Journal of Applied Sciences

ISSN 1812-5654

science alert

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Journal of Applied Sciences 12 (24): 2519-2526, 2012
ISSN 1812-5654 / DOI: 10.3923/jas.2012.2519.2526

Attitude Maneuvers of CTS-like Spacecraft Using PD based Constant-Amplitude Inputs

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Abstract: Attitude maneuvers of a Communication Technology Satellite (CTS)-like spacecraft using constant-amplitude thrusters is of great importance. The spacecraft consists of a rigid main body and two symmetrical solar panels. When the panels are large, they cannot be treated as rigid bodies anymore. They are supposed to behave structural flexibility. To discrete their motion, the finite element method is followed. Under constant-amplitude thrusts, steady-state attitude angle oscillations may occur in large amplitude after the maneuvers. Since, the spacecraft should point to the earth precisely, these oscillations must be reduced into small permissible values. To reduce residual attitude angle oscillations, Proportional Derivative (PD) based constant-amplitude input shaping logic is proposed to determine time locations of thruster switching. Then, under such inputs, attitude maneuvers of the spacecraft are simulated numerically. Results of simulations show that the precise orientation of the satellite can be achieved.

Key words: Flexible spacecraft, attitude maneuver, constant-amplitude, PD control

INTRODUCTION

Growing needs of human in communication require larger communication satellites in size. The Early Bird American communications satellite, Intelsat-1, launched on 6th April 1965 has 39 kg mass and 0.59 m height only. Then, Intelsat-4 satellites launched in 1971-1975 have 1410 kg launch mass and 5.30 m height. Besides of batteries, the both types of satellite use solar cells mounted on their bodies to generate power needs. The increasing power need then requires the bigger solar cell sizes. They cannot be mounted on the limited dimension of the satellite body anymore. The solar cells then are covered on the deployed panels. They form solar wings. Intelsat-5 satellites launched in 1980-1989 have solar wings in 15.90 m span and 2000 kg launch mass. The following generation satellites, Intelsat-7, launched between 1993-1996 have 3695 kg launch mass, 21.80 m solar wing span and 4.20 m height. The Malaysian satellite, MEASAT-3, launched in 2006 has 3220 kg mass in the beginning of life in orbit with 26.2 m solar wing span. Canadian Communication Technology Satellite (CTS) Anik F1 launched 21st November 2000 has 3,015 kg in orbit mass at the beginning of life with two solar wings in 40.4 m span. Canadian communication technology satellite (CTS) Anik F2 launched on 14th July 2004 has 3,805 kg in orbit mass at the beginning of life with two solar wings in 47.9 m span. From the above examples it can be concluded that the communication satellites become heavier and heavier, while their solar wing spans become longer and longer. Based on the limitation of weight in launching, the materials to construct the satellite must be light in weight. When the light solar wings have long span, they must be considered as flexible structures.

The communication satellites are designed to have certain attitude accuracies in operation. For example, Hwangbo (1992) mentioned that a satellite required by the Koreasat must have an antenna beam pointing error less than 0.07° in roll and pitch and less than 0.2° in yaw. To keep the precise orientation, the satellite needs frequent corrections of its attitude during its operation in space. Attitude of rigid satellites can be changed without residual vibration problems after the maneuvers. When the satellite structures are not rigid, maneuvering the attitude without regard to the system flexibility will result large amplitude steady-state oscillations, especially when the system is equipped with constant-amplitude on-off jets.

To reduce vibration in a flexible satellite system, which is equipped with on-off reaction jets, the input shaping methods have been developed by Liu and Wie (1992), Pao and Singhose (1995) and Rogers and Seering (1996). Parman and Koguchi (1998, 1999a, b, 2000) demonstrated an application of shaped commands to change roll angle of a flexible satellite with a large number of flexible modes. They studied roll and pitch maneuvers

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of the flexible satellite. They showed that the residual vibrations can be reduced drastically when the satellite is subjected to shaped inputs suppressing the vibration at two frequencies with largest vibration amplitude.

