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Analysis of Two-layered Porous Journal Bearing using the Brinkman Model

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Abstract: The present study evaluates the effects of two-layered long porous journal bearing configuration on improvement in load capacity and reduction in coefficient of friction. The Brinkman model is utilized to model the flow in the porous region. A modified form of Reynolds equation is derived considering two-layered porous region adjacent to the bearing surface. The non-dimensional pressure and shear stress expressions are obtained using the Reynolds boundary conditions. Results of non-dimensional load capacity and coefficient of friction are presented as a function of permeability and thickness of porous layers. Based on the results presented in the study, a low permeability porous layer that adheres to high permeability porous layer on bearing surface could significantly enhance load capacity and reduce coefficient of friction.

Key words: Porous layer, load capacity, coefficient of friction, non-dimensional pressure

INTRODUCTION

The performance of hydrodynamic lubricated contacts would be improved by additives which form a thin porous layer adhered to bearing surfaces due to lubricant microstructure (Oliver, 1988). Tichy (1995) developed models applicable for fluid flow through porous medium considering the effects of lubricant additives. Li (1999) derived a modified form of Reynolds equation using Brinkman-extended Darcy model which takes into account the viscous shear and viscous damping effects. Lina *et al*. (1996) applied the Brinkman model to predict the load capacity and friction parameter for flexible long porous journal bearings. Their results showed that Brinkman model which includes viscous shear effects, predicts an increase in load capacity and reduction in coefficient of friction. Li and Chu (2004) and Elsharkawy (2005) utilized porous media model and the couple stress model to study the effects of lubricant additives on the performance of hydrodynamic contacts. By modeling the microstructure of lubricating surfaces as thin porous film press fitted on bearing surfaces, the theoretical approach on the steady state performance of hydrodynamic contacts is presented to investigate the effects of lubricant additives.

Naduvinamani (1997) presented a theoretical study of double-layered porous Rayleigh-step bearings using Darcy's model and Beavers-Joseph velocity slip at the porous media/fluid film interface. Saha and Majumdar (2004) investigated steady state and stability characteristics of hydrostatic two-layered porous oil journal bearings. Two-layered porous bearing using highly permeable structural support topped by a thin layer with restriction to fluid flow gives better stability. The characteristics of flow through three layered porous media are investigated by Allan *et al.* (2009). Attia (2007) analyzed steady flow between two parallel plates in a porous medium. Amiri (2001) presented the analysis of flow through porous medium using a capillary model.

This study presents one-dimensional analysis of two-layered long porous journal bearing using Newtonian fluid as lubricant. A modified Reynolds equation is derived using Brinkman model. The nondimensional pressure and shear stress expressions are obtained. Reynolds boundary conditions are used to solve the pressure distribution. In this work, the effects of dimensionless permeability parameter and porous layer thickness on the steady state performance characteristics such as load capacity and coefficient of friction are analyzed.

MATHEMATICAL FORMULATION

The schematic of a two-layered porous journal bearing is shown in Fig. 1. The porous layer 1 of thickness δ_1 is adjacent to the stationery bearing surface while the porous layer 2 of thickness δ_2 is adjacent to the porous layer 1. In the present analysis, the variation of pressure across the porous layer and fluid film is assumed to be negligible.

The fluid motion in the two-layered porous regions (region I: $0 \le y \le \delta$ and region II: $\delta_1 \le y \le \delta_1+\delta_2$) is governed

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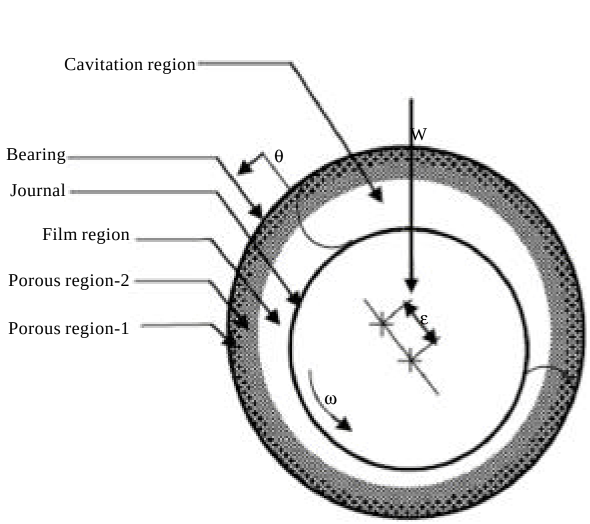

