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H_∞ Filtering for Networked Lipschitz Nonlinear System with Quantization and Packet Dropout

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Abstract: Filter is applied to estimate the state of a system disturbed by external noise. When the statistical information of the noise is not exactly known, H_∞ filter is suitable for the state estimation of the system. In this study, H_∞ filter is designed for networked Lipschitz nonlinear system with logarithmic quanitzer and packet dropout governed by Markovian Chain. The filtering error system is modeled as a Markovian jump system. Based on Lyapunov stability theory, the sufficient condition for the existence of the desired filter which guarantees the filtering error system to be stochastically stable with a given H_∞ performance is derived. The filter parameters can be obtained by the solution of a set of linear matrix inequalities. The simulation example proves that the proposed method is effective in filtering of networked Lipschitz nonlinear system.

Key words: H_∞ filtering, data quantization, packet dropout, markovian jump system, linear matrix inequality

INTRODUCTION

Control systems where the control loops are closed over a shared communication network are called networked control systems (Antsaklis and Baillieul, 2004; Hespanha *et al*., 2007). In networked control systems, the components such as sensors, filters, controllers and actuators exchange information across the communication network. So, compared with the traditional control systems using point-to-point architecture, networked control systems have the advantages of high flexibility, low cost, easy maintenance and reconfiguration and so on. Therefore, networked control systems are applied in a wide range of areas such as modern manufacturing plants, autonomous vehicles, aircrafts, etc.

When the states of the networked control systems are not measurable and disturbed by noise, filter can be designed to estimate them. If the noise is white, Kalman filter and linear filter can be used to estimate the states (Huang and Dey, 2007; Wei *et al*., 2009). However, if the statistical characteristic of the noise is difficult to determine, Kalman filter is no longer useful. In such case, H_∞ filter which regards the noise as energy bounded signal can obtain good filtering effect. Much effort has been made towards this over the past few years. Based on quadratic and parameter-dependent stability, Gao and Chen (2007) investigated robust H_∞ filtering for networked linear uncertain system with limited communication capacity. The problem of H_∞ filtering was addressed for networked continuous-time and discrete-time linear system with dynamic quantizer, transmission delay and packet dropout in the study of Jiang *et al*. (2010a) and Song *et al*. (2011), respectively. Hu and Yue (2012) designed event-based H_∞ filter for networked linear system with network-induced delay. Zhang *et al.* (2009) studied the H_∞ filtering problem for networked discrete-time linear system with packet dropout governed by Markovian chain. Zhang and Han (2010) studied non-fragile H_∞ filter for networked linear system by delay decomposition approach. It is noted that the above references are concerned with the filtering problem of networked linear system. For networked T-S fuzzy delay nonlinear system, Jiang *et al.* (2010b) designed H_∞ filter by delay probability dependent approach. Qiu *et al.* (2011) designed H_∞ filter for networked discrete-time T-S fuzzy affine nonlinear system based on piecewise-quadratic Lyapunov function, S-procedure and matrix-inequality techniques. However, to the best of the authors' knowledge, H_∞ filter for networked Lipschitz nonlinear system has not been fully investigated, which motivates us for this study.

In this study, logarithmic quantizer and packet dropout governed by Markovian chain are considered. The H_∞ filter which makes the filtering error system stochastically stable and a prescribed H_∞ attenuation level guaranteed is designed to estimate the state of

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networked Lipschitz nonlinear system. The sufficient condition for the existence of the filter is derived in terms of linear matrix inequality.

Notation: Throughout this study, R^n stands for the n-dimensional Euclidean space. $||\bullet||$ denotes the Euclidean vector norm. $Pr(\bullet)$ stands for the occurrence probability of event "•" and $E[\bullet]$ is the mathematical expectation operator. The superscript "-1" and "T" denote the inverse and the transpose of a matrix, respectively. The notation X>0 (X<0) means that the matrix X is real symmetric positive definite (negative definite). I denotes the identity matrix with appropriate dimension. The asterisk "*" in a matrix is used to represent the term that is induced by symmetry. diag {•} stands for a block-diagonal matrix.