In this study, computer simulations of attitude maneuvers of the CTS-like spacecraft equipped with on-off thruster are presented. The spacecraft consists of a rigid main body and two flexible solar wings. The spacecraft model developed by Parman and Koguchi (1998) is used and torque inputs resulted by thrusters are at constant amplitude. The solar panel offset angle is set to 30°. For this condition of the spacecraft, roll motions couple with the yaw angle motions. The constant-amplitude torque inputs based on Proportional Derivative (PD) control logic are applied to maneuver the spacecraft attitude.

CTS-LIKE SPACECRAFT

Finite element model of the spacecraft: The spacecraft studied in this study is a CTS-like spacecraft consisted of a rigid main body and two symmetrical large solar panels. Since, the solar panels are large in size but light in weight, they are supposed as flexible structures. The finite element method is used to discrete their elastic deformations. For this purpose, a finite element model of the spacecraft is developed by using the model studied by Parman and Koguchi (1998) as shown in Fig. 1. In this model, each solar panel is divided into 16 rectangular plate elements. By using such a division, each solar panel has 27 nodal points. The elements are numbered from 1 through 16 on the right-hand-side and from 17 through 32 on the left-hand-side, while their nodal points are numbered from 1 through 27 on the right-hand-side and from 28 through 54 on the left-hand-side. The solar panels are oriented towards the sun and the declination with respect the X_b-axis of rigid main body-fixed frame is identified by the offset angle δ. Only out-of-plane deformations of the solar panels are considered.

The spacecraft attitude with respect to its orbital reference frame is expressed in Bryant's angles: roll angle ϕ, pitch angle θ and yaw angle Ψ; where the definition of these angles can be seen in Fig. 2. The equations of motion of the spacecraft have been developed by Parman and Koguchi (1998) by using Lagrange's formulation. For small attitude angle displacements, they can be written in the following matrix form:

$$\begin{bmatrix} mU_3 & Q & W \\ Q^T & I & A \\ W^T & A^T & M \end{bmatrix} \begin{Bmatrix} \ddot{r} \\ \ddot{\boldsymbol{\Theta}} \\ \ddot{d} \end{Bmatrix} + \begin{bmatrix} 0_{3\times3} & Q\tilde{\boldsymbol{\omega}}_0 & W \\ 0_{3\times3} & I\tilde{\boldsymbol{\omega}}_0 & A \\ 0_{f\times3} & A^T\tilde{\boldsymbol{\omega}}_0 & M \end{bmatrix} \begin{Bmatrix} \dot{r} \\ \dot{\boldsymbol{\Theta}} \\ \dot{d} \end{Bmatrix} + \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & 0_{3\times f} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times f} \\ 0_{f\times3} & 0_{f\times3} & K \end{bmatrix} \begin{Bmatrix} r \\ \boldsymbol{\Theta} \\ d \end{Bmatrix} = \begin{Bmatrix} F_b \\ T_b \\ F_a \end{Bmatrix} \tag{1}$$

where, m is the mass of the spacecraft and Q is the coupling matrix for the translational and rotational displacements of the rigid main body. The coupling matrix for the translational displacements of rigid main body and the displacements of flexible solar panels is notated in W, while the inertia matrix of the whole spacecraft is symbolized in I. The coupling for the rotational displacements of the rigid main body and the displacements of flexible solar panels is expressed is A. The matrices M, D and K are the mass matrix, the damping matrix and the stiffness matrix of the flexible solar panels, respectively. If the angular velocity of spacecraft orbit is ω_0, then $\tilde{\omega}_0$ is its skew symmetric matrix. Vector r is the translational displacement of the rigid main body, while Θ is the rotational displacement of the rigid main body. Vector Θ is an expression vector of Bryant's angles. Vector d is stating the displacements of the flexible solar panels. F_b and T_b are external forces and torques vectors acting on the rigid main body, F_a is the vector of external forces and torques acting on the solar panels, while f is the total number of degrees of freedom of the solar panels. In this study, the flexible

