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Supplier Selection using a New Russell Model in the Presence of Undesirable Outputs and Stochastic Data

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Abstract: Supplier selection is one of the significant topics in Supply Chain Management (SCM). One of the techniques that can be used for selecting suppliers is Data Envelopment Analysis (DEA). In this study, to handle uncertainty in supplier selection problem, a new Russell model in the presence of undesirable outputs and stochastic data is developed. This study proposed a deterministic equivalent of the stochastic model and convert this deterministic problem into a quadratic programming problem. This quadratic programming problem is then solved using algorithms available for this class of problems. A numerical example is presented to demonstrate the applicability of the proposed approach.

Key words: Data envelopment analysis, chance-constrained programming, Russell measure, supplier selection, stochastic data

INTRODUCTION

Supplier selection represents one of the most significant functions to be performed by the purchasing decision makers which determines the long-term viability of the firm (Zouggari and Benyoucef, 2011). Several mathematical programming techniques have been proposed for supplier selection in the literature. However, because of the intricacy of the decision-making process involved in supplier selection, all the aforementioned references in supplier selection, except for the Data Envelopment Analysis (DEA) model; rely heavily on some sort of procedure for determining the importance weights associated with the performance criteria. These importance weights are generally subjective and it is often difficult for the decision makers to precisely assign numbers to their preferences. This is especially intimidating for the decision makers when the number of performance criteria is increased. Furthermore, these methods do not consider stochastic data in the supplier selection process (Azadi *et al.*, 2012).

DEA, developed by Charnes *et al.* (1978), provides a non-parametric methodology for evaluating the efficiency of each of a set of comparable Decision Making Units (DMUs). As Saen (2010) addresses, classical DEA models rely on the assumption that inputs have to be minimized and outputs have to be maximized. However, as Koopmans (1951) discussed earlier, the process of plant may produce bad outputs such as CO_2 emission and effluent.

As Azadi and Saen (2012) addressed, Chance-constrained Programming (CCP) developed by Charnes and Cooper (1963) is an operations research approach for optimization under uncertainty when some or all coefficients in a linear programme are random variables distributed in accordance with some probability law. In CCP, the optimization problem is concerned with identification of the value of the decision variables so that the expected loss in the criterion is minimized subject to the requirement that the probability that any given constraint is violated is not allowed to exceed some a priori specified level (Olesen, 2006). The stochastic input and output variations in DEA have been studied by Sengupta (1998, 2000), Olesen and Petersen (1995), Morita and Seiford (1999), Huang and Li (2001), Cooper *et al.* (2004) and Olesen (2006). Talluri *et al.* (2006) utilized the CCP model proposed by Land *et al.* (1993) for supplier selection.

We use CCP model proposed by Cooper *et al.* (2004) since it has benefits proposed by Land *et al.* (1993). It opens possible novel routes for sensitivity analysis. In addition, it can be solved by a deterministic equivalent.

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However, the model proposed by Talluri *et al*. (2006) does not consider undesirable factors, while the model proposed in this study takes into account the undesirable factors.

Motivated by those points, the objective of this study is to propose a model for selecting suppliers in the presence of undesirable outputs.

PAST RESEARCHES

Here, various studies on the supplier selection, DEA and undesirable outputs are briefly summarized.

Supplier selection: There are several supplier selection methods available in the literatures such as Analytical Hierarchical Process (AHP) (Chan *et al*., 2007; Ng, 2008), fuzzy programming model (Sanayei *et al*., 2010), intelligent model (Das and Shahin, 2003), Multiple Attribute Utility Approach (MAUT) (Min, 1994). Also, there are other methods for supplier selection problem such as fuzzy logic approaches (Bevilacqua and Petroni, 2002; Lee, 2009; Noorul Haq and Kannan, 2006), case-based reasoning (Choy *et al*., 2005), Multi-objective Programming (MOP) (Arunkumar *et al*., 2006), mixed integer programming (Hartmut, 2007), chance-constrained and genetic algorithm (He *et al*., 2008), DEA (Azadi and Saen, 2011; Hosseinzadeh *et al*., 2011), Analytic Network Process (ANP) (Bayazit, 2006; Gencer and Gurpinar, 2007), integrated approach (Ting and Cho, 2008), total cost of ownership approach (Bhutta and Huq, 2002), hybrid AHP (Sevkli *et al*., 2008), etc.