PROBLEM FORMULATION AND PRELIMINARIES

The networked filtering problem investigated in this study is illustrated in Fig. 1, where the plant is described by the following discrete-time Lipschitz nonlinear system:

$$\begin{aligned} x(k+1) &= Ax(k) + Ff(x(k)) + Bw(k) \\ y(k) &= Cx(k) + Gg(x(k)) + Dw(k) \\ z(k) &= Lx(k) \end{aligned} \quad (1)$$

where $x(k) \in R^n$ is the state vector, $y(k) \in R^p$ is the measured output, $z(k) \in R^q$ is the signal to be estimated, $w(k) \in R^m$ is the noise belonging to $L_2[0, \infty)$. A, F, B, C, G, D and L are constant matrices with appropriate dimensions. f(x(k)) and g (x(k)) are nonlinear functions which are assumed to be Lipschitz with respect to x(k), i.e., there exist known appropriately dimensioned matrices F_1 and G_1 such that for all $x_1(k)$, $x_2(k)$ there hold:

$$\begin{aligned} & f(0) = 0,\ g(0) = 0 \\ & \|f(x_1(k)) - f(x_2(k))\| \le \|F_1(x_1(k) - x_2(k))\|, \\ & \|g(x_1(k)) - g(x_2(k))\| \le \|G_1(x_1(k) - x_2(k))\| \end{aligned}$$

Some problems such as data quantization, network-induced delay and data packet dropout arise due to the introduction of network in control loops. The measured output y (k) is quantized before transmitted to the remote filter through network to save network bandwidth. The following logarithmic quantizer is used in this study:

$$Q(v) = \begin{cases} u_i, & \frac{1}{1+\delta}u_i < v \le \frac{1}{1-\delta}u_i,\ v > 0 \\ 0, & v = 0 \\ -Q(-v), & v > 0 \end{cases}$$

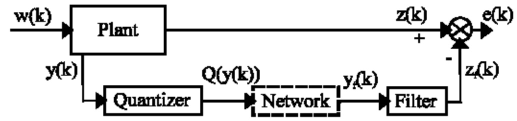

Fig. 1: Diagram of networked filtering system

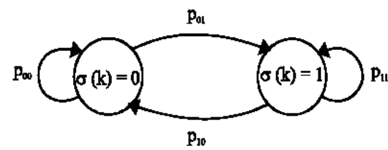

Fig. 2: Data transmission process

where, the set of quantized levels $U = \{\pm u_i, u_i = \rho^i u_0, i = \pm 1, \pm 2, ...\}\ U\ \{\pm u_0\} U\ \{0\}$, $0<\rho<1$, $u_0>0$, $\delta = 1-\rho/1+\rho$, ρ is the quantization density of Q(v) (Fu and Xie, 2005).

It is assumed that the network-induced delay is small enough to be negligible. During transmission, data packet may be dropped out due to the disturbance or network congestion which is common in wireless network (Smith and Seiler, 2003). The data packet transmission process is described by a two-state Markovian chain shown in Fig. 2. $\sigma(k) = 0$ means that data packet is transmitted successfully. $\sigma(k) = 1$ indicates that data packet is dropped out. The Markovian chain has a transition probability matrix $P = [p_{ij}]$, where $p_{ij} = Pr(\sigma(k+1) = j|\sigma(k) = i)$, $\forall i, j \in \{1, 2\}$, $p_{ij} \ge 0$:

$$\sum\nolimits_{j=0}^{1} p_{ij} = 1$$

The following filter is used to estimate the signal z (k):

$$\begin{aligned} & x_f(k+1) = A_{f\sigma(k)}x_f(k) + B_{f\sigma(k)}y_f(k) \\ & z_f(k) = C_{f\sigma(k)}x_f(k), \\ & \sigma(k) = i \in \{0, 1\} \end{aligned} \quad (2)$$

where, $x_f(k) \in R^n$ is the filter state vector, $y_f(k) \in R^p$ is the input of the filter, $z_f(k) \in R^q$ is the estimation of z(k). $A_{f\sigma(k)}$, $B_{f\sigma(k)}$ and $C_{f\sigma(k)}$ are appropriately dimensioned filter matrices to be designed.

