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Optimal Solution of Transportation Problem Using Linear Programming: A Case of a Malaysian Trading Company

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Abstract: Managers regularly make decisions pertaining to the effective and efficient allocation of resources to various activities in meeting organizational objective. The task of deciding the optimum plan for distributing goods at the lowest cost possible is a case in point. Minimizing cost of transportation is fundamental for companies in the midst of highly competitive business environment. This study highlights the application of linear programming and spreadsheet that facilitate managers in a Malaysian Trading Company in determining the optimum transportation plan that leads to the lowest transportation cost of polymer from four plants to four demand destinations. It also discusses sensitivity technique in analyzing the impact of uncertainty of unit shipping cost to the total shipping cost of the trading company.

Key words: Transportation problem, transportation cost, linear programming, spreadsheet, sensitivity technique

INTRODUCTION

Effective and efficient movement of products or services from point of supply to points of demand is crucial for any businesses. Transporting finished products to the market at the lowest possible cost leads to huge potential of cost saving and consequently maximize the company's profit (Mandel, 2004). As such, company seeks to optimize their distribution plan for their products in relation to the cost of transportation. In a very competitive business environment, the conversion cost and ex-factory price of the products are almost the same everywhere, leaving delivered price to vary with the distance between consumers and suppliers.

The choice of the quantity to be supplied, the location to be delivered and the right and most economical means of transportation are widely referred to as transportation problem. The purpose is to minimize the cost of shipping goods from one location to another so that the needs of each arrival area are met while every supply location operates within its capacity (Mandel, 2004; Reeb and Leavengood, 2002).

Improvement in the transportation plan has significant impact to a company's bottom-line. Research has shown that a five percent reduction in transportation cost has similar impact as a 30 percent increase in sales. It is a means of sustain cost savings. Furthermore, improvement in transportation usually leads to better service levels (Mandel, 2004; Reeb and Leavengood, 2002).

Generally, distribution cost ranges from nine percent to 14 percent of sales. Among others, it includes all logistics related expenses such as warehousing, personnel and transportation expenses. By and large, transportation cost alone takes up almost 50% of the total distribution cost (Reeb and Leavengood, 2002). The aim is getting the right quantity of products to the right place, at the right time and in the desired condition, while making the greatest contribution to the company.

Quantitative analyses have been successfully applied in obtaining optimal solution to many transportation problems (Phillips *et al*., 1987; Lapin, 1985). The most favorable solutions to the business operations were established using the graphical method (Reeb and Leavengood, 1998a), simplex method (Reeb and Leavengood, 1998b) and duality and sensitivity analysis to interpret linear programming solutions (Reeb and Leavengood, 2000) to name a few.

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Against this background, the objective of this study is to establish a network representation, mathematical formulation and spreadsheet model and analysis the case of a transportation problem of a Malaysian trading company. Linear Programming and spreadsheet are used to determine the minimum shipping cost of poly vinyl chloride polymer from four supply points to four demand destinations. Sensitivity technique is employed to analyze the impact of uncertainty of unit shipping cost to the total shipping cost of the trading company.

TRANSPORTATION PROBLEM WITH LINEAR PROGRAMMING

By and large, transportation problem is concerned with the task of distribution of goods from any supply points to any demand destinations at the lowest total distribution cost possible. Each supply point has a certain supply capacity and each destination has a certain level demand that has to be fulfilled. The cost of transportation from one supply point to one destination varies linearly with the quantity supplied. Indeed, transportation problem is approached as a linear programming problem which can be solved by simplex method using linear programming.

Linear Programming is a powerful problem solving tool that aids management in making decisions. The basic approach is to formulate a mathematical model as a linear programming model that represents the problem and then to analyze this model using spreadsheet. Any linear programming model includes decision variables that represent the decisions to be made, constraints that represent the restrictions on the feasible values of these decision variables and an objective function that expresses the overall measure of performance for the problem (Taylor III, 2010; Dieter and Schmidt, 2009; Hiller and Hiller, 2004; Reeb and Leavengood, 2002).