Data Envelopment Analysis (DEA): DEA is a non-parametric linear programming method. It has been employed for assessing the relative efficiency of a homogeneous set of DMUs in both profit and non-profit organizations and a number of extensions and applications have been reported (Niknafs and Parsa, 2011; Laha and Kuri, 2011; Koc *et al*., 2011; Keramidou *et al*., 2011; Zandieh *et al*., 2009; Ghorbani *et al*., 2010; Ergulen and Torun, 2009; Hatami-Marbini *et al*., 2009; Rayeni *et al*., 2010; Rayeni and Saljooghi, 2010; Mirhedayatian *et al*., 2011; Hosseinian *et al*., 2009; Asharafi and Jaafar, 2011; Jahanshahloo and Afzalinejad, 2007; Taher and Malek, 2009). Amongst the characteristics that make DEA a powerful tool is its ability to deal with multiple outputs and multiple inputs without requiring any assumptions about the functional form relating inputs to outputs; focus on the efficiency frontier and not on the central trend of the production units and free the decision maker from the necessity to use separate indices, such as labor productivity, capital productivity, etc.

In 1978, basic DEA model was proposed by Charnes, Cooper and Rhodes (CCR). The CCR model is presented in Model 1. Table 1 presents the nomenclatures used in this study. By solving Model 1 n times (each time evaluating a different DMU), relative efficiency scores for all the DMUs are obtained. These measures divide the DMUs into two categories: those with score of 1 (efficient) and those with scores less than 1 (inefficient):

$$\max E_o = \frac{\sum_{r=1}^{s} u_r y_{rk}}{\sum_{i=1}^{m} v_i x_{ik}}$$
$$\text{s.t.} \quad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1, \qquad j=1, 2,..., n \tag{1}$$
$$u_r\ v_i \ge 0, \quad r=1, 2,...., s, \quad i=1, 2,..., m$$

DEA models may be generally classified into radial and non-radial models. The radial models include the CCR ratio form (the radial model under constant RTS technology, where RTS stands for returns to scale) and the BCC model (the radial model under variable RTS). This type of efficiency measure needs a separate treatment between output-orientation and input-orientation. The non-radial models include an additive model,

Table 1: The nomenclatures

$j \in J$,	$j = 1, ..., n$	collection of DMUs
r	=	$1,..., s$ the set of outputs
i	=	$1,..., m$ the set of inputs
DMU_o	=	The DMU under investigation
$A \in R^{m \times n}$	=	A matrix with m rows and n columns
X	=	$[x_1,..., x_n] \in R^{m \times n}$
Y^g	=	$[Y_1^g,...,Y_n^g] \in R^{s1 \times n}$
Y^b	=	$[Y_1^b,...,Y_n^b] \in R^{s2 \times n}$
s^g_r	=	The shortfalls of rth good output
s^b_r	=	The shortfalls of rth bad output
y_{rj}	=	The rth output of jth DMU
x_{ij}	=	The ith input of jth DMU
y_{ro}	=	The rth output of oth DMU
x_{io}	=	The ith input of *o*th DMU
σ_r	=	Small positive number reflecting the ratio of the possible minimum of $\{y_{rj} \mid j = 1,..., n\}$ to its possible maximum
$\hat{y}_{rj}$	=	The rth output of jth DMU after scale transformation
θ_i	=	Radial input shrinkage factor
ϕ_r	=	Radial output augmentation factor
$\lambda = [\lambda_j]$	=	Vector of DMU loadings, determining best practice for the DMU_o
u_r	=	Weight of the rth output
λ_j	=	Reference weights associated with DMU
θ_o	=	Radial input shrinkage factor
μ_r	=	Weight given to output r
v_i	=	weight given to input i
θ	=	The best possible relative efficiency achieved by DMU
$-\Phi^{-1}$	=	Inverse of standard normal distribution function
s_i^+	=	rth output shortfalls
s_i^-	=	ith input excess
σ^O_r	=	Standard deviation of rth output
σ^I_i	=	Standard deviation of ith input
α	=	risk that is between zero and 1
Var y_{io}	=	rth output variance of the DMU_o
Var x_{io}	=	ith input variance of the DMU_o
ξ, ς and ζ	=	Enternal slacks

multiplication model, Range-adjusted Measure (RAM) (Cooper *et al*., 1999) and slack-based measure. This group of efficiency measure does not need any special treatment on the output/input orientation. Both are aggregated into a single efficiency measure (Sueyoshi and Sekitani, 2007).

