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Flood Frequency Analysis Based on t-copula for Johor River, Malaysia

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Abstract: A copula based methodology is presented in this study for the flood frequency analysis of Johor river basin in southern Malaysia. The objective of this study was to find the best-fit distributions to the flood characteristics and finding their joint probability for flood frequency analysis. The joint dependence structures of the flood characteristics (peak flow (Q_p), flood volume (V) and flood duration (D) were modelled using an Archimedean Copula (t-Copula). The distribution methods were tested to identify the best distribution that would fit the distributions of various flood characteristics. Based on Kolmogorov-Smirnov (K-S) test the Generalized Pareto distribution is the best-fit distribution for peak flow. On the other hand, General Extreme Value (GEV) is the best-fit distribution for the flood duration and flood volume. The best fit distributions were then used to develop the joint Cumulative Distribution Function (CDF) of the flood characteristics based on t-Copula. Peak flow-volume, volume-duration and peak flow-duration pairs were found to be negatively related. It is expected that the bivariate distributions formulated is useful for flood risk assessment and design of hydraulic structures in Malaysia.

Key words: Flood frequency analysis, goodness-of-fit test, t-copula, bivariate probability distribution

INTRODUCTION

Flood incidents have to be comprehensively understood for the design of hydraulic structures and to develop, plan and manage water resources. To do this, flood frequency analyses are usually conducted to generate the probability of annual occurrence of a flood or peak river discharge. The detail has to be as precise as possible with the flood duration, volume and peak flow clearly defined (Renard and Lang, 2007). For instance, a flood event with 100-year return period is logically more damaging than a 10-year return period flood, even when both flood peak and flood volume are similar. Therefore, advanced method that can give a joint probability of different characteristics viz., flood duration, flood volume and peak flood flow needs to be developed and this has been carried out by some researchers (Gonzalez and Valdes, 2003; Keef *et al*., 2013; Li *et al*., 2013; Requena *et al*., 2013; Salvadori and De Michele, 2013).

However, if the above-mentioned analysis is done using traditional bivariate models, not all types of probability distribution function can be used as the marginal distribution. Fortunately, this can be overcome by using the copula method. Using a copula, a univariate marginal can be linked to its full multivariate distribution. Moreover, it can link some flood properties together using a joint-probability function. This increasingly popular method, first initialized by De Michele and Salvadori (2003), has now been used to simulate the relationships in various hydrological events (Chung and Salas, 2000; Kim *et al*., 2003; Shiau and Shen, 2001; Cancelliere and Salas, 2004; Salvadori and De Michele, 2004; Shiau *et al*., 2007; Zhang and Singh, 2007; McNeil and Neslehova, 2009; Ghosh, 2010; Guangtao and Zoran, 2012; Xie and Wang, 2013). In spite of this, the technique has not attempted in Malaysia yet.

The Johor river basin in southern Peninsular Malaysia which takes up about 14% of the Johor state has rivers and tributaries that act as important water supply sources not only for the state itself, but also for Singapore. Flood is a common phenomenon in the river basin; during the past twenty years, the basin had been

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severely flooded for five times. This further magnifies the need to generate joint flood characteristics probability for the welfare of mankind.

The objective of this study was to find the best-fit distribution to the flood variables and also finding the joint probability of the flood characteristics using a frequency analysis that was incorporated with the Archimedean Copula. The parameters under scrutiny were annual peak flow, flood volume and flood duration. The final copula, i.e., the t-copula or student Copula, was used to model the joint dependence of peak flow-volume, volume-duration and peak flow-duration.

MATERIALS AND METHODS

Study Area: The Johor River basin, covering an area of 2700 km^2, is located in the southeast of Peninsular Malaysia as seen in “Fig. 1”. The river originates in Mount Gemuruh, flows in the north-south direction and terminates into the Strait of Johor. The total length of the river is approximately 122.7 km. Geographically, the basin extends from 1°27'N-1°49'N (latitude) and 103°42' N to 104°01' N (longitude). The topography of the basin is undulating, but quite steep at the upstream. The highest point in the Johor basin is a mountain-Gunung Ledang (1276 m). The basin is primarily covered with forest, rubber and oil palm plantations can also be found.