LINEAR PROGRAMMING MODEL

Network representation: The Linear Programming model is setup to minimize the shipping cost and meet each of the demand while not exceeding the maximum capacity of the polymer producing petrochemical plants. In general, the following information is specified:

- A set of n supply points from which product is shipped. Supply point i can supply at most s_i units
- A set of m demand destinations to which the product is shipped. Demand destination j must receive at least d_j units of the shipped product
- Each plant produced at supply point i and shipped to demand destination j incur a variable cost of C_{ij}

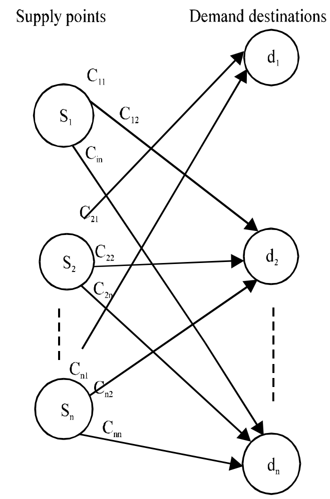

Fig. 1: The network representation of a general transportation problem

Formulation of mathematical model: Based on the network in Fig. 1, linear programming mathematical model is formulated. Let x_{ij} be the units shipped from supply point i to demand destination j. Then the formulation of a transportation problem with the objective of minimizing transportation cost is represented by:

$$\text{Min} \sum_{i=1}^{i=m} \sum_{j=1}^{j=n} C_{ij} x_{ij} \quad (1)$$

Formula (1) is subjected to two constraints namely:

Supply constraints:

$$\sum_{j=1}^{j=n} x_{ij} \le s_i \quad (I = 1, 2, \ldots, n) \quad (2)$$

Demand constraints:

$$\sum_{i=1}^{i=m} x_{ij} = d_j \quad (j = 1, 2, \ldots, n) \quad (3)$$

Since all the x_{ij} must be non-negative, the above need to meet the following additional restriction:

where,

$$x_{ij} \ge 0 \ (i = 1, 2, \ldots, n; j = 1, 2, \ldots, n)$$

The objective is to choose the value of x_{ij} so as to:

$$\text{Minimize the shipping cost} = C_{11}x_{11}+C_{12}x_{12}+\ldots+C_{1n}x_{1n}+C_{21}x_{21}+C_{22}x_{22}+\ldots+C_{2n}x_{2n}+\ldots+C_{n1}x_{n1}+C_{n2}x_{n2}+\ldots+C_{nn}x_{nn}$$

Subject to the supply constraint:

- Supply point 1, S_1: $x_{11}+x_{12}+...+x_{1n}$
- Supply point 2, S_2: $x_{21}+x_{22}+...+x_{2n}$
- Supply point n, S_n: $x_{n1}+x_{n2}+...+x_{nn}$

And subject to demand constraint:

- Demand destination 1, d_1: $x_{11}+x_{21}+...+x_{n1}$
- Demand destination 2, d_2: $x_{12}+x_{22}+...+x_{n2}$
- Demand destination n, d_n: $x_{1n}+x_{2n}+...+x_{nn}$

Formulation of spreadsheet model: Generally a linear programming mathematical model has a large number of variables that need to be evaluated. The process of calculation is simplified using a spreadsheet. Similarly, mathematical model of the transportation problem (Fig. 1) that involves many variables can be solved easily using a spreadsheet as shown in Fig. 2.

Detail calculation for the spreadsheet cell G11, G12, G13 and G14 are shown in Table 1.

Similarly, the calculation for cell C15, D15, E15 and F15 are summarized in Table 2.

Finally, the minimum total shipping cost is computed and the results are indicated in Table 3. It is the product of unit cost by shipment quantity. Spreadsheet gives the optimal solution shown in the changing cells Shipment Quantity (C11:F14) for the Unit Cost of shipping of the product from each supply point (C4:F7); subject to supply and demand constraints.

	A	B	C	D	E	F	G	H	I
1									
2			Destinations						
3	Unit cost		D_{11}	D_2		D_n			
4	$Source_1$		C_{11}	C_{12}		C_{1n}			
5	$Source_2$		C_{21}	C_{22}		C_{2n}			
6									
7	$Source_n$		C_{n1}	C_{n2}		C_{nn}			
8									
9	Shipment quantity			Destinations			Producer @ source		
10			D_1	D_2		D_n			Capacity
11	$Source_1$		X_{11}	X_{12}		X_{1n}		≤	S_1
12	$Source_2$		X_{21}	X_{22}		X_{2n}		≤	S_2
13									
14	$Source_n$		X_{n1}	X_{n2}				≤	S_n
15	Total								
16			=	=		=			Total cost
17	Demand		d_1	d_2		d_n			

Fig. 2: A spreadsheet model

Table 1: Formulae for cells G11, G12, G13 and G14

Cells	G (produced @ source)
G11	= SUM (C11:F11)
G12	= SUM (C12:F12)
G13	= SUM (C13:F13)
G14	= SUM (C14:F14)

CASE STUDY

A trading company, wholly-owned subsidiary of a Malaysian petrochemical company is involved in buying and selling poly vinyl chloride polymer. The polymer is produced by four petrochemical plants in Malaysia and is exported to four destinations namely China, the Middle East, Europe and South East Asia. The available production capacity of each polymer producing petrochemical plant and the demand of the customers are indicated in Table 4 and 5, respectively. Each plant has a fixed capacity per annum that shall be distributed to the customers. Similarly, each destination has a fixed demand per annum that must be fulfilled from the various plants.