Undesirable outputs: In accordance with the global environmental conservation awareness, undesirable outputs of productions and social activities, e.g., air pollutants and hazardous wastes, are being increasingly recognized as dangerous and undesirable. Thus, development of technologies with less undesirable outputs is an important subject of concern in every area of production. DEA usually assumes that producing more outputs relative to less input resources is a criterion of efficiency. In the presence of undesirable outputs, however, technologies with more good (desirable) outputs and less bad (undesirable) outputs relative to less input resources should be recognized as efficient (Cooper *et al*., 2007).

Authors believe that this study has a significant contribution to an important and very much under-researched topic. The contributions of proposed model are as follows:

- The proposed model considers undesirable outputs
- The proposed model considers stochastic data
- The proposed model considers both undesirable outputs and stochastic data, simultaneously

PROPOSED MODEL

As Cooper *et al*. (2007) address, Russell Measure (RM) reflects nonzero slacks in inputs and outputs when they are present. In this way we avoid limitations of the radial measures which cover only some of the input or output inefficiencies and hence measure only weak efficiency. Following, Fare and Lovell (1978) and Cooper *et al*. (2007); the Russell Measure (RM) for the jth DMU (j = 1, 2, ..., n) can be formulated as follows:

$$\begin{aligned}
&\min \frac{1}{m+s}\left(\sum_{i=1}^{m}\theta_i + \sum_{r=1}^{s}\frac{1}{\phi_r}\right)\\
&\text{s.t.} -\sum_{j=1}^{n} x_{ij}\lambda_j + \theta_i x_{io} \ge 0 \quad (i = 1,\ldots,m),\\
&\sum_{j=1}^{n} y_{rj}\lambda_j - \phi_r y_{ro} \ge 0 \quad (r = 1,\ldots,s),\\
&\theta_i \le 1\ (i = 1,\ldots,m),\quad 1 \le \phi_r\ (r = 1,\ldots,s),\quad 0 \le \lambda_j\ (j = 1,\ldots,n)
\end{aligned} \tag{2}$$

where, the variables (θ_i and ϕ_r) indicate the level of efficiency related to the ith input and the rth output, respectively. The variables (λ_j for j = 1, ..., n) are used for a structural connection among DMUs in the input-output space.

In examining Model 2, Cooper *et al*. (1999, 2007) discuss that it is difficult to compute and to interpret the RM. Then, they proposed to use Enhanced Russell Graph Measure (ERGM) to overcome such difficulties. The ERGM is formulated as follows:

$$\begin{aligned}
&\min \left(\sum_{i=1}^{m}\theta_i \Big/ m\right)\left(\sum_{r=1}^{s}\phi_r \Big/ s\right)\\
&\text{s.t.} -\sum_{j=1}^{n} x_{ij}\lambda_j + \theta_i x_{io} \ge 0 \quad (i = 1,\ldots,m),\\
&\sum_{j=1}^{n} x_{rj}\lambda_j + \phi_r y_{ro} \ge 0 \quad (r = 1,\ldots,s),\\
&\theta_i \le 1 \quad (i = 1,\ldots,m),\quad 1 \le \phi_r \quad (r = 1,\ldots,s),\quad 0 \le \lambda_j \quad (j = 1,\ldots,n)
\end{aligned} \tag{3}$$

