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Unbiased Phase Delay Estimator with Negative Frequency Contribution for Real Sinusoids

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Abstract: The problem of estimating the phase delay between two real sinusoids with unknown frequency is discussed. It is indicated that the negative frequency contribution is the essential and internal cause of the bias brought by the Discrete-time Fourier Transform (DTFT)-based phase delay estimator for real signals, when the sinusoidal frequency is quite low or close to the Nyquist frequency. Based on the DTFT-based estimator, a simple unbiased phase delay estimator with negative frequency contribution considered, is proposed for real signals with unknown frequency. The new formula for phase delay calculation is derived and detailed steps of the proposed estimator are presented accordingly. Simulation results show that the proposed estimator has removed the bias of the DTFT-based one and can attain optimum performance even when the sinusoidal frequency is quite low or close to the Nyquist frequency. Furthermore, the proposed estimator is proved to be particularly effective when there are not enough sampled data available for DTFT calculation. So the estimator is expected to be helpful to those engaged in phase/time delay estimation for many areas such as radar, machinery fault diagnosis and industrial measurement.

Key words: Phase delay estimation, negative frequency, discrete-time fourier transform

INTRODUCTION

Accurate estimation of propagation delay between signals received at two spatially separated sensors is involved in many areas such as radar, underwater acoustics, machinery fault diagnosis and industrial measurement (Etter and Stearns, 1981; Carter, 1993). The Generalized Cross Correlation (GCC) method (Knapp and Carter, 1976; Hertz and Azaria, 1985) is usually used for time delay estimation by locating the cross correlation peak of the two received signals. However, the method requires a prior statistics of the received signals and is not suitable when the noises are spatially correlated.

For single complex sinusoids, a Discrete-time Fourier Transform (DTFT)-based delay estimator (So, 2001) is developed by calculating the phase difference of the DTFTs of the received signals at the sinusoidal frequency. For real-valued sinusoids with known frequency, a Quadrature Delay Estimator (QDE) (Maskell and Woods, 2002) is proposed by using the in-phase and quadrature-phase components of the received signals. Using the idea of (So, 2001; Maskell and Woods, 2002), two modified discrete-time phase delay estimators for real signals with known frequency have been recently developed (So, 2005). The first estimator removes the bias of the QDE and the second estimator calculates the phase difference of the DTFTs of two complex sinusoids derived from the real signals. In the second estimator, the real tone is transformed to a complex tone with the use of known frequency information. As a result, there is no negative frequency component. However, the estimator is not suitable for real signals with unknown frequency.

Among the phase delay estimators for real signals with unknown frequency, the discrete Fourier transform (DFT)-based estimator and the DTFT-based estimator are two typical ones. As for the DFT-based estimator, the phase delay estimate is given by the phase difference of the DFTs of two real sinusoids at the maximum spectral line. As for the DTFT-based estimator, the phase delay estimate is given by the phase difference of the DTFTs of two real signals at the estimated signal frequency. However, both estimators have neglected the contribution of negative frequency when estimating the phase delay between two real sinusoids. When the sinusoidal frequency is quite low or close to the Nyquist frequency, both estimators will bring about significant biases or even become ineffective. The same thing will occur when there are not enough sampled data available for DFT/DTFT calculation.

To remove the bias of the DTFT-based estimator, a new unbiased phase delay estimator with negative frequency contribution considered, which can attain optimum accuracy even when the sinusoidal frequency is quite low or close to the Nyquist frequency, is developed based on the DTFT-based estimator in this study.

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PRINCIPLE OF THE DTFT-BASED DELAY ESTIMATOR FOR REAL SIGNALS WITH UNKNOWN FREQUENCY

Among the phase delay estimators for real signals with unknown frequency, the DTFT-based estimator is the primary one. First the signal frequency should be estimated and then the DTFTs of two sinusoids at the estimated signal frequency are calculated to obtain the DTFT phases. The phase delay estimate is finally given by the difference between two DTFT phases.