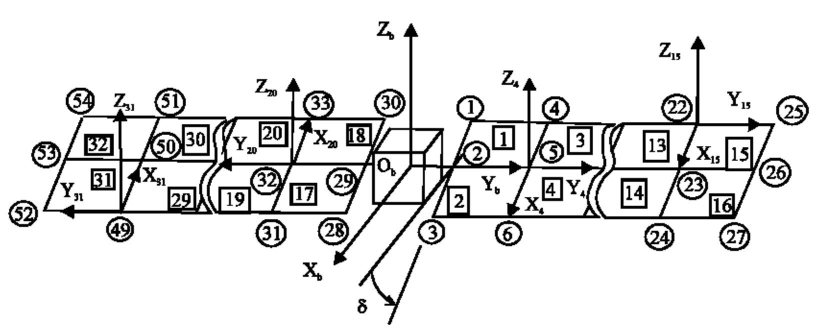

Fig. 1: The Model of communication technology satellite (CTS)-like spacecraft, Element numbers are in rectangles and nodal point numbers are in circles, $X_jY_jZ_j$ (j = 1, 2, ..., 32) are local reference frames for solar panel elements

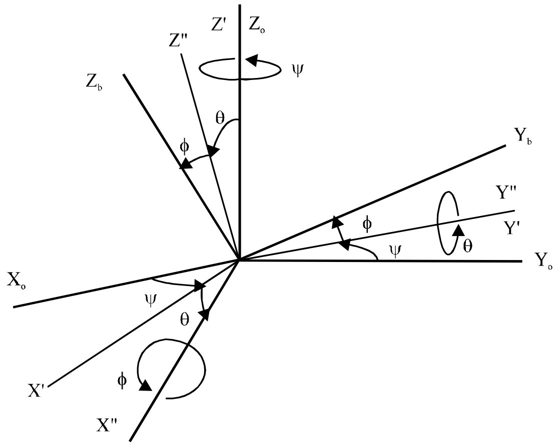

Fig. 2: The rotations from an observation reference frame F_o ($X_oY_oZ_o$) to the main body-fixed reference frame F_b ($X_bY_bZ_b$)

structural subsystems are supposed to have no dissipation properties, so that D = 0.

Rest-to-rest attitude maneuver of CTS-Like spacecraft using "Fast" constant-amplitude feed-forward torques: The main body of real spacecraft contains cameras, control devices, electronics, antennas, etc. They are placed in suitable positions in the main body structure related to their functions. For control consideration, the locations of these components are also trying to follow symmetrical conditions with respect to the main body reference axes. For simulations, the main body of spacecraft is modeled as 6 lumped masses at certain positions as shown in Table 1. Following this table, total mass of the main body is 3,100 kg. The parameters of flexible solar panels can be seen in Table 2. The offset angle of solar panels is taken to be 30 degrees. For this configuration, the total mass of the spacecraft becomes 3,446 kg. The origin of the rigid main body fixed reference frame coincides with the centre of mass of the whole spacecraft in the undeformed state, I_{xx} = 42,140 kg m^{-2}, I_{yy} = 1,431 kg m^{-2}, I_{zz} = 41,498 kg m^{-2}, I_{xy} = I_{yz} = 0 and I_{xz} = 112 kg m^{-2}. The orbital frame moves relative to the inertial frame with constant angular velocity:

$$\omega_o = -\omega_o \, j_i \tag{2}$$

where, j_i is the unit vector in Y_i-axis direction, ω_o = 7.29 × 10^{-5} rad sec^{-1}, so that F_o performs in F_i one rotation per sidereal day.

Using the above parameters, the motion of the spacecraft is expressed in 168 generalized coordinates. The first six coordinates represent translations and rotations of the main body. The system has six natural frequencies in zero values and 162 ones in nonzero values. The zero values are relating to the rigid body modes: three in translation and three in rotations. The nonzero values of natural frequencies range from 0.042 Hz until 34.6 Hz. The natural frequencies of the system and components of related eigenvectors for roll, pitch and yaw angles are plotted in Fig. 3. It can be seen in this figure that the large values of eigenvector's components are related to low natural frequencies of the system. The high natural frequencies are relating to the small values of eigenvector's components. So, if the system oscillates, the large vibrations happen at low frequencies.