The input of the filter can be expressed as $y_f(k) = Q(y(k)) = (I+H(k))\ y(k)$, $||H(k)|| \le \delta$ when data packet is transmitted over the network successfully (Fu and de Souza, 2009). Otherwise when data packet is dropped out during the transmission, the input of the filter is assumed to hold the previous value, i.e., $y_f(k) = y_f(k-1)$. Thus, the input of the filter can be written as:

$$y_f(k) = (1-\sigma(k))(I+H(k))y(k) + \sigma(k)y_f(k-1) \quad (3)$$

Defining $\tilde{x}(k)=[x^T(k)\ \ x_f^T(k)\ \ y_f^T(k)]^T$, $\eta(k)=[f^T(k,x(k))\ \ g^T(k,x(k))]^T$, and $e(k)=z(k)-z_f(k)$, the filtering error system can be described by:

$$\begin{aligned}&\tilde{x}(k+1)=\tilde{A}_{\sigma(k)}\tilde{x}(k)+\tilde{F}_{\sigma(k)}\eta(k)+\tilde{B}_{\sigma(k)}w(k)\\&e(k)=\tilde{C}_{\sigma(k)}\tilde{x}(k),\\&\sigma(k)=i\in\{0,1\}\end{aligned}\tag{4}$$

Where:

$$\tilde{A}_0=\hat{A}_0+\hat{D}H(k)E_1,$$

$$\tilde{F}_0=\hat{F}_0+\hat{D}H(k)E_2,$$

$$\tilde{B}_0=\hat{B}_0+\hat{D}H(k)E_3,$$

$$\tilde{C}_0=\begin{bmatrix}L & -C_f & 0\end{bmatrix},$$

$$\hat{A}_0=\begin{bmatrix}A & 0 & 0\\ B_{f0} & A_{f0} & 0\\ C & 0 & 0\end{bmatrix},$$

$$\hat{F}_0=\begin{bmatrix}F & 0\\ 0 & B_{f0}G\\ 0 & G\end{bmatrix},$$

$$\hat{B}_0=\begin{bmatrix}B\\ B_{f0}D\\ D\end{bmatrix},$$

$$\hat{D}=\begin{bmatrix}0\\ B_{f0}\\ I\end{bmatrix},$$

$$E_1=\begin{bmatrix}C & 0 & 0\end{bmatrix},$$

$$E_2=\begin{bmatrix}0 & G\end{bmatrix},$$

$$E_3=D,$$

$$\tilde{A}_1=\begin{bmatrix}A & 0 & 0\\ 0 & A_{f1} & B_{f1}\\ 0 & 0 & I\end{bmatrix},$$

$$\tilde{F}_1=\begin{bmatrix}I & 0\\ 0 & 0\\ 0 & 0\end{bmatrix},$$

$$\tilde{B}_1=\begin{bmatrix}B\\ 0\\ 0\end{bmatrix},$$

$$\tilde{C}_1=\begin{bmatrix}L & -C_f & 0\end{bmatrix}$$

Before giving the main results, the following two definitions and a lemma which will be used in the sequel are introduced.

Definition 1 (Shi *et al.*, 1999): When w (k) = 0, the filtering error system (4) is said to be stochastically stable if for every initial state ($\tilde{x}_0$, σ_0):

$$E\left[\sum_{k=0}^{\infty}\|\tilde{x}(k)\|^2\right]<\infty$$

Definition 2 (Shi *et al.*, 1999): Given a scalar γ>0, the filtering error system (4) is said to have an H_∞ performance γ, if:

$$\sum_{k=0}^{\infty}E\left[e^T(k)e(k)\right]<\gamma^2\sum_{k=0}^{\infty}E\left[w^T(k)w(k)\right]$$

for all nonzero w (k)$\in L_2$ [0, ∞) under zero initial condition.

Lemma 1 (Xie, 1996): Given matrice Γ_1, Γ_2 and Γ_3 of appropriate dimensions with $\Gamma_1=\Gamma_1^T$, then, $\Gamma_1+\Gamma_2\Delta(k)\Gamma_3+\Gamma_3^T\Delta^T(k)\Gamma_2^T<0$ holds for all $\Delta(k)$ satisfying $\Delta^T(k)\Delta(k)\le I$, if and only if there exists some ε>0 such that $\Gamma_1+\varepsilon^{-1}\Gamma_2\Gamma_2^T+\varepsilon\Gamma_3^T\Gamma_3<0$.

DESIGN OF H_∞ FILTER

The following theorem gives a sufficient condition for the existence of H_∞ filter in the form of (2).