Johor has a tropical rainforest climate with monsoon rain from November until February, blowing from the South China Sea. The average annual rainfall is 2,470 mm with average temperatures ranging between 25.5°C (78 °F) and 27.8°C (82°F). Humidity is between 82 and 86%. Floods triggered by heavy rainfall are common phenomena in the basin. A continuous heavy downpour in December 2006 caused severe flood in the basin during the end of 2006 to the beginning of 2007. A large point of the basin were flooded with water levels as high as 10 feet (3.0 m). The basin was deemed as a suitable candidate for flood analysis because of these properties.

Hourly stream flow data recorded at Rantau Panjang gauging station of Johor river for the time period of 1965-2010 had been collected from the Department of Irrigation and Drainage, Malaysia. The location of the river gauging station is shown in “Fig. 1”.

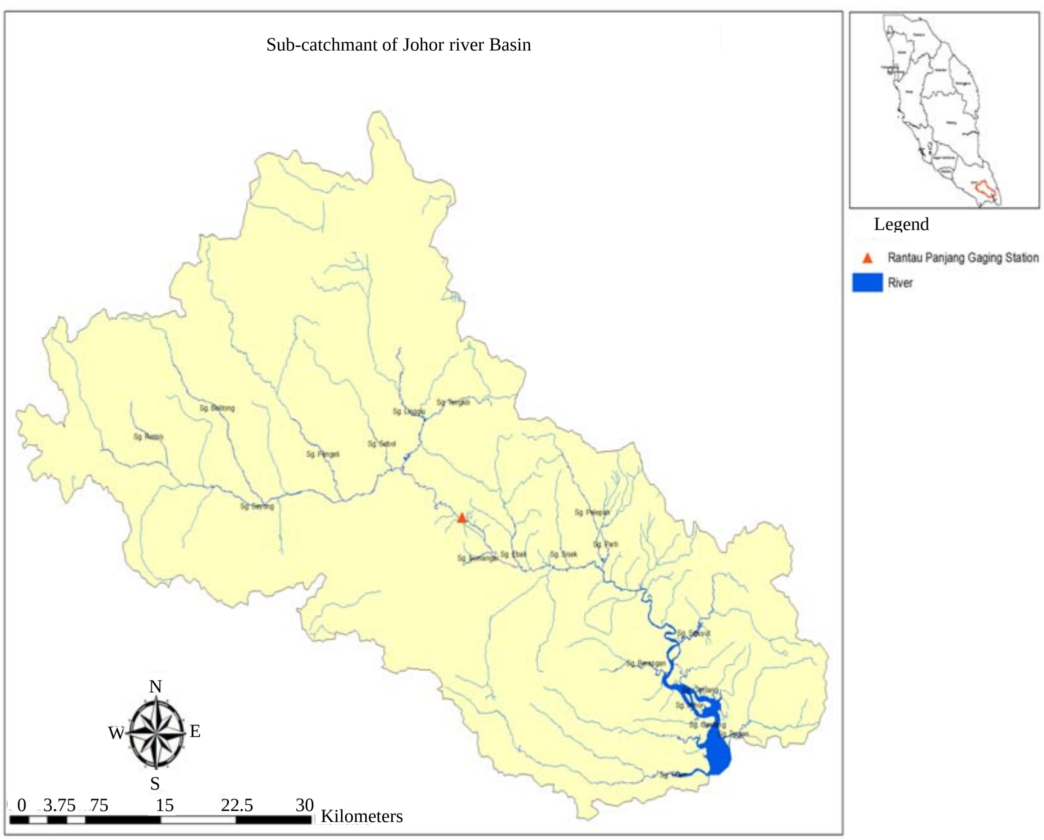


Fig. 1: Map of Johor River in south of peninsular Malaysia

Modeling the peak flow, flood duration and flood volume: This study had used the Generalized Pareto, Pearson, Exponential, Beta and the Generalized Extreme Value (GEV) distributions to model the flood variables' distributions. The cumulative distribution function (CDF) is defined as:

$$F(x) = \int_{-\infty}^{x} f(t)dt \quad (1)$$

The theoretical CDF is displayed as a continuous curve. The empirical CDF is denoted by:

$$F_n(x) = \frac{1}{n}[\text{Number of observations} \le x] \quad (2)$$

where, x is the random variable representing the hourly rainfall intensity.