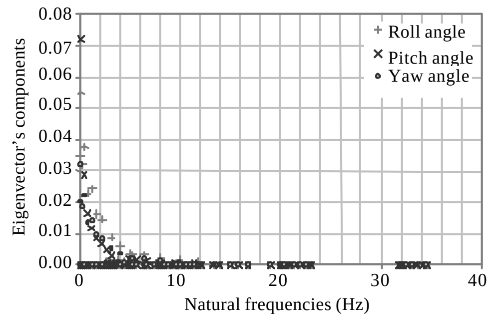

Fig. 3: Plot of eigenvector's components in roll, pitch and yaw angles vs. Natural frequencies

Table 1: Lumped masses consisting of the rigid main body

	Position (m)		
Mass (kg)	X_b	Y_b	Z_b
400	0.50	0.00	0.00
400	-0.50	0.00	0.00
550	0.00	0.70	0.00
550	0.00	-0.70	0.00
600	0.00	0.00	0.90
600	0.00	0.00	-0.90

Table 2: Parameters of the solar panels of spacecraft

Description	Values
Number of solar panels	2
Dimension of each solar panel (m^3)	16×3×0.03
Young's modulus, E (N m^{-2})	0.6×10^8
Poisson ratio, υ	0.3
Mass density, ρ (kg m^{-3})	120
Number of elements in each solar panel	16
Dimension of each element, b×a×c (m^3)	2×1.5×0.03
Offset angle, δ (degrees)	30
Distance between panel's root and O_b (m)	1.80

A system is called to be in a rest condition if the velocity of the system is zero. For a flexible system, the rest condition doesn't mean the velocity to be zero, but it means that the average velocity of the oscillating system is zero. The maneuver is referred to as rest-to-rest if the conditions of the system before and after the maneuver process to be in rest. For the system having translational motion, the rest-to-rest maneuver is to move it from one initial rest position to another rest position. The rest-to-rest maneuver of the rotational system is to change its attitude angle, from one rest condition to another rest condition.

A time-optimal input is an input given to a certain maneuver in the shortest time duration. For an input consisted of a series of alternating-sign constant-amplitude pulses, the input to slew the system in rest-to-rest maneuver is a bang-bang in maximum amplitude. The bang-bang input will be two sequence pulses in the alternating sign in the same width.

In this study, the observed spacecraft is supposed to have no control and no damping properties on the flexible solar panels. The control inputs are only applied to the rigid main body at the center of mass of spacecraft, as constant amplitude force or torque pulses resulted by on-off reaction jets. For such a system, under control or external torques only, remembering Eq. 1, the resultant attitude angle acceleration of spacecraft as a rigid body motion can be written as:

$$\ddot{\boldsymbol{\Theta}} = \begin{Bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{Bmatrix} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}^{-1} \begin{Bmatrix} T_{b_x} \\ T_{b_y} \\ T_{b_z} \end{Bmatrix} \tag{3}$$

where I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}, and I_{yz} are components of the inertia matrix I of the whole spacecraft and T_{bx}, T_{by} and T_{bz} are components of the torque input vector T_b on the rigid main body. By integrating Eq. 3 with respect to time we get an expression of desired attitude angle velocity:

$$\dot{\boldsymbol{\Theta}}_d = \begin{Bmatrix} \dot{\phi}_d \\ \dot{\theta}_d \\ \dot{\psi}_d \end{Bmatrix} = \int \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}^{-1} \begin{Bmatrix} T_{b_x} \\ T_{b_y} \\ T_{b_z} \end{Bmatrix} dt \tag{4}$$

and integrating once more gives a desired roll angle displacement:

$$\boldsymbol{\Theta}_d = \begin{Bmatrix} \phi_d \\ \theta_d \\ \psi_d \end{Bmatrix} = \iint \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}^{-1} \begin{Bmatrix} T_{b_x} \\ T_{b_y} \\ T_{b_z} \end{Bmatrix} dt\,dt \tag{5}$$

The spacecraft has 4° roll angle, 7° pitch angle and -4° yaw angle at initial condition. These attitude angles must be corrected to 0°.

Torqueses used to maneuver the spacecraft being studied here are resulted by on-off reaction jets, so the amplitude of torques is constant. The shortest duration time of constant-amplitude feed-forward command for rest-to-rest slew maneuver is a bang-bang input. Constraint equations that must be satisfied for this rest-to-rest slew maneuver are $\{\dot{\phi}_d, \dot{\theta}_d, \dot{\psi}_d\}^T = \{0, 0, 0\}^T$ and $\{\phi_d, \theta_d, \Psi_d\}^T = \{-0.0698, -0.1222, 0.0698\}^T$. If the amplitude of command, either T_{bx}, T_{by}, or T_{bz} is determined to be 20 N m, the profile of bang-bang torques needed consist of 24.29 seconds long of T_{bx}, 5.91 seconds long of T_{by} and 24.10 sec long of T_{bz} bang-bangs. First, the spacecraft is subjected to the roll torque. Then, the pitch torque is applied as the roll torque finished. Finally, it is subjected the yaw torque input.