Theorem: Given scalar γ>0 and quantization density ρ, the filtering error system is stochastically stable with an H_∞ performance γ, if there exist matrices $P_i^T=P_i>0$, $M_i^T=M_i>0$, $R_i^T=R_i>0$, N_i, V_i, B_{fi}, $i=0,1$ scalars ε_1>0 and ε_2>0 such that the following LMIs hold:

$$\begin{bmatrix}\Pi_{11} & * & * & * & * & * & *\\ \Pi_{21} & \Pi_{22} & * & * & * & * & *\\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * & * & * & *\\ \Pi_{41} & \Pi_{42} & \Pi_{43} & \Pi_{44} & * & * & *\\ \Pi_{51} & \Pi_{52} & \Pi_{53} & 0 & \Pi_{55} & * & *\\ \Pi_{61} & 0 & 0 & 0 & 0 & \Pi_{66} & *\\ 0 & 0 & 0 & \Pi_{74} & \Pi_{75} & 0 & \Pi_{77}\end{bmatrix}<0\tag{5}$$

$$\begin{bmatrix} \Omega_{11} & * & * & * & * & * \\ 0 & \Omega_{22} & * & * & * & * \\ 0 & 0 & \Omega_{33} & * & * & * \\ \Omega_{41} & \Omega_{42} & \Omega_{43} & \Omega_{44} & * & * \\ \Omega_{51} & \Omega_{52} & \Omega_{53} & 0 & \Omega_{55} & * \\ \Omega_{61} & 0 & 0 & 0 & 0 & \Omega_{66} \end{bmatrix} < 0 \qquad (6)$$

Where:

$$\Pi_{11} = \mathbf{diag}\{\Pi_{110}, \quad -M_0, \quad -R_0\}, \Pi_{110} = -P_0 + \varepsilon_1(F_1^T F_1 + G_1^T G_1) + \varepsilon_2 \delta^2 C^T C,$$
$$\Pi_{21} = \begin{bmatrix} 0 & 0 & 0 \\ \varepsilon_2 \delta^2 G^T C & 0 & 0 \end{bmatrix}, \Pi_{22} = \mathbf{diag}\{-\varepsilon_1 I, \ -\varepsilon_1 I + \varepsilon_2 \delta^2 G^T G\},$$
$$\Pi_{31} = \begin{bmatrix} \varepsilon_2 \delta^2 D^T C & 0 & 0 \end{bmatrix}, \Pi_{32} = \begin{bmatrix} 0 & \varepsilon_2 \delta^2 D^T G \end{bmatrix}, \Pi_{33} = -\gamma^2 I + \varepsilon_2 \delta^2 D^T D,$$
$$\Pi_{41} = \begin{bmatrix} P_0 A & 0 & 0 \\ B_{f0} C & N_0 & 0 \\ R_0 C & 0 & 0 \end{bmatrix}, \Pi_{42} = \begin{bmatrix} P_0 F & 0 \\ 0 & B_{f0} G \\ 0 & R_0 G \end{bmatrix}, \Pi_{43} = \begin{bmatrix} P_0 B \\ B_{f0} D \\ R_0 D \end{bmatrix},$$
$$\Pi_{44} = \mathbf{diag}\{-p_{00}^{-1} P_0, \quad -p_{00}^{-1} M_0, \quad -p_{00}^{-1} R_0\}, \Pi_{51} = \begin{bmatrix} P_1 A & 0 & 0 \\ B_{f0} C & N_0 & 0 \\ R_1 C & 0 & 0 \end{bmatrix},$$
$$\Pi_{52} = \begin{bmatrix} P_1 F & 0 \\ 0 & B_{f0} G \\ 0 & R_1 G \end{bmatrix}, \Pi_{53} = \begin{bmatrix} P_1 B \\ B_{f0} D \\ R_1 D \end{bmatrix}, \Pi_{55} = \mathbf{diag}\{-p_{01}^{-1} P_1, \quad -p_{01}^{-1} M_1, \quad -p_{01}^{-1} R_1\},$$
$$\Pi_{61} = \begin{bmatrix} L & -V_0 & 0 \end{bmatrix}, \Pi_{66} = -I, \Pi_{74} = \begin{bmatrix} 0 & B_{f0}^T & I \end{bmatrix}, \Pi_{75} = \Pi_{74}, \Pi_{77} = -\varepsilon_2 I,$$
$$\Omega_{11} = \mathbf{diag}\{\Omega_{110}, -M_1, -R_1\}, \Omega_{110} = -P_1 + \varepsilon_1(F_1^T F_1 + G_1^T G_1), \ \Omega_{22} = -\varepsilon_1 I, \ \Omega_{33} = -\gamma^2 I$$
$$\Omega_{41} = \begin{bmatrix} P_0 A & 0 & 0 \\ 0 & N_1 & B_{f1} \\ 0 & 0 & R_0 \end{bmatrix}, \ \Omega_{42} = \begin{bmatrix} P_0 F & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \Omega_{43} = \begin{bmatrix} P_0 B \\ 0 \\ 0 \end{bmatrix},$$
$$\Omega_{44} = \mathbf{diag}\{-p_{10}^{-1} P_0, -p_{10}^{-1} M_0, -p_{10}^{-1} R_0\}, \Omega_{51} = \begin{bmatrix} P_1 A & 0 & 0 \\ 0 & N_1 & B_{f1} \\ 0 & 0 & R_1 \end{bmatrix},$$
$$\Omega_{52} = \begin{bmatrix} P_1 F & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \Omega_{53} = \begin{bmatrix} P_1 B \\ 0 \\ 0 \end{bmatrix}, \ \Omega_{55} = \mathbf{diag}\{-p_{11}^{-1} P_1, -p_{11}^{-1} M_1, -p_{11}^{-1} R_1\},$$
$$\Omega_{61} = \begin{bmatrix} L & -V_1 & 0 \end{bmatrix}, \ \Omega_{66} = -I$$