The Probability Density Function (PDF) is the probability that the variate has the value x:

$$\int_a^b f(x)dx = P(a \le X \le b) \quad (3)$$

For discrete distributions, the empirical (sample) PDF is displayed as vertical lines representing the probability mass at each integer X:

$$f(x) = P(X = x) \quad (4)$$

The empirical PDF is represented as a histogram with equal-width vertical bars (bins). The bins represent the number of sample data that belong to a certain interval divided by the total number of data points. Ideally, a continuous curve can be properly scaled to the number of intervals to form a continuous curve.

The following explains the Generalized Extreme value (GEV) and Generalized Pareto models of the PDF and CDF.

Generalized extreme value (GEV): The general extreme value with continuous shape parameter (κ), continuous scale parameter (σ) and continuous location parameter (μ) have PDF and CDF given as below:

$$f(x) = \begin{cases} \frac{1}{\sigma}\exp(-(1+kz)^{-1/k})(1+kz)^{-1-1/k} & k \ne 0 \\ \frac{1}{\sigma}\exp(-z-\exp(-z)) & k = 0 \end{cases} \quad (5)$$

$$F(x) = \begin{cases} \exp(-(1+kz)^{-1/k}) & k \ne 0 \\ \exp(-\exp(-z)) & k = 0 \end{cases} \quad (6)$$

Where:

$$z \equiv \frac{x-\mu}{\sigma}$$

$$1 + k\frac{(x-\mu)}{\sigma} \rangle 0 \quad \text{for} \quad k \ne 0$$

$$-\infty \langle x \langle +\infty \quad \text{for} \quad k = 0$$

Generalized Pareto distribution: The Generalized Pareto distribution with continuous shape parameter (κ), continuous scale parameter (σ>0) and continuous location parameter (μ) have PDF and CDF as below:

$$F(x) = \begin{cases} \frac{1}{\sigma}(1+\kappa\frac{(x-\mu)}{\sigma}^{-1-1/k}) & k \ne 0 \\ \frac{1}{\sigma}\exp(-\frac{(x-\mu)}{\sigma}) & k = 0 \end{cases} \quad (7)$$

and

$$F(x) = \begin{cases} \frac{1}{\sigma}(1+\kappa\frac{(x-\mu)}{\sigma}^{-1-1/k}) & k \ne 0 \\ \frac{1}{\sigma}\exp(-\frac{(x-\mu)}{\sigma}) & k = 0 \end{cases} \quad (8)$$

where:

$$\mu \le x < +\infty \quad \text{for} \quad \kappa \ge 0$$

$$\mu \le x \le \mu - \frac{\sigma}{\kappa} \quad \text{for} \quad \kappa < 0$$

Goodness-of-fit test: Goodness-of-fit (GOF) tests measure the extent to which a random sample is compatible with a theoretical probability distribution function. In this study, the GOF tests, or more specifically the Kolmogorov-Smirnov tests, were carried out at 5% level of significance. Details of the Kolmogorov-Smirnov test are as follows:

Kolmogorov-Smirnov (K-S) test: This nonparametric test measures the distance between an empirical distribution and the CDF of the selected distribution. It can also measure the differences between two empirical distributions. For a random variable, X and sample (x_1, x_2, x_3,.........,x_n) the empirical CDF of X [$F^x(x)$] is:

$$F(x) = \frac{1}{n}\sum_{i=1}^{n} I(x_i \le x) \quad (9)$$

where,
I (condition) = 1 if true and 0 otherwise.