For simulations done in this paper, the second-order linear differential equations system, Eq. 1, is solved using the Newmark Method (Smith and Griffiths, 2005). As results, under these inputs, all attitude angles can be changed to 0°. After the torques were removed, the roll and yaw angles still oscillate with an amplitude dominantly at the period of 23.62 sec which relates to natural frequency resulted in calculation 0.266 rad sec^{-1}. The dominant pitch angle oscillation has 6.06 sec period, relating to natural frequency 1.037 rad sec^{-1}, with the 0.5° total amplitude. The total amplitude of residual oscillation is 10.4° for roll angle and 6.1° for yaw angle as shown in Fig. 4. These oscillations are larger than the desired attitude angle displacements. Also, all residual attitude angles oscillations are larger than the permitted attitude errors of such a spacecraft required by Koreasat.

Shaping constant-amplitude torque input based on PD control: The spacecraft studied here is equipped with on-off reaction jets and it cannot produce variable amplitude actuation force. The spacecraft must be maneuvered with constant amplitude torques resulted by thruster pulses. For this purpose, the input is shaped by following PD control. If the desired angular displacement and velocity vectors of the spacecraft main body are Θ_d and $\dot{\Theta}_d$, respectively, then at time t the vectors of angular displacement and velocity errors are:

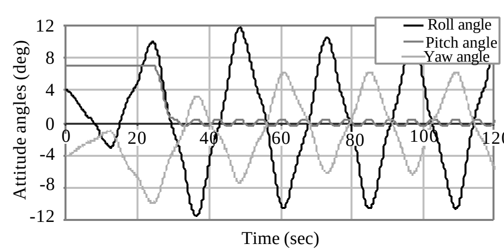

Fig. 4: Time responses under the bang-bang torque inputs: attitude angle displacement of the main body

$$e_\Theta = \Theta(t) - \Theta_d \quad (6)$$

and

$$e_{\dot\Theta} = \dot{\Theta}(t) - \dot{\Theta}_d \quad (7)$$

respectively. In standard PD control, the torque input can be written as:

$$\breve{T}_b = -\mathbf{K}_\Theta e_\Theta - K_{\dot\Theta} e_{\dot\Theta} \quad (8)$$

where, K_Θ and $K_{\dot\Theta}$ are displacement and velocity gain constant matrices, respectively. Equation 8 produces variable values of control command in roll, pitch and yaw directions. For application of PD control on the torque commands resulted by constant-amplitude thrusters, the following logic is defined to shape T_b in Eq. 1:

$$T_b = \begin{cases} 0, & \text{if } |e_\Theta| < e_{\Theta 0} \text{ and } |e_{\dot\Theta}| < e_{\dot\Theta 0} \\ 0, & \text{if } |\breve{T}_b| < T_{switch} \\ T_{max}\,\mathrm{sgn}(\breve{T}_b), & \text{if } |e_\Theta| > e_{\Theta 0} \text{ or } |e_{\dot\Theta}| > e_{\dot\Theta 0} \end{cases} \quad (9)$$

In Eq. 9, $e_{\Theta 0}$ and $e_{\dot\Theta 0}$ are setting points of attitude angle and angular velocity errors, respectively. T_{switch} is the setting point of T to switch on the thruster. In this study, it is selected that:

$e_{\Theta 0}$ = $\{10^{-4}, 10^{-4}, 10^{-4}\}^T$ deg
$e_{\dot\Theta 0}$ = $\{10^{-5}, 10^{-5}, 10^{-5}\}^T$ deg sec^{-1}
T_{max} = $\{20, 20, 20\}^T$ Nm and
T_{switch} = $\{10, 10, 10\}^T$ Nm

For example, at certain time, when:

$e_{\Theta 0}$ = $\{-12\times10^{-4}, -0.2\times10^{-4}, 20\times10^{-4}\}^T$ deg
$e_{\dot\Theta 0}$ = $\{20\times10^{-5}, 0.2\times10^{-5}, -45\times10^{-5}\}^T$ deg sec^{-1}
$\breve{T}_b$ = $\{8, -0.2, -11\}^T$ Nm

then

T_b = $\{0, 0, -20\}^T$ Nm

Although, the roll angle and velocity errors are greater than the error setting points, but since the magnitude of roll torque following PD control is less than the thruster switching point, then the roll thruster will be switched off.

