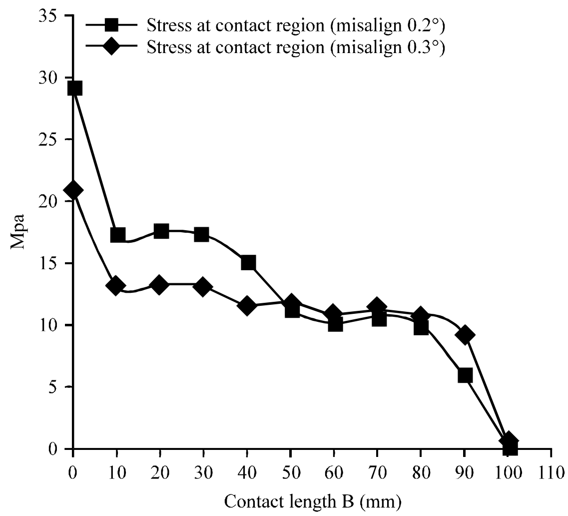


Fig. 11: Monitoring of tooth stiffness and load variation over face width in misalign condition

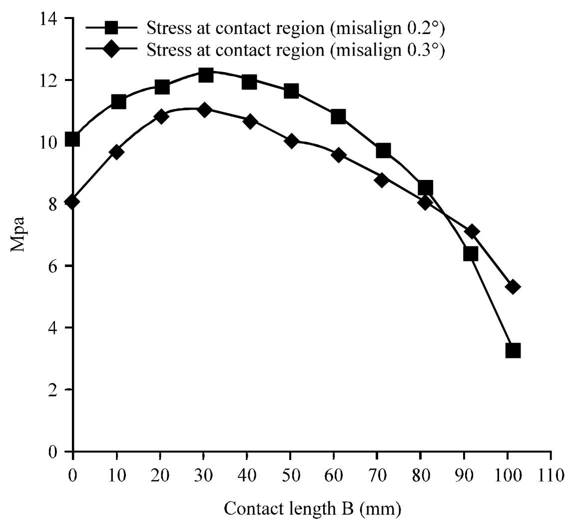


Fig. 12: Monitoring of tooth stiffness and load variation over face width in misalign condition

when uninformed load distribution over gear face width is taking into consideration and average value of the load distribution over face width or calculation that neglects this influence and keep all other calculation requirements unmodified. So, the face load factor KHβ- for contact stress and KFβ- for tooth root stress at surface contact region the face load factor can be calculated as:

$$K_{F\beta} = \left(\frac{\sigma_{max/facewidth}}{\sigma_{avg/facewidth}}\right) = \frac{21.34}{15.61} = 1.3670 \quad (9)$$

$$K_{H\beta} = \left(\frac{\sigma_{max/facewidth}}{\sigma_{avg/facewidth}}\right) = \frac{14.16}{13.18} = 1.0743 \quad (10)$$

Figure 10a and b showed the equivalent stress at the contact region with angle 0.2° and 0.3° misalignment. It is clearly seen that the stress distribution is acting to the subside of one end at the gear face width and increase maximum at the edge of the gear flank. This result is nearly similar to the contour analysis of FEA result from Hassan (2010).

In Fig. 11, the effect of misalignment mesh to the stress distribution at the root is presented. The numerical result from ANSYS showed the deviation on subside end of the gear root in similar contour pattern as the effect on the contact region of the face width. The equivalent stress increases almost twice as the equivalent stress at the contact region. At both the entering and leaving corners of tooth root, the stress levels get extremely exaggerated due to what is called corner contact (Houser *et al*., 2006). This corner contact may be harmful or may simply get polished out so that the stress reduces closely to the surrounding values.

Graph in Fig. 12 is the contact pattern plotted with consideration of influence of angular misalignment at the contact region of the gear tooth. From the expression we can see that the face load factor at contact region KFβ = 1.8304 and face load factor at tooth root KHβ = 1.248 calculated from Eq. 9 the value of the factor is higher than the value when the gear in align condition. Pattern of the contact stress is skewed to the end of corner contact at the tooth flank of the gear. This value is increased with the misalignment angle 0.3^0 is measure.

CONCLUSION

Gear mesh misalignment comes in various modes from a number of sources that can lead to a fatigue breakage to the gear tooth. In this paper the axial misalignment effect of the tooth root and contact region at the flank of gear surface have been identified. Some compromises parameter such as dynamic load and the deviation angle of contact defined as a major contributor of increasing of stress on the tooth root. The values of equivalent stresses and its distribution change with the changing of misalignment angle, where the stress concentration is increased at the contact region and on the tooth root proportional with increasing the misalignment angle, this is occurring in the side of subside the load and decreasing in the other side of the gear face. The face load factor calculation described in this paper, also with the simultaneous incorporation of this influence in gear load capacity calculations. This result also validates the previous result from the other researcher where the increasing of the deviation misalignment angle and load will cause the increasing of the face load factor and probably could lead to a fatigue initiation at the maximum stress region and finally leads to breakage of the gear.

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