Fig. 1: Geometry of two-layered porous journal bearing

by Brinkman equation considering viscous shear and viscous damping effects as:

$$\frac{1}{\mu}\frac{dp}{dx} = -\frac{u_j}{k_j} + \frac{d^2u_j}{dy^2} \quad \text{for } j = 1, 2 \tag{1}$$

The fluid motion in the film region obeys the conventional governing Newtonian flow equations:

$$\frac{1}{\mu}\frac{dp}{dx} = \frac{d^2u_j}{dy^2} \quad \text{for } j = 3 \tag{2}$$

The boundary conditions are: (1) no-slip boundary conditions at the bearing and journal surface and (2) velocities and viscous shear stresses match at the porous layer 1/porous layer 2 and porous layer 2/fluid film interface:

$$y = 0: \quad u_1 = 0 \tag{3}$$

$$y = \delta_1: u_1 = u_2 = u_{12} \text{ and } \frac{du_1}{dy} = \frac{du_2}{dy} \tag{4}$$

$$y = \delta_1 + \delta_2: \; u_2 = u_3 = u_{23} \text{ and } \frac{du_2}{dy} = \frac{du_3}{dy} \tag{5}$$

$$y = h: \quad u_3 = u_j \tag{6}$$

Integrating the Eq. 1-2 using the boundary conditions in Eq. 3-6, the dimensionless velocity distribution is expressed as:

$$0 \le Y \le \Delta_1: \quad U_1 = U_{12}\frac{\sinh\left(\frac{Y}{\sqrt{K_1}}\right)}{\sinh\left(\frac{\Delta_1}{\sqrt{K_1}}\right)} - K_1\frac{dP}{d\theta}C_1(Y) \tag{7}$$

$$\Delta_1 \le Y \le \Delta_1 + \Delta_2: \; U_2 = -U_{12}\frac{\sinh\left(\frac{Y-\Delta_1-\Delta_2}{\sqrt{K_2}}\right)}{\sinh\left(\frac{\Delta_2}{\sqrt{K_2}}\right)} + U_{23}\frac{\sinh\left(\frac{Y-\Delta_1}{\sqrt{K_2}}\right)}{\sinh\left(\frac{\Delta_2}{\sqrt{K_2}}\right)} - K_2\frac{dP}{d\theta}C_2(Y) \tag{8}$$

$$\Delta_1 + \Delta_2 \le Y \le H: \; U_3 = U_{23} + (1 - U_{23})\left(\frac{Y-\Delta_1-\Delta_2}{H-\Delta_1-\Delta_2}\right) + \frac{1}{2}\frac{dP}{d\theta}(Y-\Delta_1-\Delta_2)(Y-H) \tag{9}$$

Where:

$$C_1(Y) = 1 - \frac{\sinh\left(\frac{Y}{\sqrt{K_1}}\right)}{\sinh\left(\frac{\Delta_1}{\sqrt{K_1}}\right)} + \frac{\sinh\left(\frac{Y-\Delta_1}{\sqrt{K_1}}\right)}{\sinh\left(\frac{\Delta_1}{\sqrt{K_1}}\right)},$$
$$C_2(Y) = 1 + \frac{\sinh\left(\frac{Y-\Delta_1-\Delta_2}{\sqrt{K_2}}\right)}{\sinh\left(\frac{\Delta_2}{\sqrt{K_2}}\right)} - \frac{\sinh\left(\frac{Y-\Delta_1}{\sqrt{K_2}}\right)}{\sinh\left(\frac{\Delta_2}{\sqrt{K_2}}\right)} \tag{10}$$

$$U_{12} = F_1 - \frac{dP}{d\theta}F_2, \quad U_{23} = F_3 - \frac{dP}{d\theta}F_4 \tag{11}$$

$$F_1 = \frac{-E_{12}E_{231}}{E_{11}E_{22} - E_{21}E_{12}}, F_2 = \frac{E_{22}E_{13} - E_{12}E_{232}}{E_{11}E_{22} - E_{21}E_{12}}, F_3 = \frac{E_{11}E_{231}}{E_{11}E_{22} - E_{21}E_{12}},$$
$$F_4 = \frac{-E_{21}E_{13} + E_{11}E_{232}}{E_{11}E_{22} - E_{21}E_{12}} \tag{12}$$

$$E_{11} = \frac{1}{\sqrt{K_1}}\coth\left(\frac{\Delta_1}{\sqrt{K_1}}\right) + \frac{1}{\sqrt{K_2}}\coth\left(\frac{\Delta_2}{\sqrt{K_2}}\right),$$
$$E_{22} = \frac{1}{\sqrt{K_2}}\coth\left(\frac{\Delta_2}{\sqrt{K_2}}\right) + \frac{1}{(H-\Delta_1-\Delta_2)},$$
$$E_{12} = E_{21} = -\frac{1}{\sqrt{K_2}}\operatorname{csch}\left(\frac{\Delta_2}{\sqrt{K_2}}\right) \tag{13}$$