Attitude maneuvers of spacecraft under PD based feedback constant-amplitude inputs: The values of gain constant matrices used for simulations presented in this paper are:

$$\mathbf{K}_\Theta = \begin{bmatrix} 10000 & 0 & 0 \\ 0 & 10000 & 0 \\ 0 & 0 & 10000 \end{bmatrix}$$

and

$$\mathbf{K}_{\dot\Theta} = \begin{bmatrix} 20000 & 0 & 0 \\ 0 & 20000 & 0 \\ 0 & 0 & 20000 \end{bmatrix}$$

A thruster has a certain capability in switching: from switch off to switch on, from switch on to switch off, or from switch on in one direction to switch on to the opposite direction. The torques resulted by the spacecraft thrusters consist of roll, pitch and yaw simultaneously. If the minimum time interval of sequence thruster switching is a second, the time history of the spacecraft attitude angles can be seen in Fig. 5a. It can be seen in this case that the spacecraft has very poor attitude accuracies after the maneuvers. The roll torque input used is shown in Fig. 5b.

For 0.5 sec minimum switching time interval, the responses of the spacecraft attitude angles can be seen in Fig. 6. The maneuvers are completed in all directions after t = 100 sec. Now, after the maneuvers, the oscillations of attitude angles can be reduced better than the bang-bang input case. However, they are still big enough in amplitude. The pitch angle oscillates in 0.4 deg resultant amplitude, while the amplitudes of roll and yaw angle oscillations are still in 0.9 and 0.5 deg, respectively.

When the minimum switching time interval used is reduced to 0.1 sec, the responses of the spacecraft attitude angles can be seen in Fig. 7. The spacecraft needs about 92 sec to complete the maneuvers. Here, the attitude angle oscillations can be reduced slightly. The pitch angle oscillation happens in 0.03 deg amplitude. However, the amplitudes of roll and yaw angles after t = 100 sec are still about 0.13 and 0.08 degrees, respectively.

A 0.02 sec minimum time interval of thruster switching results better accuracies of the spacecraft attitude after the maneuvers. To complete the maneuvers, the spacecraft consumes about 88 sec. The time histories of the spacecraft attitude angles for this case are shown

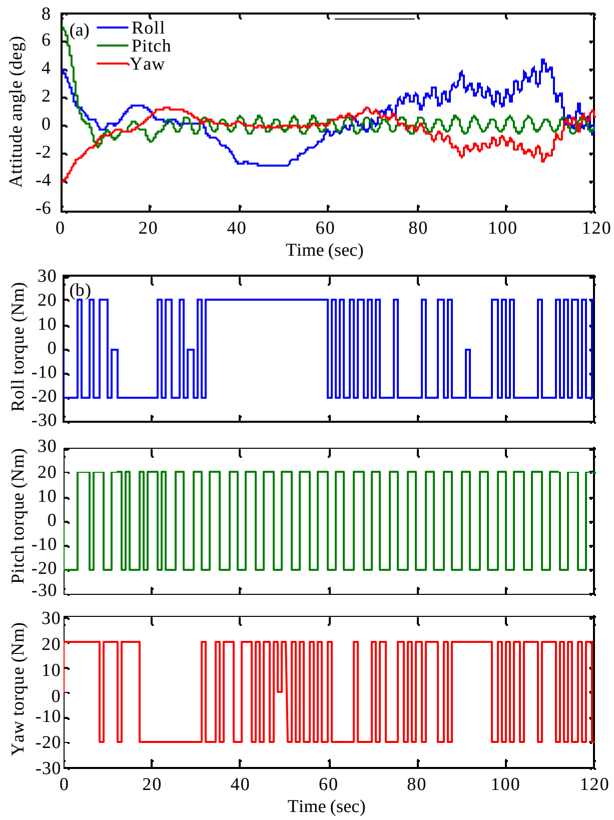

Fig. 5(a-b): Time histories of attitude angles and torques for 1 sec minimum switching time interval of thrusters