Given two cumulative probability functions F_x and F_y, the Kolmogorov-Smirnov statistic test (D and D.) are:

$$D_+ = \max(F_x(x) - F_y(x))$$

$$D_- = \max(F_y(x) - F_x(x))$$

Theoretical aspects of copula: A copula captures the dependence of two or more random variables. The Sklar's theorem (Sklar, 1959) states that the joint behaviour of random variables (X, Y) with continuous marginal distributions of $u = F_x(x) = P(X \le x)$ and $v = F_y(y) = P(Y \le y)$ can be characterized uniquely by its associated dependence function or copula, C. For 2-dimensional cases, all (u,v) relationships can be written as:

$$F_{X,Y}(X,Y) = C[F_X(x), F_Y(y)] = C(u,v) \tag{10}$$

where, $F_{X,Y}(x,y)$ is the joint CDF of random variables X and Y and also $\forall\ x,y \in R$. When $I = [0,1]$, the bivariate copula has a distribution function of $C = I^2 \rightarrow I$ which normally satisfies the following basic properties:

- **The boundary conditions:** $C(t,0) = C(0,t) = 0$ and $C(t,1) = C(1,t) = t, \forall t = I$
- **Increasing property:** $C(u_2, v_2) - C(u_2, v_1) - C(u_1, V_2) + C(u_1, v_1) \ge 0$, $\forall u_1, u_2, v_1, v_1 \in I$ such that $u_1 \le u_2$ and $v_1 \le v_2$

The bivariate copula density, c(u,v), is the double derivative of C with respect to its marginals and can be written as:

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v}$$

Archimedean copulas: The copula function, $C: [0,1]^2 \rightarrow [0,1]$, is called the bivariate Archimedean copula and can be called (Nelsen, 2006):

$$C(u,v) = \phi^{-1}(\phi(u)) + \phi(v))\ u, v \in [0,1] \tag{11}$$

where, $\phi(u)$ and $\phi(v)$ are known as the generator functions of the copula and ϕ^{-1} is the inverse of $\phi(u)$ and $\phi(v)$. The generator, $\phi: I \rightarrow R^+$, is a decreasing continuous convex function such that $\phi(1) = 0$ and $\phi(0) = \infty$.

In this study, a single Archimedean copula function, i.e., t or student copula was used. The generator functions, expressions and other properties of this Archimedean copula functions can be found in Table 1 (Nelsen 2006). It should be noted here that application of a copula family is bounded by the flood variables' relationships (e.g., when the Kendall's rank correlation (τ) dependence measure is used.

t-Copula: This copula is invariant when the marginal distributions are standardized and remains so even when the random vector X has transforming components that are strictly increasing. The copula of $t_d(v,\mu,\Sigma)$ is identical

Table 1: The summary statistics of flood parameters

	Peakflow (m^3/sec)	Duration (h)	Volume (mm)
Maximum	725	600	231
Minimum	77	144	20
Average	248	349	105
SD	164	126	49

to that of $t_d(v,0,P)$ where P is the correlation matrix represented by the dispersion matrix, Σ. The unique copula then becomes:

$$C^t_{v,P}(u) = \int_{-\infty}^{t_v^{-1}(u_1)} \cdots \int_{-\infty}^{t_v^{-1}(u_d)} \frac{\Gamma(\frac{v+2}{2})}{\Gamma(\frac{v}{2})\sqrt{(\pi v)^d |P|}} (1 + \frac{x' P^{-1} X}{v})^{-\frac{v+2}{2}} dx \tag{12}$$

where, $t^{-1}{}_v$ is the quantile function of a standard univariate, t_v, distribution. For a bivariate, this is denoted as $C^t{}_{v,\rho}$ where ρ is the off-diagonal element of P.