The shipping costs from the polymer producing petrochemical plants to the various destinations are shown in Table 6. The unit cost of shipping varies as a result of differences in among others distance and also currencies exchange rates. Hence, allocating the production capacities to the various demand destinations in the optimal way to minimize total cost of shipping is crucial for the trading company.

Table 2: Formulae for cells C15, D15, E15 and F15

C15	D15	E15	F15
= SUM (C11:C14)	= SUM (D11:D14)	= SUM (E11:E14)	= SUM (F11:F14)

Table 3: Formula for minimum total shipping cost

Parameter	I
Total cost	= SUMPRODUCT (C4:F7,C11:F14)

Table 4: Production capacity

Plant	Production capacity (thousands ton per annum)
P1	110
P2	75
P3	95
P4	125

Table 5: Shipment quantity

Destination	Shipment quantity (thousands ton per annum)
China (D1)	200
Middle East (D2)	90
South East Asia (D3)	40
Europe (D4)	45

Table 6: Shipping costs

	Unit cost of shipping (RM' 000)			
Plant	China	Middle East	South East Asia	Europe
P1	200	300	100	600
P2	400	350	150	650
P3	300	250	150	600
P4	500	400	200	700

Mathematical and spreadsheet formulation: The formulation of the transportation problem for this case study is aimed at determining the optimum allocation of production capacity to each demand destination to minimize the cost of shipping. There are four polymer producing plants and four demand destination under consideration. Hence, i = 1, 2, 3, 4 and j = 1, 2, 3, 4.

Let x_{ij} = the amount (tonne/annum) to be shipped from polymer producing plants i to demand destination j.

C_{ij} = unit cost of shipping (RM'000/tonne) from polymer producing plants i to demand destination j.

The objective function is mathematically expressed as follows:

$$\text{Minimize shipping cost} = C_{11}x_{11}+C_{12}x_{12}+C_{13}x_{13}+C_{14}x_{14}+C_{21}x_{21}+C_{22}x_{22}+C_{23}x_{23}+C_{24}x_{24}+C_{31}x_{31}+C_{32}x_{32}+C_{33}x_{33}+C_{34}x_{34}+C_{41}x_{41}+C_{42}x_{42}+C_{43}x_{43}+C_{44}x_{44}$$

The trading company faces two constraints. Firstly, the total supply by each plant cannot exceed the plant's capacity. Secondly, each destination will receive the required quantity of polymer to meet its demand. Therefore, the above restrictions are expressed by Linear Programming constraints as follows:

Supply constraint:

- 110 Supply from P1: $x_{11}+x_{12}+x_{13}+x_{14}$
- 75 Supply from P2: $x_{21}+x_{22}+x_{23}+x_{24}$
- 95 Supply from P3: $x_{31}+x_{32}+x_{33}+x_{34}$
- 125 Supply from P4: $x_{41}+x_{42}+x_{43}+x_{44}$

Demand constraint:

- 200 demand for D1: $x_{11}+x_{21}+x_{31}+x_{41}$
- 90 demand for D2: $x_{12}+x_{22}+x_{32}+x_{42}$
- 40 demand for D3: $x_{13}+x_{23}+x_{33}+x_{43}$
- 45 demand for D4: $x_{14}+x_{24}+x_{34}+x_{44}$

Since all the x_{ij} must be non-negative, the above need to meet the following additional restriction as well:

$$x_{ij} \geq 0 \; (i = 1, 2, 3, 4; j = 1, 2, 3, 4)$$

Combining the objective function, supply constraint, demand constraint and sign restriction yields the following Linear Programming formulation:

$$\text{Minimum shipping cost} = 200x_{11}+300x_{12}+100x_{13}+600x_{14}+400x_{21}+350x_{22}+150x_{23}+650x_{24}+300x_{31}+250x_{32}+150x_{33}+600x_{34}+500x_{41}+400x_{42}+200x_{43}+700x_{44}$$

	A	B	C	D	E	F	G	H
1								
2		Destinations						
3	Unit cost	D1	D2	D3	D4			
4	P1	200	300	100	600			
5	P2	400	350	150	650			
6	P3	300	250	150	600			
7	P4	500	400	200	700			
8		Destinations						
9	Quantity	D1	D2	D3	D4	Prod.		Capacity
10	P1	110	0	0	0	110	≤	110
11	P2	0	30	0	45	75	≤	75
12	P3	90	5	0	0	95	≤	95
13	P4	0	55	40	0	95	≤	125
14	Total	200	90	40	45			
15		=	=	=	=			Tot. cost
16	Demand	200	90	40	45			120,000

Fig. 3: Spread sheet formulation and solver output

RESULTS

Mathematical and spreadsheet formulation: The output of the spreadsheet formulation for the trading company transportation problem is as shown in Fig. 3. The supply and demand constraints are shown in cells Capacity (H10:H13) and cells Demand (B16:E16), respectively. The unit shipping cost from the polymer producing plants to the various demand destinations are included in cells Unit Cost (B4:E7).