Consider two real sinusoids with the same frequency:

$$\begin{aligned} s_1(t) &= A_1\cos(2\pi f_0 t + \theta_1) \\ s_2(t) &= A_2\cos(2\pi f_0 t + \theta_2) \end{aligned} \tag{1}$$

where, A_1, A_2 is amplitudes, f_0 is signal frequency, θ_1, θ_2 is initial phases. The phase delay is defined as $\Delta\theta = \theta_2 - \theta_1$. The sampling sequences of two sinusoids can be formulated as follows:

$$\begin{aligned} s_1(n) &= A_1\cos(\omega_0\cdot n + \theta_1) \\ s_2(n) &= A_2\cos(\omega_0\cdot n + \theta_2) \end{aligned}, n = 0,1,\cdots,N-1 \tag{2}$$

where, ω_0 is digital frequency, $\omega_0 = 2\pi f_0/f_s$, f_s θ_1, θ_2 is sampling frequency, $f_s \geq 2f_0$, N is Number of sampled points. Mark $\hat{\omega}_0$ as the estimated value of ω_0 and then the DTFT of $s_1(n)$ at $\hat{\omega}_0$ is computed as (De Vegte, 2002):

$$S_1(\hat{\omega}_0) = \sum_{n=0}^{N-1} A_1\cos(\omega_0\cdot n + \theta_1)\cdot e^{-j\hat{\omega}_0 n} \tag{3}$$

According to Euler's formula, a real sinusoid can be formulated as the sum of two exponential signals with positive and negative frequencies respectively. Neglect the exponential signal with negative frequencies in Eq. 3 and then:

$$S_1(\hat{\omega}_0) \approx \sum_{n=0}^{N-1}\frac{A_1}{2}e^{j(\omega_0\cdot n+\theta_1)}\cdot e^{-j\hat{\omega}_0 n} = \begin{cases} \dfrac{A_1}{2}\cdot\dfrac{\sin[(\omega_0-\hat{\omega}_0)N/2]}{\sin[(\omega_0-\hat{\omega}_0)/2]}\cdot e^{j[\theta_1+\frac{(\omega_0-\hat{\omega}_0)N}{2}-\frac{\omega_0-\hat{\omega}_0}{2}]}, \text{when } \hat{\omega}_0 \neq \omega_0 \\ \dfrac{A_1}{2}\cdot N\cdot e^{j\theta_1}, \text{when } \hat{\omega}_0 = \omega_0 \end{cases} \tag{4}$$

So the phase of $s_1\hat{\omega}_0$ can be approximated as:

$$\phi_1 \approx \theta_1 + (\omega_0 - \hat{\omega}_0)N/2 - (\omega_0 - \hat{\omega}_0)/2 \tag{5}$$

Similarly, the DTFT phase of $s_2(n)$ at $\hat{\omega}_0$ can be approximated as:

$$\phi_2 \approx \theta_2 + (\omega_0 - \hat{\omega}_0)N/2 - (\omega_0 - \hat{\omega}_0)/2 \tag{6}$$

From Eq. 5 and 6, the estimate of $\Delta\theta$, denoted by $\Delta\hat{\theta}$, is obtained from the difference between ϕ_1 and ϕ_2:

$$\Delta\hat{\theta} \approx \phi_2 - \phi_1 \tag{7}$$

That is to say, the phase delay of two sinusoids equals approximately the subtraction of two DTFT phases at the estimated signal frequency. We refer to this phase delay estimator as DTFT-based method.

Note that the DTFT-based estimator referenced here is different from the modified DTFT method of So (2005). The former is suitable for real signals with unknown frequency, while the latter is suitable for real signals with known frequency. The key idea of the modified DTFT method of So (2005) is to transform the real tone to a complex tone with the use of known frequency information. In contrast, the phase delay estimate of the DTFT-based estimator in this section is given by the phase difference of the DTFTs of two real sinusoids at the estimated signal frequency.

UNBIASED PHASE DELAY ESTIMATOR WITH NEGATIVE FREQUENCY CONTRIBUTION

As the exponential signal with negative frequencies is neglected in Eq. 4, it is no longer the DTFT of the real sinusoid but is that of the complex one. In other words, the contribution of negative frequency components in the spectrum is neglected in Eq. 4. When the sinusoidal frequency is quite low (for instance, below a frequency resolution of DFT) or close to the Nyquist frequency, the negative frequency interference in the spectrum becomes remarkable (De Vegte, 2002; Xie *et al*., 1998) which is why the DTFT results obtained from Eq. 4 obviously deviate from the true values of signal spectra. As a result, it brings about significant biases in phase delay estimation. The same thing will occur when a small number of sampled data are taken in DTFT calculation.

Therefore, a new phase delay estimator which is expected to remove the bias of the DTFT-based estimator, is proposed based on DTFT with negative frequency contribution considered. The new formula for phase delay calculation is derived and detailed steps of the proposed estimator are presented accordingly.