The difference between Model 2 and 3 can be found in only their objective functions. It is clear that Model 3 is a nonlinear programming problem and hence, it is still difficult to solve the problem. To enhance the computational capability of Model (3), Cooper *et al*. (2007) have proposed a transformation from Model 3 to a linear programming equivalence via the well-known treatment of fractional programming (Charnes and Cooper, 1962). To briefly review their treatment applied to Model 3, a new variable:

$$\beta = \left(\sum_{r=1}^{s}\phi_r \Big/ s\right)$$

is included into Model 3. Here, the variable satisfies both $0 \le \beta \le 1$ and:

$$\beta = \left(\sum_{r=1}^{s}\phi_r \Big/ s\right)$$

Then, all the variables in Model 3 can be transformed as follows: $u_i = \beta\theta_i$ (i = 1,..., m), $v_r = \beta\phi_r$ (r = 1,..., s) and $t_j = \beta\lambda_j$ (j = 1,..., n). Using these transformed variables, Model 3 can be reformulated as follows:

$$\begin{aligned}
&\min \sum_{i=1}^{m} u_i \Big/ m\\
&\text{s.t.} \sum_{r=1}^{s} v_r = 1,\\
&\quad -\sum_{j=1}^{n} x_{ij} t_j + u_i x_{io} \ge 0 \quad (i = 1,\ldots,m)\\
&\sum_{j=1}^{n} y_{rj} t_j - v_r y_{ro} \ge 0 \quad (r = 1,\ldots,s),\\
&u_i \le \beta \quad (i = 1,\ldots,m) \quad \beta \le v_r\ (r = 1,\ldots,s),\ 0 \le t_j\ (j = 1,\ldots,n),\ 0 \le \beta \le 1
\end{aligned} \tag{4}$$

As a result of the transformation, Model 4 is reformulated as a linear programming problem. Therefore, it can be easily solved by any linear programming software. However, the ERGM proposed by Cooper *et al.* (1999) and Cooper *et al.* (2007) can provide an approximate of efficiency score for RM. Such an effort cannot perfectly solve the computation issue of the RM measurement. Sueyoshi and Sekitani (2007) proposed to use Second-order Cone Programming (SOCP) that can directly solve the RM without depending upon the ERGM approximation. Moreover, the SOCP approach makes it possible to formulate a dual model of the RM. After the SOCP is applied to reformulate the RM, then the primal and dual models can be established within the computational framework of the interior point method (not Simplex method). As a result of the dual development, the type of RTS is determined which is an economic implication under the RM. The model proposed by Sueyoshi and Sekitani (2007) is as follows:

$$\begin{aligned}
&\min \sum_{i=1}^{m} u_i \Big/ m \\
&\text{s.t.} \sum_{r=1}^{s} v_r = 1, \\
&\quad -\sum_{j=1}^{n} x_{ij} t_j + u_i x_{io} \geq 0 \quad (i = 1,\ldots,m) \\
&\sum_{j=1}^{n} y_{rj} t_j - v_r y_{ro} \geq 0 \quad (r = 1,\ldots,s), \\
&\sum_{j=1}^{n} t_j = \beta, \\
&u_i \leq \beta \ (i = 1,\ldots,m) \ \ \beta \leq v_r \ (r = 1,\ldots,s), \ 0 \leq t_j \ (j = 1,\ldots,n), \ 0 \leq \beta \leq 1
\end{aligned} \tag{5}$$

At this juncture, the new model is developed. Assume we have n DMUs each consuming m inputs and producing p outputs. The outputs corresponding to indices 1, 2,..., k are desirable and the outputs corresponding to indices k+1, k+2,..., p are undesirable outputs. We like to produce desirable outputs as much as possible and not to produce undesirable outputs. Let $X \epsilon R_+^{m \times n}$ and $Y \epsilon R_+^{P \times n}$ be the matrices, consisting of non-negative elements, containing the observed input and output measures for the DMUs. Korhonen and Luptacik (2004) decomposed matrix Y into two parts:

$$Y = \begin{pmatrix} Y^g \\ Y^b \end{pmatrix}$$

where a k×n matrix Y^g is standing for desirable outputs (good) and a (p-k)×n matrix Y^b is standing for undesirable outputs (bad). We further assume that there are no duplicated units in the data set. We denote by x_j (the jth column of X) the vector of inputs consumed by DMU_j and by x_{ij} the quantity of input i consumed by DMU_j. A similar notation is used for outputs. Occasionally, we decompose the vector y_j into two parts:

$$y_j = \begin{pmatrix} y_j^g \\ y_j^b \end{pmatrix}$$

where the vectors y_j^g and y_j^b refer to the desirable and undesirable output. When it is not necessary to emphasize the different roles of inputs and (desirable/undesirable) outputs, we denote

$$u = \begin{pmatrix} y^g \\ -y^b \\ -x \end{pmatrix}$$

and:

$$U = \begin{pmatrix} Y^g \\ -Y^b \\ -X \end{pmatrix}$$

Furthermore, we denote $1 = [1,\ldots,1]^T$ and refer by e_i to the ith unit vector in R^n We consider set $T = \{u | u = U\lambda, \lambda \epsilon \Lambda\}$, where $\Lambda = \{\lambda | \lambda \epsilon \Re_+^k \text{ and } A\lambda \leq b\}, e_i \epsilon \Lambda$, i = 1,..., n. Furthermore, consider matrix $A \epsilon \Re^{k \times n}$ and vector $B \epsilon \Re^k$ which are used to specify the feasible values of λ variables.

Model (5) is combined with undesirable output:

$$\begin{aligned}
&\min \sum_{i=1}^{m} u_i \Big/ m \\
&\text{s.t.} \sum_{r=1}^{s} v_r = 1, \\
&\quad -\sum_{j=1}^{n} x_{ij} t_j + u_i x_{io} \geq 0 \quad (i = 1,\ldots,m) \\
&\sum_{j=1}^{n} y_{rj}^{\ g} t_j - s_r^g = v_r y_{ro}^{\ g} \quad (r = 1,\ldots,s), \\
&\sum_{j=1}^{n} y_{rj}^{\ b} t_j + s_r^b = v_r y_{ro}^{\ g}, \quad (r = 1,\ldots,s), \\
&\sum_{j=1}^{n} t_j = \beta \\
&u_i \leq \beta \ (i = 1,\ldots,m) \ s_r^g \geq 0, \ s_r^b \geq 0, \\
&\beta \leq v_r \ (r = 1,\ldots,s), \ 0 \leq t_j \ (j = 1,\ldots,n), \quad 0 \leq \beta \leq 1
\end{aligned} \tag{6}$$

Model 6 is structured under variable RTS technology, depending upon β. Now, the novel model of stochastic ERGM-undesirable output is developed which permits the possible existence of stochastic variability in the data. The proposed model can deal with both undesirable outputs and stochastic data in ERGM context, simultaneously. There is not any model that discusses supplier selection in the presence of both undesirable outputs and stochastic data in ERGM context. The proposed model is the first and unique model.

As we know, the typical DEA models do not permit stochastic variations in input and output; hence, DEA efficiency measurement may be sensitive to such variations. For instance, a DMU which is measured as efficient relative to other DMUs, might turn inefficient if such random variations are considered. In what follows, stochastic version of the output-oriented undesirable model is presented which allows for the possibility of stochastic alterations in input and output data. We suppose that all inputs and outputs are random variables with a multivariate normal distribution and known parameters.

Assume that ξ_i represents 'external slack' for the ith input. We select its value to satisfy:

$$p = \left\{ \sum_{j=1}^{n} \tilde{x}_{ij} t_j - u_i \tilde{x}_{io} \leq 0 \right\} = (1-\alpha) + \xi_i \tag{7}$$

There must then exist a positive number $s_i^- > 0$ such that:

$$p = \left\{ \sum_{j=1}^{n} \tilde{x}_{ij} t_j - s_i^- \leq u_i \tilde{x}_{io} \right\} = 1-\alpha \tag{8}$$

Such a positive value of s_i^- permits a reduce in $\tilde{x}_{io}$ for any sample devoid of worsening any other input or output to the indicated probabilities. It is easy to demonstrate that $\xi_i = 0$ if and only if $s_i^- = 0$.