The changing cells Quantity (B10:E13) show the optimal shipping plan of the trading company. The resulting total cost is given in the target cell Total Cost (H16). This solution minimizes the total cost of shipping of the polymer from the four polymer producing plants to four demand destinations.

Sensitivity analysis: Sensitivity analysis is used to investigate the sensitivity of the total cost of shipping to changes in unit shipping cost of each supply point namely Plant 1, Plant 2, Plant 3 and Plant 4 to the various demand destinations. It is a non-probabilistic methodology to provide information pertaining to the impact of uncertainty of unit shipping cost to the total shipping cost. The relative magnitude of change in the

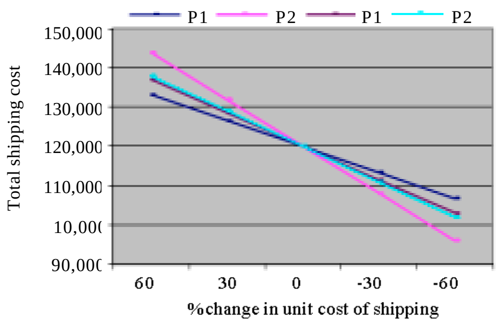

Fig. 4: Sensitivity graph of total cost of shipping

Table 7: Sensitivity analysis of total cost of shipping

Change in unit cost of shipping (%)	Total cost of shipping (million RM) Supply points P1	P2	P3	P4
60	133,200	143,850	136,950	138,000
30	126,600	131,925	128,475	129,000
0	120,000	120,000	120,000	120,000
-30	113,400	108,075	111,525	111,000
-60	106,800	96,1500	103,050	102,000

total shipping cost is investigated by varying the unit shipping cost of one supply point at a time, while the other remain the same. Table 7 shows the resulting values of total shipping cost as each unit cost of shipping is varied over a range of ±60% from the most-likely estimate.

The impact of the resultant changes in the total shipping cost is made explicit by drawing sensitivity graph as illustrated by Fig. 4. The relative degree of sensitivity of the total shipping cost to the variation in the unit shipping cost is indicated by the slope of the curves in the graph (Sullivan *et al*., 2012; Park, 2004). The steeper the slope of the curve, the more sensitive the total shipping cost is to the changes in that unit shipping cost.

DISCUSSION

Th e case study is concerned with distribution of polymer from four polymer producing plants in Malaysia to four demand destinations namely China, Middle East, South East Asia and Europe. The objective is to determine the distribution plan that leads to the lowest total shipping cost to the trading company that is entrusted to deliver the polymer. It is subjected to two constraints. Each source of supply has a fixed production capacity and each demand destination has a fixed quantity for the polymer. Furthermore, the unit cost of shipping from the plants to the various destinations varies. Using Linear Programming and spreadsheet an optimal solution was obtained to meet the objective of minimizing the cost of shipping for the polymer from the plant to the market.

The results show that 200,000 ton/annum supply for China market should be arranged from Plant 1 (110,000 ton/annum) and Plant 3 (90,000 ton/annum). The demand for Middle East of 90,000 ton/annum must be delivered from three sources namely Plant 2 (30,000 ton/annum), Plant 3 (5,000 ton/annum) and Plant 4 (55,000 ton/annum). The delivery of polymer for South East Asia market should be arranged from Plant 4 (40,000 ton/annum). Finally, the requirement for Europe market of 45,000 ton/annum shall be arranged from Plant 2.

The resulting total shipping cost is RM 120,000 million. This solution minimizes the total cost of shipping of the polymer from the four polymer producing plants to four demand destinations. The results of these analyses are in accordance with the statements posited by Reeb and Leavengood (2002) and Mandel (2004). Based on the sensitivity graph illustrated in Fig. 4, curve P2 has the greatest slope. Hence, it can be concluded that the variation in the unit shipping cost of Plant 2 has the greatest impact to the total cost of shipping of the trading company.

CONCLUSION

This study has highlighted the applications of Linear Programming and spreadsheet in a case study of a transportation problem of a Malaysian trading company. Optimum plan and solution to minimize the total cost of transportation were formulated and analyzed. Certainly, linear programming is an alternative decision tool available to engineers and managers alike in ensuring their operations are conducted effectively and efficiently at the lowest cost possible and consequently maximize the company's profit.

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