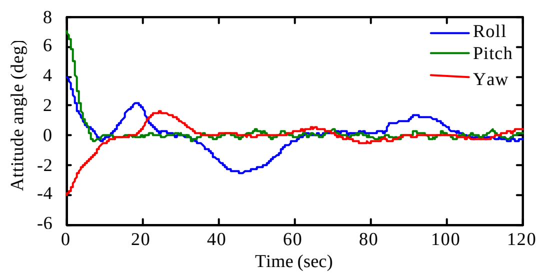

Fig. 6: Time responses for 0.5 sec minimum switching time interval of thrusters

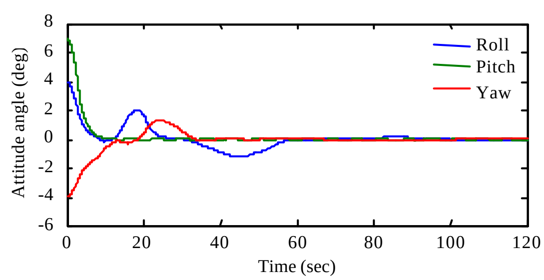

Fig. 7: Time responses for 0.1 sec minimum switching time interval of thrusters

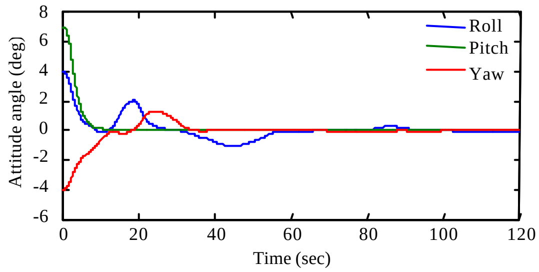

Fig. 8: Time responses for 0.02 sec minimum switching time interval of thrusters

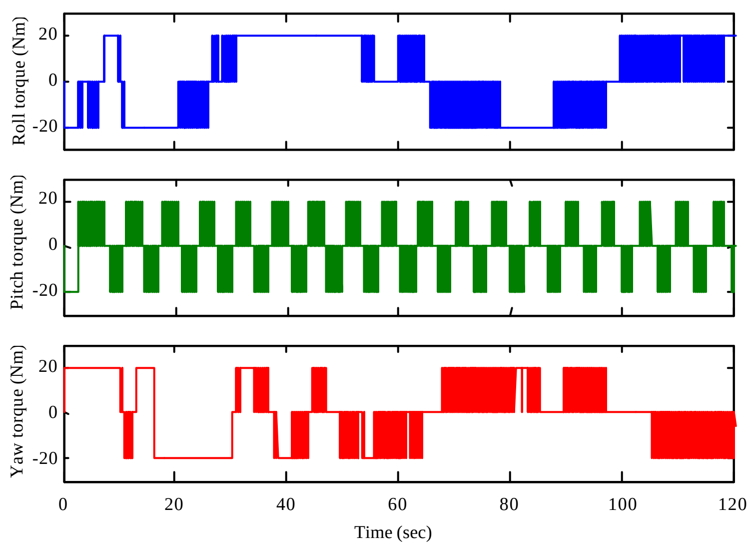

Fig. 9: Time history of torques for 0.02 sec minimum switching time interval of thrusters

in Fig. 8. The figure shows that the oscillation amplitudes of roll, pitch and yaw angles after t = 100 sec are 0.07, 0.03 and 0.06 degrees, respectively. These attitude angle oscillations are small enough and all satisfy the attitude accuracy required by Koreasat as stated by Hwangbo (1992). The roll, pitch and yaw torque inputs needed for the maneuvers are shown in Fig. 9. In reality, after completing the maneuvers, all thrusters can be switched off and the attitude of the spacecraft will be maintained by passive stabilization devices.

ACKNOWLEDGMENTS

This study is supported by MOSTI ScienceFund (Project Number: 04-02-02-SF0097). The authors are thankful to Universiti Teknologi PETRONAS for providing the research facilities.

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