In a similar manner, ζ_r is the external slack for the rth output. Via external slack, we refer to slack outside the braces. We can select the value of this external slack which is not stochastic, so it satisfies:

$$p = \left\{ \sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g} \geq 0 \right\} = (1-\alpha) + \zeta_r \tag{9}$$

There must then exist a positive number $s_r^g > 0$ such that:

$$p = \left\{ \sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g} \geq s_r^g \right\} = 1-\alpha \tag{10}$$

This positive value of s_r^g permits a still further raise in $\tilde{y}_{ro}^{\ g}$ for any set of sample observations devoid of worsening any other input or output. It is easy to see that $\zeta_r = 0$ if and only if $s_r^g = 0$.

Also, for constraint 4 of Model 6 we have:

$$p = \left\{ \sum_{j=1}^{n} \tilde{y}_{rj}^{\ b} t_j + v_r \tilde{y}_{ro}^{\ b} \geq 0 \right\} = (1-\alpha) + \varsigma_r \tag{11}$$

Consequently:

$$p = \left\{ \sum_{j=1}^{n} \tilde{y}_{rj}^{\ b} t_j + v_r \tilde{y}_{ro}^{\ b} \geq s_r^b \right\} = 1-\alpha \tag{12}$$

Using Relations 7-12, can replace Model 6 with following model:

$$\begin{aligned}
&\min \ \sum_{i=1}^{m} u_i \Big/ m \\
&\text{s.t.} \ \sum_{r=1}^{s} v_r = 1, \\
&\quad p = \left\{ \sum_{j=1}^{n} \tilde{x}_{ij} t_j + s_i^- \leq u_i \tilde{x}_{io} \right\} = 1-\alpha \quad (i = 1,\ldots, m) \\
&\quad p = \left\{ \sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g} \geq s_r^g \right\} = 1-\alpha \quad (r = 1,\ldots, s) \\
&\quad p = \left\{ \sum_{j=1}^{n} \tilde{y}_{rj}^{\ b} t_j + v_r \tilde{y}_{ro}^{\ b} \geq s_r^b \right\} = 1-\alpha \quad (r = 1,\ldots, s) \\
&\sum_{j=1}^{n} t_j = \beta \\
&u_i \leq \beta \ \ (i = 1,\ldots, m) \ s_r^g \geq 0, \ \ s_r^b \geq 0, \\
&\beta \leq v_r \ \ (r = 1,\ldots, s), \ 0 \leq t_j \ (j = 1,\ldots, n), \quad 0 \leq \beta \leq 1
\end{aligned} \tag{13}$$

For the 3rd constraint in Model 13, we have:

$$\sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g} \geq s_r^g$$

$$p\left\{ \frac{\sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g} - E\left(\sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g}\right)}{\sqrt{\operatorname{var}\left\{\sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g}\right\}}} \leq \frac{s_r^g - E\left(\sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g}\right)}{\sqrt{\operatorname{var}\left\{\sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g}\right\}}} \right\} = \alpha$$

Note that the conversion process is discussed for constraint 3 in Model 13 and the same process could be repeated for constraints 2 and 4.

For the sake of simplicity, we indicate:

$$\sqrt{\operatorname{var}\left\{ \sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g} \right\}}$$

by $\sigma_r^{go}(t)$. Hence:

$$p\left(\frac{\sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j - v_r \tilde{y}_{ro}^{\ g} - \sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j + v_r \tilde{y}_{ro}^{\ g}}{\sigma_r^{go}(t)} \leq \frac{s_r^g - \sum_{j=1}^{n} \tilde{y}_{rj}^{\ g} t_j + v_r \tilde{y}_{ro}^{\ g}}{\sigma_r^{go}(t)} \right) = \alpha$$

In other words:

$$p\left(Z \le \frac{s_r^g - \sum_{j=1}^{n} \tilde{y}_{rj}{}^g t_j + v_r \tilde{y}_{ro}{}^g}{\sigma_r^{go}(t)} = \alpha\right)$$

where, Z is a normal standard variable and we have:

$$\phi\left(\frac{s_r^g - \sum_{j=1}^{n} \tilde{y}_{rj}{}^g t_j + v_r \tilde{y}_{ro}{}^g}{\sigma_r^{go}(t)}\right) = \alpha$$