The simulation of t copula is not difficult because a multivariate t-distributed random vector, $X \approx t_d(v,0,P)$, can be generated with normal mixture construction to generate another vector, $U = (t_v(X_1, \ldots\ldots, t_v(X_d))'$; in here, t_v is the df of a standard univariate, t. For estimation purposes, the t copula can be calculated as:

$$c^t_{v,P}(u) = \frac{f_{v,P}(t_v^{-1}(u_1), \ldots\ldots t_v^{-1}(u_d))}{\prod_{i=1}^d f_v(t_v^{-1}(u_i))}, \quad u \in (0,1)^d \tag{13}$$

where, $f_{v,p}$ is the joint density of $t_d(v,0,P)$-a distributed random vector. f_V is the density of the univariate standard t-distribution with v degrees of freedom.

According to Li *et al.* (2012), the copula with two dependence parameters for the bivariate t-distribution with v degrees of freedom and correlation, ρ, Eq. 14 is:

$$C^t(u, v; \theta_1, \theta_2) = \int_{-\infty}^{t_{\theta_1}^{-1}(u)} \int_{-\infty}^{t_{\theta_1}^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\theta_2^2)}} \times \left\{1 + \frac{(s^2 - 2\theta_2 st + t^2}{\upsilon(1-\theta_2^2)}\right\}^{-(\theta_1+2)/2} ds\, dt \tag{14}$$

where, $t^{-1}{}_{\theta 1}$ denotes the inverse of the standard univariate t-distribution's CDF with θ_1 degrees of freedom. The two dependence parameters are (θ_1, θ_2). The parameter θ_1 controls the heaviness of the tails. For $\theta_1 < 3$, the variance does not exist and for $\theta_1 < 5$, the fourth moment does not exist. As $\theta_1 \rightarrow \infty$, $C^t(u,v; \theta_1, \theta_2) \rightarrow \Phi_G(u,v; \theta_2)$.

RESULTS AND DISCUSSION

The summary statistics of the flood parameters are given in Table 1. The averages of peak flow, flood duration and flood volume at the study site were 248 m^3/sec, 349 hours and 105 mm, respectively. Table 2 presents the shape parameter (κ), continuous scale parameter (σ) and continuous location parameter (μ) of various distributions used to fit the distribution of flood variables. Based on the (K-S) GOF tests, it is found that the Generalized Pareto distribution best fit the peak flow distribution. On the other hand, GEV is the best-fit distribution for the flood duration and flood volume distributions.

Measures of dependence are common to summarize a complicated dependence structure in a single number. There are three important concepts for dependence measures viz. the classical linear correlation (ρ), rank correlation (κ) and the coefficients of tail dependence (v). These measures are good enough to give sensible measures for any dependence structure. The values of ρ, τ and v for each pair of flood variables are given in Table 3.

The joint cumulative distribution function of the peak flow and duration, peak flow and volume and duration and volume are illustrated in Fig. 2-4, respectively. By horizontally cutting the joint CDF, a set of counters lines can be obtained. Here, it should be noted that for a given joint distribution, there may exist more than one possible flood variable combinations. Hence, the contour lines of joint distribution of each pairs of flood characteristics are illustrated in separate Fig. 2b.

The contour lines of joint cumulative distribution of the peak flow and duration are depicted in "Fig. 2". For each cumulative distribution contour, there is an inverse relationship between peak flow and duration. Either can be as low as needed for any value of cumulative distribution contour, but then the other becomes large. Each becomes somewhat higher than cumulative distribution contour if the other one is low. It means that if flood peak is high, the flood duration will be low or vice versa. Also, with higher cumulative distribution contour, allows higher peak flow and flood duration. Therefore, the joint probability graph presented in "Fig. 2" refers to the chance of two conditions viz. peak flow and flood duration occurring at the same time. Figure shows joint distribution contours for flood volume and flood duration at 0.2, 0.4, 0.6, 0.8 and 1.0. The contour line marked 0.2 means that if the peak discharge is more than 231 m^3/sec, the flood duration will be less than 108 h or vice versa. Other contour lines can be interpreted in similar way.

The contour lines of joint distribution of the peak flood flow and flood volume are presented in "Fig. 3". For hydraulic designing and hydraulic infrastructure operation, combined occurrence of these two flood characteristics is often important. The joint probability graph presented in Fig. 3 refers to the chance of two flood characteristics viz. peak flood flow and flood volume occurring at the same time. "Fig. 3" shows cumulative distribution of flood volume and flood duration for probabilities 0.2, 0.4, 0.6, 0.8 and 0.9.