Furthermore, the parameters of the filter $A_{fi} = N_i M_i^{-1}$ and $C_{fi} = V_i M_i^{-1}$.

Proof: Consider the following Lyapunov functional candidate:

$$V[\tilde{x}(k), \sigma(k) = i] = x^T(k) P_i x(k) + x_f^T(k) Q_i x_f(k)$$
$$+ y_f^T(k-1) R_i y_f(k-1) = \tilde{x}^T(k) S_i \tilde{x}(k)$$

where, S_i = diag $\{P_i, Q_i, R_i\}$.

Taking the difference of V $[\tilde{x}(k), \sigma(k) = i]$ along the trajectory of system (4) with w (k) = o results in:

$$\Delta V(k) = E\{V[\tilde{x}(k+1), \sigma(k+1)] \mid \tilde{x}(k), \sigma(k) = i\} - V[\tilde{x}(k), \sigma(k) = i]$$
$$= \tilde{x}^T(k+1) \sum_{j=0}^{1} p_{ij} S_j \tilde{x}(k+1) - \tilde{x}^T(k) S_i \tilde{x}(k)$$
$$= \tilde{x}^T(k) \tilde{A}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_i \tilde{x}(k) + 2\tilde{x}^T(k) \tilde{A}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_i \eta(k)$$
$$+ \eta^T(k) \tilde{F}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_i \eta(k) - \tilde{x}^T(k) S_i \tilde{x}(k)$$

As:

$$\eta^T(k)\eta(k) = f^T(k, x(k)) f(k, x(k)) + g^T(k, x(k))$$
$$g(k, x(k)) \le x^T(k)(F_1^T F_1 + G_1^T G_1) x(k)$$

there exists $\varepsilon_1 > 0$ such that:

$$\varepsilon_1 x^T(k)(F_1^T F_1 + G_1^T G_1) x(k) - \varepsilon_1 \eta^T(k)\eta(k) \ge 0$$

Then:

$$\Delta V(k) \le \tilde{x}^T(k) \tilde{A}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_i \tilde{x}(k) + 2\tilde{x}^T(k) \tilde{A}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_i \eta(k)$$
$$+ \eta^T(k) \tilde{F}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_i \eta(k) - \tilde{x}^T(k) S_i \tilde{x}(k) + \varepsilon_1 x^T(k)$$
$$(F_1^T F_1 + G_1^T G_1) x(k) - \varepsilon_1 \eta^T(k)\eta(k)$$
$$= \xi^T(k) \Theta_i \xi(k)$$

Where:

$$\xi(k) = \begin{bmatrix} \tilde{x}^T(k) & \eta^T(k) \end{bmatrix}^T,$$

$$\Theta_i = \begin{bmatrix} \tilde{A}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_i - S_i + W & \tilde{A}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F} \\ \tilde{F}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_i & \tilde{F}_i^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_i - \varepsilon_1 I \end{bmatrix}$$

$W = [I\ 0\ 0]^T \, \boldsymbol{\varepsilon}_1 (F_1^T F_1 + G_1^T G_1) \, [I\ 0\ 0]$.