$$E_{13} = H_1^* + H_2^*, E_{231} = \frac{1}{(H-\Delta_1-\Delta_2)}, E_{232} = H_2^* + \frac{1}{2}(H-\Delta_1-\Delta_2) \tag{14}$$

$$H_i^* = \sqrt{K_i}\left[\coth\left(\frac{\Delta_i}{\sqrt{K_i}}\right) - \operatorname{csch}\left(\frac{\Delta_i}{\sqrt{K_i}}\right)\right] \text{ for } i = 1, 2 \tag{15}$$

$$H = (1+\varepsilon\cos\theta) \tag{16}$$

The equation of continuity across the film is:

$$Q = \int_0^{\Delta_1} U_1 dY + \int_{\Delta_1}^{\Delta_1+\Delta_2} U_2 dY + \int_{\Delta_1+\Delta_2}^{H} U_3 dY \quad (17)$$

Simplifying the equation of continuity across the film, yields the non-dimensional pressure gradient term for a two-layered porous journal bearing as:

$$\frac{dP}{d\theta} = \frac{G_1 - Q}{G_2} \quad (18)$$

Where:

$$G_1 = F_1 H_1^* + (F_1 + F_3) H_2^* + \frac{1}{2}(F_3 + 1)(H - \Delta_1 - \Delta_2) \quad (19)$$

$$G_2 = F_2 H_1^* + (F_2 + F_4) H_2^* + \left[\frac{1}{2}F_4 + \frac{1}{12}(H - \Delta_1 - \Delta_2)^2\right] (H - \Delta_1 - \Delta_2) + K_1(\Delta_1 - 2H_1^*) + K_2(\Delta_2 - 2H_2^*) \quad (20)$$

For $\Delta_2 = 0$, G_1 and G_2 in Eq. 19-20 reduce to:

$$G_1 = F_5 H_1^* + \frac{1}{2}F_5(H - \Delta_1) + \frac{1}{2}(H - \Delta_1) \quad (21)$$

$$G_2 = F_6 H_1^* + \frac{1}{2}F_6(H - \Delta_1) + \frac{1}{12}(H - \Delta_1)^3 + K_1(\Delta_1 - 2H_1^*) \quad (22)$$

Where:

$$F_5 = \frac{1}{(H - \Delta_1)\left[\frac{1}{\sqrt{K_1}}\coth\left(\frac{\Delta_1}{\sqrt{K_1}}\right) + \frac{1}{(H - \Delta_1)}\right]} \quad (23)$$

$$F_6 = \left[\frac{H_1^* + \frac{1}{2}(H - \Delta_1)}{\frac{1}{\sqrt{K_1}}\coth\left(\frac{\Delta_1}{\sqrt{K_1}}\right) + \frac{1}{(H - \Delta_1)}}\right] \quad (24)$$

The Reynolds boundary conditions are:

$$P\big|_{\theta=0} = 0, \quad P\big|_{\theta=\theta_r} = 0 \quad \text{and} \quad \frac{dP}{d\theta}\bigg|_{\theta=\theta_r} = 0 \quad (25)$$

Integrating the Eq. 18 and substituting the first boundary condition given in Eq. 25, yields the nondimensional pressure profile as:

$$P = \int_0^{\theta} \frac{G_1}{G_2} d\theta - Q\int_0^{\theta} \frac{1}{G_2} d\theta \quad (26)$$

Substitution of the Reynolds boundary conditions for nondimensional pressure at film rupture in Eq. 26 and simplifying results in Q as:

$$Q = \frac{\int_0^{\theta_r} \frac{G_1}{G_2} d\theta}{\int_0^{\theta_r} \frac{1}{G_2} d\theta} \quad (27)$$

Substituting the pressure gradient boundary condition given in Eq. 23 in the expression for nondimensional pressure gradient in Eq. 18, results in:

$$Q = G_1\big|_{\theta=\theta_r} \quad (28)$$

The Newton-Raphson iterative procedure is used to solve simultaneously both θ_r and Q using Eq. 27 and 28.