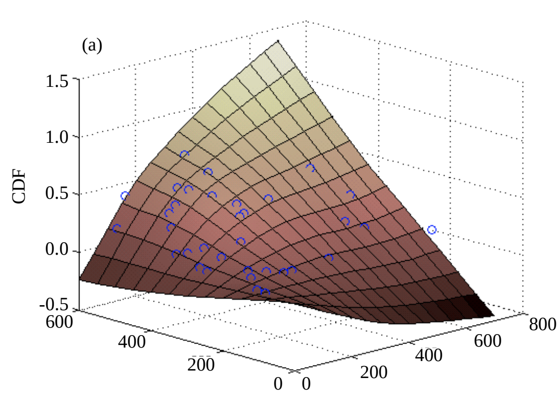
Table 2 : Fitting result parameters for various distributions of flood variables

Flood variable	Best fitted distribution	Parameters
Peakflow (P)	Gen. pareto	κ = -0.033905
		σ = 184.4800
		μ = 70.68400
Duration (D)	Gen. extreme value (GEV)	κ = -0.20041
		σ = 122.450
		μ = 299.350
Volume (V)	Gen. extreme value (GEV)	κ = -0.0740
		σ = 42.820
		μ = 83.017

Based on the Kolmogorov–Smirnov test, the GEV distribution is the best fit to flood volume and duration and Gen. Pareto distribution is the best for peak flow

Table 3: An estimate Rho and Nu of the matrix of linear correlation parameters for a t copula

	Peakflow-duration	Peakflow-volume	Duration-volume
ρ	0.0165	0.6455	0.4397
v	5.5585	7.7138	2.6615
τ	0.4720	0.0150	0.3330



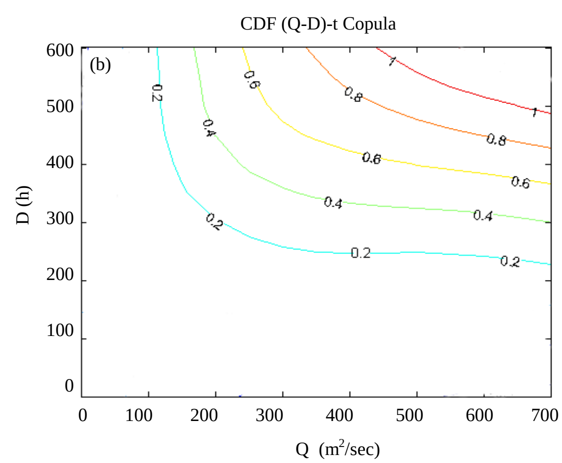


Fig. 2(a-b): (a) Joint cumulative distribution function of peakflow and flood duration (b) Contours showing the two-dimensional view of joint cumulative distribution function of peakflow and flood duration