Let $M_0 = Q_0^{-1}$, $M_1 = Q_1^{-1}$, $N_0 = A_{f0} M_0$, $V_0 = C_{f0} M_0$, $N_1 = A_{f1} M_1$, $V_1 = C_{f1} M_1$. Pre-and post-multiplying both sides of inequality (5) by diag $\{I, Q_0, I, I, I, I, P_0^{-1}, I, R_0^{-1}, P_1^{-1}, I, R_1^{-1}, I, I\}$ yields:

$$\Phi_1 + \varepsilon_2 \Phi_2 \Phi_2^T + \varepsilon_2^{-1} \delta^2 \Phi_3^T \Phi_3 < 0$$

Where:

$$\Phi_1 = \begin{bmatrix} -S_0 + W & * & * & * & * & * \\ 0 & -\varepsilon_1 I & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ \hat{A}_0 & \hat{F}_0 & \hat{B}_0 & -p_{00}^{-1} S_0^{-1} & * & * \\ \hat{A}_0 & \hat{F}_0 & \hat{B}_0 & 0 & -p_{01}^{-1} S_1^{-1} & * \\ \tilde{C}_0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$
$$\Phi_2 = \begin{bmatrix} 0 & 0 & 0 & \hat{D}^T & \hat{D}^T & 0 \end{bmatrix}^T$$
$$\Phi_3 = [E_1 E_2 E_3\ 0\ 0\ 0]$$

According to lemma 1, the above inequality can be written as:

$$\begin{bmatrix} -S_0 + W & * & * & * & * & * \\ 0 & -\varepsilon_1 I & * & * & * & * \\ 0 & 0 & -\gamma^2 I & * & * & * \\ \hat{A}_0 & \hat{F}_0 & \hat{B}_0 & -p_{00}^{-1} S_0^{-1} & * & * \\ \hat{A}_0 & \hat{F}_0 & \hat{B}_0 & 0 & -p_{01}^{-1} S_1^{-1} & * \\ \tilde{C}_0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} < 0$$

Then, by Schur complement, it can be obtained that:

$$\Xi_0 = \begin{bmatrix} \Xi_{011} & * & * \\ \Xi_{021} & \Xi_{022} & * \\ \Xi_{031} & \Xi_{032} & \Xi_{033} \end{bmatrix} < 0$$

Where:

$$\Xi_{011} = \tilde{A}_0^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_0 - S_0 + W + \tilde{C}_0^T \tilde{C}_0,$$

$$\Xi_{021} = \tilde{F}_0^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_0,$$

$$\Xi_{022} = \tilde{F}_0^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_0 - \varepsilon_1 I,$$

$$\Xi_{031} = \tilde{B}_0^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_0,$$

$$\Xi_{032} = \tilde{B}_0^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_0,$$

$$\Xi_{033} = \tilde{B}_0^T \sum_{j=0}^{1} p_{ij} S_j \tilde{B}_0 - \gamma^2 I,$$

The above inequality implies that:

$$\Theta_0 = \begin{bmatrix} \tilde{A}_0^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_0 - S_0 + W & * \\ \Xi_{021} & \Xi_{022} \end{bmatrix} < 0$$

Similarly, inequality (6) leads to:

$$\Xi_1 = \begin{bmatrix} \Xi_{111} & * & * \\ \Xi_{121} & \Xi_{122} & * \\ \Xi_{131} & \Xi_{132} & \Xi_{133} \end{bmatrix} < 0$$

Where:

$$\Xi_{111} = \tilde{A}_1^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_1 - S_1 + W + \tilde{C}_1^T \tilde{C}_1,$$

$$\Xi_{121} = \tilde{F}_1^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_1,$$

$$\Xi_{122} = \tilde{F}_1^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_1 - \varepsilon_1 I,$$

$$\Xi_{131} = \tilde{B}_1^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_1,$$

$$\Xi_{132} = \tilde{B}_1^T \sum_{j=0}^{1} p_{ij} S_j \tilde{F}_1,$$

$$\Xi_{133} = \tilde{B}_1^T \sum_{j=0}^{1} p_{ij} S_j \tilde{B}_1 - \gamma^2 I$$

Since:

$$\Xi_1 < 0$$

$$\Theta_1 = \begin{bmatrix} \tilde{A}_1^T \sum_{j=0}^{1} p_{ij} S_j \tilde{A}_1 - S_1 + W & * \\ \Xi_{121} & \Xi_{122} \end{bmatrix} < 0$$