Or:

$$-\sum_{j=1}^{n} \tilde{y}_{rj}{}^g t_j + v_r \tilde{y}_{ro}{}^g + s_r^g - \phi^{-1}\ \sigma_r^{go}(t) = 0$$

The deterministic equivalent for Model 13 is as follows:

$$\begin{aligned}
&\min \ {\textstyle\sum_{i=1}^{m} u_i}/_{m} \\
&\text{s.t.} \sum_{r=1}^{s} v_r = 1, \\
&-\sum_{j=1}^{n} \tilde{x}_{ij} t_j - s_i^- - \phi^{-1}(\alpha) - \sigma_i^I(t) = -u_i \tilde{x}_{io}, \quad i = 1,\ldots,m \\
&-\sum_{j=1}^{n} \tilde{y}_{rj}{}^g t_j + v_r \tilde{y}_{ro}{}^g + s_r^g - \phi^{-1} \sigma_r^{go}(t) = 0, \quad r = 1,\ldots,s^g \qquad (14) \\
&-\sum_{j=1}^{n} \tilde{y}_{rj}{}^b t_j + v_r \tilde{y}_{ro}{}^b + s_r^b - \phi^{-1} \sigma_r^{bo}(t) = 0, \quad r = 1,\ldots,s^b \\
&u_i \le \beta \ \ (i = 1,\ldots,m)\ s_r^g \ge 0, \ \ s_r^b \ge 0, \\
&\beta \le v_r \ \ (r = 1,\ldots,s), \ 0 \le t_j \ (j = 1,\ldots,n), \qquad 0 \le \beta \le 1
\end{aligned}$$

To derive equations for $\sigma_i^I(t)$ note that:

$$\begin{aligned}
\sigma_i^I(t)^2 &= \operatorname{var}\left\{\sum_{j=1}^{n} \tilde{x}_{ij} t_j - u\tilde{x}_{io}\right\} = \operatorname{Var}\left\{\sum_{j=1}^{n} x_{ij} t_j + (t_o - 1) u_i x_{io}\right\} \\
&= \operatorname{var}\left(\sum_{j=1}^{n} x_{it} t_j\right) + \operatorname{Var}\left((t_o - 1) u_i x_{io}\right) \\
&\quad + 2\operatorname{Cov}, \left(\sum_{j=1}^{n} x_{it} t_j + (t_o - 1) u_i x_{io}\right)
\end{aligned}$$

Therefore:

$$\sigma_i^I(t)^2 = \sum_{t\neq 0}\sum_{k\neq 0} t_i t_k \operatorname{Cov}(\tilde{x}_{it}, \tilde{x}_{ik}) + 2(t_0 - 1)\sum_{t\neq 0} t_j \operatorname{Cov}(\tilde{x}_{it}, \tilde{x}_{ik}) + (\tilde{x}_{it}, \tilde{x}_{ik})^2 \operatorname{Var}(\tilde{x}_{io})$$

Similarly, for the constraints 3 and 4 of Model 14, we have:

$$\left(\sigma_r^{go}(t)\right)^2 = \sum_{t\neq 0}\sum_{k\neq 0} t_i t_j \operatorname{Cov}(\tilde{y}_{ri}, \tilde{y}_{rt}) + 2(t_0 - 1) \times \sum_{t\neq 0} t_i \operatorname{Cov}(\tilde{y}_{ri}, \tilde{y}_{ro}) + (t_0 - 1)^2 \operatorname{Var}(\tilde{y}_{ro})$$

$$\left(\sigma_r^{bo}(t)\right)^2 = \sum_{t\neq 0}\sum_{k\neq 0} t_i t_j \operatorname{Cov}(\tilde{y}_{ri}, \tilde{y}_{rt}) + 2(t_0 - 1) \times \sum_{t\neq 0} t_i \operatorname{Cov}(\tilde{y}_{ri}, \tilde{y}_{ro}) + (t_0 - 1)^2 \operatorname{Var}(\tilde{y}_{ro})$$

It is obvious, from the forms of σ^I_i (t), $\sigma_r{}^{go}$ (t) and $\sigma_r{}^{bo}$ (t), that Model 14 is a nonlinear programme.