The radial and tangential nondimensional load capacity obtained by integration of nondimensional pressure along and perpendicular to line of centers are expressed as:

$$W_\varepsilon = -\int_0^{\theta_r} P\cos\theta d\theta, \quad W_\phi = \int_0^{\theta_r} P\sin\theta d\theta \quad (29)$$

The nondimensional load capacity is expressed as:

$$W = \sqrt{W_\varepsilon^2 + W_\phi^2} \quad (30)$$

The nondimensional shear stress in the journal bearing at Y = H is obtained as:

$$\Pi\big|_{Y=H} = \frac{dU_3}{dY}\bigg|_{Y=H} \quad (31)$$

The nondimensional friction force on the journal surface is obtained by integrating the shear stress along the journal surface as:

$$F = \int_0^{\theta_r} \Pi d\theta \quad (32)$$

The nondimensional friction coefficient is calculated as:

$$C_f = \left(\frac{R}{C}\right)\frac{f}{w} = \frac{F}{W}$$

NUMERICAL STUDIES AND DISCUSSION

A two-layered porous journal bearing is considered in the analysis. The influence of permeability and thickness of porous layers on the nondimensional load capacity and coefficient of friction are presented. The parameters used in the analysis are: eccentricity ratio (ε) = 0.5; nondimensional permeability of porous layers 1

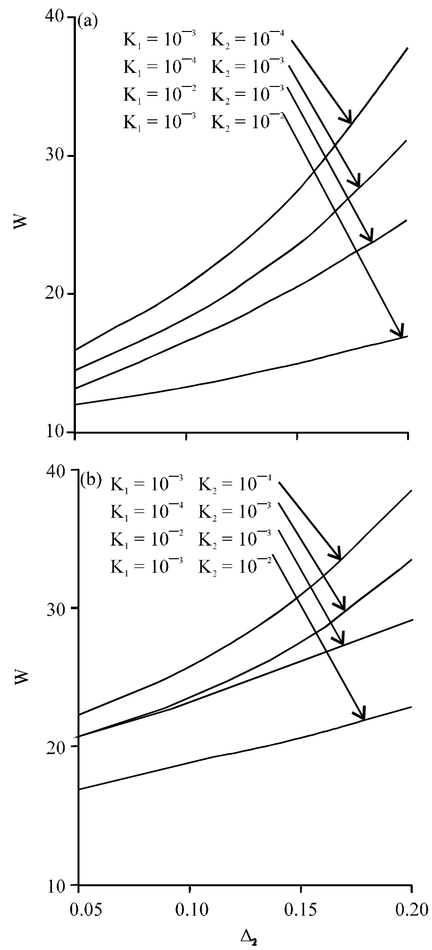

Fig. 2(a-b): Nondimensional load capacity, (a) $\varepsilon = 0.5$, $\Delta_1 = 0.2$ and (b) $\varepsilon = 0.5$, $\Delta_2 = 0.2$

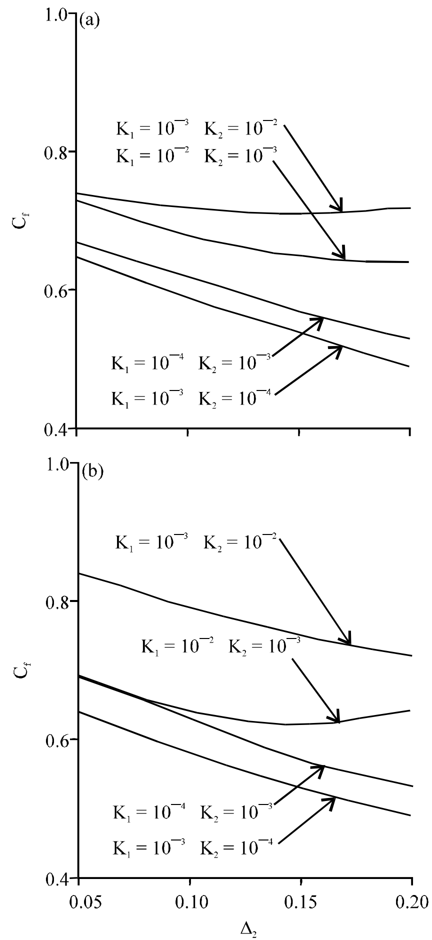

Fig. 3(a-b): Coefficient of friction, (a) $\varepsilon = 0.5$, $\Delta_1 = 0.2$ and (b) $\varepsilon = 0.5$, $\Delta_2 = 0.2$

and 2, respectively (K_1, K_2) = 10^{-2}, 10^{-3} and nondimensional thickness of porous layers 1 and 2, respectively (Δ_1, Δ_2)= 0.05, 0.1, 0.15, 0.2.