Thus:

$$\begin{aligned} &E\{V[\tilde{x}(k+1), \sigma(k+1)] \mid \tilde{x}(k), \sigma(k) = i\} - V[\tilde{x}(k), \sigma(k) = i] \\ &\le \xi^T(k)\Theta_i \xi(k) \le -\beta \xi^T(k)\xi(k) < 0 \end{aligned}$$

where, $\beta = \min_{i \in S} [\lambda_{\min}(-\Theta_i)] > 0$ is the least eigenvalue of $-\Theta_i$. For $N \ge 1$, it can be obtained that:

$$\begin{aligned} &E\{V[\tilde{x}(N+1), \sigma(N+1)]\} - V(\tilde{x}_0, \sigma_0) \le \sum_{k=0}^{N} -\beta E[\xi^T(k)\xi(k)] \\ &\le \sum_{k=0}^{N} -\beta E[\tilde{x}^T(k)\tilde{x}(k)] \end{aligned}$$

which yields:

$$\sum_{k=0}^{\infty} E[\tilde{x}^T(k)\tilde{x}(k)] < \beta^{-1} V(\tilde{x}_0, \sigma_0) < \infty$$

Therefore, it follows from definition 1 that the filtering error system (4) is stochastically stable.

Next, the filtering error system (4) is proved to have an H_∞ performance γ. Consider the following performance index:

$$J_N = E\left\{\sum_{k=0}^{N} [e^T(k)e(k) - \gamma^2 w^T(k)w(k)]\right\}$$

Under zero initial condition, it can be obtained that:

$$\begin{aligned} J_N &= E\left\{\sum_{k=0}^{N} \left[e^T(k)e(k) - \gamma^2 w^T(k)w(k) + \Delta V(k)\right]\right\} - E\left\{\sum_{k=0}^{N} \Delta V(k)\right\} \\ &= \sum_{k=0}^{N} \varphi^T(k)\Xi_i\varphi(k) - V[\tilde{x}(N+1), \sigma(N+1)] \\ &< \sum_{k=0}^{N} \varphi^T(k)\Xi_i\varphi(k) < 0 \end{aligned}$$

Where:

$$\varphi(k) = [\tilde{x}^T(k) \;\; \eta^T(k) \;\; w^T(k)]^T$$

As $N \to \infty$, one has:

$$J_\infty = E\left\{\sum_{k=0}^{\infty} \left[e^T(k)e(k) - \gamma^2 w^T(k)w(k)\right]\right\} < 0$$

According to definition 2, the filtering error system has an H_∞ performance γ. This completes the proof.

SIMULATION EXAMPLE

In this section, a numerical example is given to demonstrate the effectiveness of the method proposed in this study.