NUMERICAL EXAMPLE

The idea for this example is taken from Azadi and Saen (2012). The example contains specifications on twenty suppliers (DMUs). These DMUs consume two inputs to produce two outputs. The data are available in Table 2. The performance measures utilized were number

Table 2: Related attributes for 20 suppliers

	Inputs				Desirable output		Undesirable output	
	Number of personnel		Average time for serving customers (hours)		Profit margin ($1000)		Number of dissatisfied customers	
Supplier (DMU)	Mean	Var	Mean	Var	Mean	Var	Mean	Var
Esfehan	6	0.5	70	7	25	3	10	2
Arak	5	1	130	8	17	2	12	3
Tabriz	11	2	125	5	15	1	50	4
Khorasan	8	1	100	4	25	2	55	5
Khozestan	9	1	90	1	30	3	70	7
Zanjan	6	2	75	5	50	5	15	5
Shiraz	18	1	150	10	14	1	35	5
Karaj	25	1	280	20	65	0.5	42	2
Kerman	12	1	160	10	50	3	60	4
Kermanshah	10	1	135	9	40	2	70	9
Gilan	12	1	120	4	10	4	75	10
Mazandaran	10	2	95	2	5	1	45	2
Hamedan	7	1	70	2	12	2	43	10
Yazd	11	2	140	5	30	1	5	4
Shargh	20	3	140	20	80	2	5	2
Gharb	23	2	150	25	65	4	8	2
Shomal	25	3	120	15	78	3	7	2
Qazvin	10	1	70	1	40	2	25	1
Bandarabas	12	1	115	5	5	1	65	4
Omran	5	2	80	5	17	1	10	3

Table 3: The efficiency scores for the 20 suppliers with $\alpha = 0.05$

Supplier (DMU)	Efficiency scores ($\alpha = 0.05$)
Esfehan	-1.36
Arak	-2.57
Tabriz	-0.84
Khorasan	-4.34
Khozestan	0
Zanjan	-1.27
Shiraz	-2.13
Karaj	-2.45
Kerman	-3.38
Kermanshah	-1.17
Gilan	-.118
Mazandaran	0
Hamedan	-.216
Yazd	-.174
Shargh	-.484
Gharb	0
Shomal	-1.318
Qazvin	-3.894
Bandarabas	-1.21
Omran	0

of personnel, average time for serving customers, profit margin and number of dissatisfied customers. Number of personnel and average time for serving customers were used in some way as inputs for the DEA model. The desirable output utilized in the study is profit margin. The undesirable output is number of dissatisfied customers. Note that the inputs and outputs selected in this study are not exhaustive by any means but are some general measures that can be utilized to evaluate suppliers.

The computational results from using Model 14 with $\alpha = 0.05$ are shown in Table 3. The efficient suppliers are Khozestan, Mazandaran, Gharb and Omran. These suppliers are efficient.

CONCLUSION

In today's fierce competitive environment characterized by thin profit margins, high consumer expectations for quality products and short lead-times, companies are forced to take the advantage of any opportunity to optimize their business processes. To reach this aim, academics and practitioners have come to the same conclusion: for a company to remain competitive, it has to work with its supply chain partners to improve the chain's total performance. Thus, being the main process in the upstream chain and affecting all areas of an organization, the purchasing function is taking an increasing importance. Thus Supply Chain Management (SCM) and the supplier (vendor) selection process is an issue that received relatively large amount of attention in both academia and industry (Sanayei *et al*., 2010).

In this study, a new approach was proposed to assist the decision makers to determine the most efficient suppliers in the presence of undesirable outputs and stochastic data in ERGM context.

The problem considered in this study is at the initial stage of investigation and further research can be done based on results of this paper. Some of them are as follows:

- Similar research can be repeated for supplier selection in the presence of both stochastic data and fuzzy data
- Similar research can be performed for supplier selection in the presence of both stochastic data and slightly non-homogeneous DMUs
- This study applied the proposed model to a supplier selection problem. The proposed model is generic and can be applied to additional problem domains, such as personnel selection decisions and location planning decisions

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