Figure 2a and b show the non-dimensional load capacity (W) of two-layered porous journal bearing with nondimensional thickness of porous layers 1 and 2, respectively (Δ_1, Δ_2) for different values of nondimensional permeability of porous layers 1 and 2 (K_1, K_2). The non-dimensional load capacity (W) increases with (1) decrease in the nondimensional permeability of porous layer and (2) increase in the nondimensional thickness of porous layer. For all the parameters of nondimensional thickness of porous layers considered in the study, the higher non-dimensional load capacity (W) is obtained with reduction in the nondimensional permeability of porous layer 2 ($K_1 = 10^{-3}$, $K_2 = 10^{-2}$-10^{-4}) compared to reduction in the nondimensional permeability of porous layer 1 ($K_1 = 10^{-3}$, $K_2 = 10^{-2}$-10^{-4}).

Figure 3a and b show the coefficient of friction (C_f) of two-layered porous journal bearing as a function of nondimensional thickness of porous layers 1 and 2, respectively (Δ_1, Δ_2) for different values of nondimensional permeability of porous layers 1 and 2 (K_1, K_2). The coefficient of friction (C_f) decreases with (1) decrease in the nondimensional permeability of porous layer and (2) increase in the nondimensional thickness of porous layer. Lower coefficient of friction (C_f) is obtained with reduction in the nondimensional permeability of porous layer 2 ($K_1 = 10^{-3}$, $K_2 = 10^{-2}$-10^{-4}) compared to reduction in the nondimensional permeability of porous layer 1 ($K_1 = 10^{-3}$, $K_2 = 10^{-2}$-10^{-4}).

CONCLUSION

The present study evaluates the influence of permeability and thickness of porous layers on improvement in load capacity and reduction in coefficient of friction for a two-layered long porous journal bearing. A modified Reynolds equation is derived considering permeability and thickness of two-layered porous layers using Brinkman model. The conclusions based on the analysis are:

- Higher non-dimensional load capacity (W) and lower coefficient of friction (C_f) are obtained with (1) decrease in the permeability and (2) increase in the thickness of porous layer
- For a given thickness of porous layers 1 and 2 (Δ_1, Δ_2 in the range 0.05-0.2), a low permeability porous layer 2 with high permeability porous layer 1 ($K_1 = 10^{-3}$, $K_2 = 10^{-4}$; $K_1 = 10^{-2}$, $K_2 = 10^{-3}$) would result in higher non-dimensional load capacity (W) and lower coefficient of friction (C_f) compared to a low permeability porous layer 1 with high permeability porous layer 2 ($K_1 = 10^{-4}$, $K_2 = 10^{-3}$; $K_1 = 10^{-3}$, $K_2 = 10^{-2}$), respectively

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NOMENCLATURE

C	=	Radial clearance (m)
f	=	Friction force (N) $F = fC/\mu u_j RL$
h, H	=	Film thickness (m) $H = h/C$
ki, i = 1, 2	=	Permeability of porous layer 1 and porous layer 2, respectively (m^2) $K_i = k_i/C_2$
L	=	Length of the journal bearing (m)
p	=	Pressure distribution (N m^{-2}); $P = pC^2/\mu u_j R$
q	=	Volume flow rate per unit length along film thickness (m^2 sec^{-1}) $Q = q/u_j C$
R	=	Journal radius (m)
u	=	Velocity component along θ direction (m sec^{-1}; $U = u/u_j$)
u_i, i = 1, 2, 3	=	Local mean velocity components along θ direction in porous layer 1 and 2 and velocity component along θ direction in fluid film, respectively (m sec^{-1})
u_{12}, u_{23}	=	Velocity component along θ direction at the interface of porous layer 1-porous layer 2 and porous layer 2-fluid film, respectively (m sec^{-1})
u_j	=	Journal velocity along θ direction (m sec^{-1})
w	=	Static load, N ($W = wC^2/\mu u_j R^2 L$)
W_ε, W_ϕ	=	Nondimensional radial and tangential static load for journal bearing
x	=	Coordinate along circumferential (x) direction (m) $\theta = x/R$
y	=	Coordinate along radial (y) direction (m) $Y = y/C$
δ_i, i = 1, 2	=	Thickness of porous layer 1 and porous layer 2, respectively (m) $\Delta_i = \delta_i/C$
ε	=	Journal bearing eccentricity ratio
μ	=	Fluid viscosity (Nsec m^{-2})
θ	=	Angular coordinate measured from the position of maximum film thickness in journal bearing
θ_r	=	Angular extent of film rupture for journal bearing
τ	=	Shear stress component (N m^{-2}; N m^{-2}) $\Pi = \tau C/\mu u_j$
ω	=	Angular velocity of journal bearing (rad sec^{-1})

Subscripts:

r	=	Extent of outlet film in journal bearing measured
ε	=	Long the radial direction
ϕ	=	Along the tangential direction

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