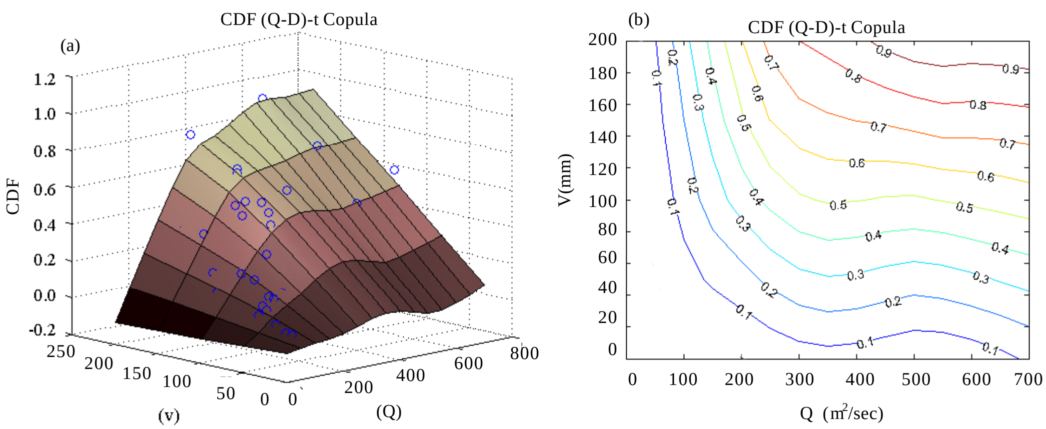


Fig. 3(a-b): (a) the joint cumulative distribution function of peakflow and flood volume; (b) contours showing the two-dimensional view of joint cumulative distribution function of peakflow and flood volume

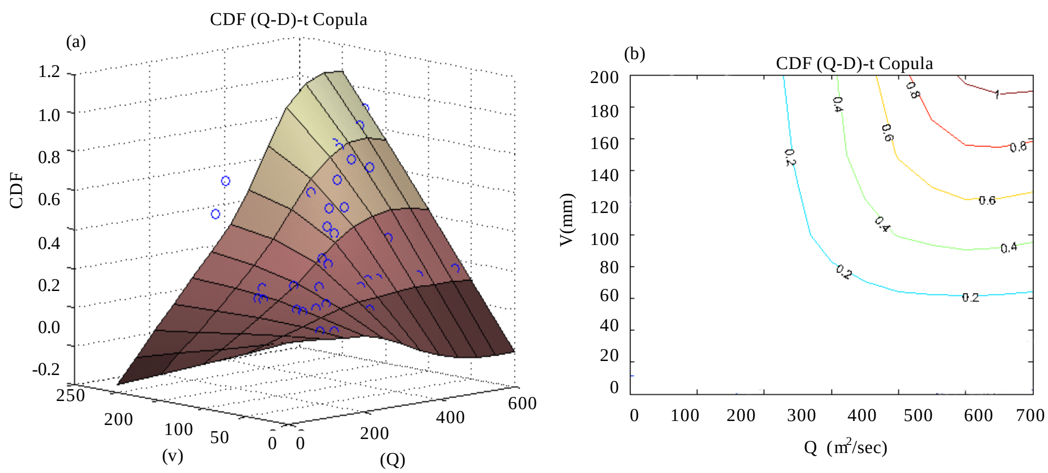


Fig. 4(a-b): (a) Joint cumulative distribution function of flood volume and flood duration (b) Contours showing the two-dimensional view of joint cumulative distribution function of flood volume and flood duration

The contour lines of joint cumulative distribution of the flood duration and flood volume are shown in "Fig. 4". Similar to other joint distributions shown in Fig. 2 and 3, the graph presented in "Fig. 4" notify the chance of two flood characteristics viz. peak duration and flood volume occurring at the same time.

Figures 2-4 indicate that the proposed method can contribute meaningfully in solving many problems of hydrological engineering designs and management problems, particularly when a single variable flood analysis cannot provide the answers. For example, given a flood event peak flow, it is possible to obtain various occurrence combinations of flood duration and flood volume and vice versa which can be helpful for flood risk management and designing of hydrological structures. Therefore, it can be remarked that the use of copulas has greatly improved the modelling of dependencies in this study. The use of copulas has successfully overcome the disadvantages of correlation and has provided a mathematically consistent model of dependence.

CONCLUSION

Information related to the peak flow, flood duration and flood volume are necessary to design hydraulic structures for water resources development and management as well as flood assessment and mitigation. An Archimedean copula known as t-copula (student copula) has been proposed in this study for the modelling of joint dependence of these flood characteristics viz. peak flow-volume, volume-duration and peak flow-duration for the Johor river basin. Malaysia's flood events' frequency and severity have been increasing. As it is not possible to change the natural course of events, concerted actions at a political and institutional level will certainly help to build the capacity and reduce people's vulnerability to flood impacts. A major outcome of the study is the production of joint distribution functions of flood characteristics. It is hoped that the study will be beneficial to a number of stakeholders in the country, particularly water resources development and management authorities and the disaster management and development/planning authorities in understanding the basin's flood risks.

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