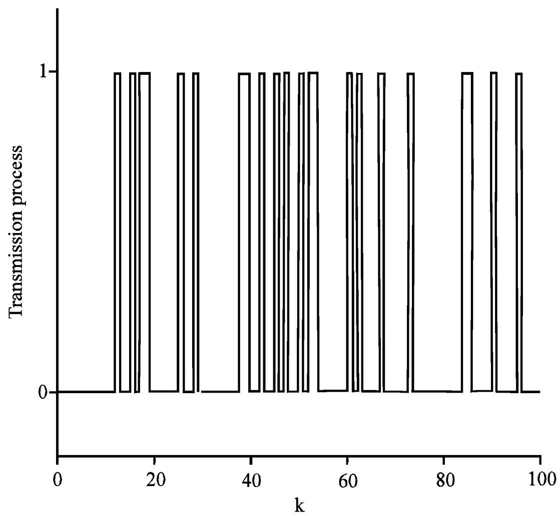

Fig. 3: Data packet transmission process

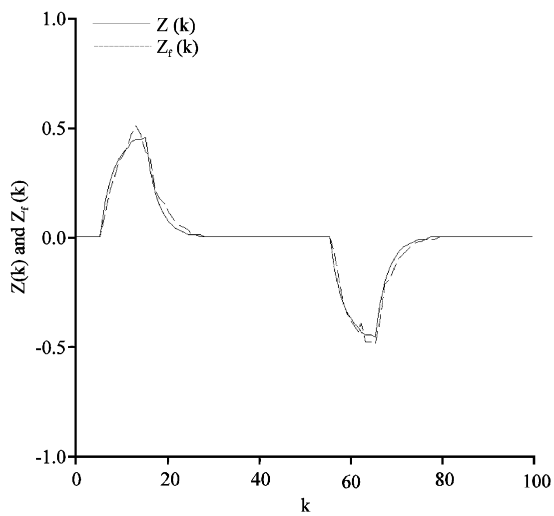

Fig. 4: Trajectories of z and z_f

Consider the Lipschitz nonlinear system (1) with the following parameters:

$$A = \begin{bmatrix} 0.1 & 0.2 \\ 0.3 & 0.5 \end{bmatrix}$$

$$F = \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & 0.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}$$

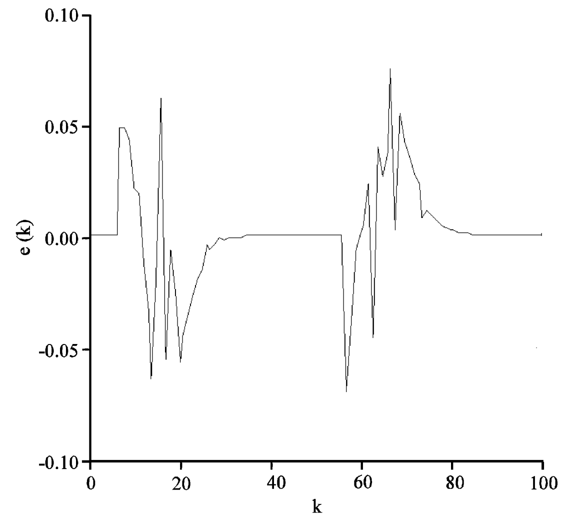

Fig. 5: Filtering error e(k)

$$D = 0.3$$

$$L = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$f(k,x(k)) = 0.1\sin(x(k))$$

$$g(k,x(k)) = 0.1\sin(x(k))$$

$$\rho = 0.5,\ \gamma = 1.8$$

$$w(k) = \begin{cases} 0.5, & 5 < k \le 15 \\ -0.5, & 55 < k \le 65 \\ 0, & \text{otherwise} \end{cases}$$

the transition probability matrix:

$$P = \begin{bmatrix} 0.8 & 0.2 \\ 0.7 & 0.3 \end{bmatrix}$$

and the data packet transmission process is shown as Fig. 3.

The inequalities (5) and (6) are solved by the MATLAB LMI toolbox. Then the parameters of the filter are:

$$A_{f0} = \begin{bmatrix} 0.104\,1 & 0.440\,9 \\ 0.189\,8 & 0.508\,2 \end{bmatrix},\ B_{f0} = \begin{bmatrix} 2.912\,1 \\ 5.593\,5 \end{bmatrix},\ C_{f0} = \begin{bmatrix} 0.091\,3 & 0.080\,1 \end{bmatrix}$$

$$A_{f1} = \begin{bmatrix} 0.102\,5 & 0.043\,9 \\ 0.248\,9 & 0.380\,7 \end{bmatrix},\ B_{f1} = \begin{bmatrix} 5.508\,7 \\ 10.149\,5 \end{bmatrix},\ C_{f1} = \begin{bmatrix} 0.099\,5 & 0.077\,0 \end{bmatrix}$$

The trajectories of z(k) and $z_f(k)$ are shown in Fig. 4. The response of the filtering error e(k) is plotted in Fig. 5. It can be seen that the proposed filter design method is effective.

CONCLUSION

The H_∞ filtering problem has been investigated for networked Lipschitz nonlinear system in this study. The logarithm quantizer and packet dropout governed by Markovian Chain are taken into consideration. Sufficient condition is derived for the existence of H_∞ filter which guarantees the filtering error system to be stochastically stable with a prescribed H_∞ performance level. The desired filter parameters can be computed by solving a set of linear matrix inequalities. Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

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