Take account of the exponential signal with negative frequencies in Eq. 3 and then:

$$\begin{aligned} S_1(\hat{\omega}_0) &= \sum_{n=0}^{N-1}\frac{A_1}{2}e^{j(\omega_0 n+\theta_1)}\cdot e^{-j\hat{\omega}_0 n} + \sum_{n=0}^{N-1}\frac{A_1}{2}e^{-j(\omega_0 n+\theta_1)}\cdot e^{-j\hat{\omega}_0 n} \\ &= \frac{A_1}{2}e^{j\theta_1}\sum_{n=0}^{N-1}e^{j(\omega_0-\hat{\omega}_0)n} + \frac{A_1}{2}e^{-j\theta_1}\sum_{n=0}^{N-1}e^{-j(\omega_0+\hat{\omega}_0)n} \end{aligned} \tag{8}$$

Assuming that $\omega_0 \neq \hat{\omega}_0$, we can obtain the following equation by deduction:

$$\tan\theta_1 = \frac{c_1 \cdot \tan\varphi_1 - c_2}{c_3 \cdot \tan\varphi_1 + c_4} \quad (9)$$

Where:

$c_1 = \sin\alpha_1 \sin\alpha_2 \cos(\alpha_1\text{-}\alpha_3)+\sin\alpha_3 \sin\alpha_4 \cos(\alpha_4\text{-}\alpha_2)$
$c_2 = \sin\alpha_1 \sin\alpha_2 \sin(\alpha_1\text{-}\alpha_3)+\sin\alpha_3 \sin\alpha_4 \sin(\alpha_4\text{-}\alpha_2)$
$c_3 = \sin\alpha_1 \sin\alpha_2 \sin(\alpha_1\text{-}\alpha_3)+\sin\alpha_3 \sin\alpha_4 \sin(\alpha_4\text{-}\alpha_2)$
$c_4 = \sin\alpha_1 \sin\alpha_2 \sin(\alpha_1\text{-}\alpha_3)+\sin\alpha_3 \sin\alpha_4 \sin(\alpha_4\text{-}\alpha_2)$
$\alpha_1 = N(\omega_0\text{-}\hat{\omega}_0)/2; \alpha_2 = (\omega_0\text{-}\hat{\omega}_0)/2; \alpha_3 = (\omega_0\text{-}\hat{\omega}_0)/2$
$\alpha_4 = N(\omega_0\text{-}\hat{\omega}_0)/2$, ϕ_1 is the phase of $S_1(\hat{\omega}_0)$

Similarly, for $s_2(n)$:

$$\tan\theta_2 = \frac{c_1 \cdot \tan\phi_2 - c_2}{c_3 \cdot \tan\phi_2 + c_4} \quad (10)$$

where, ϕ_2 is the DTFT phase of $s_2(n)$ at $\hat{\omega}_0$.

From Eq. 9 and 10, the following formula for phase delay calculation is deduced:

$$\Delta\theta = \arctan\left[\frac{m_1(\tan\varphi_2 - \tan\varphi_1)}{m_2 + m_3(\tan\varphi_1 + \tan\varphi_2) + m_4 \tan\varphi_1 \tan\varphi_2}\right] \quad (11)$$

Where:

$$m_1 = c_1c_4 + c_2c_3,\ m_2 = c_2^2 + c_4^2,\ m_3 = c_3c_4 - c_1c_2,\ m_4 = c_1^2 + c_3^2$$

Since the true value of signal frequency is unknown, we can't work out the phase delay estimate by Eq. 11. On this account, we suggest removing the unknown variable ω_0 from Eq. 11 by approximation. When the signal-to-noise Ratio (SNR) of signals is not very low, the signal frequency estimated by discrete spectrum correction (Zhang *et al*., 2001; Kang *et al*., 2000; Qi and Jia, 2001; Chen *et al*., 2004) is generally quite close to the true value, i.e., $\hat{\omega}_0 \approx \omega_0$. It follows that $\sin\alpha_1/\sin\alpha_3 \approx N$. Synchronously dividing the numerator and the denominator of Eq. 11 by $\sin\alpha_3$, Eq. 11 can approximately be expressed as follows:

$$\Delta\theta = \arctan\left[\frac{m_5(\tan\varphi_2 - \tan\varphi_1)}{m_6 + m_7(\tan\varphi_1 + \tan\varphi_2) + m_8 \tan\varphi_1 \tan\varphi_2}\right] \quad (12)$$

Where:

$m_5 = (N \sin\hat{\omega}_0)^2\text{-}(\sin\beta)^2$
$m_6 = (N \sin\hat{\omega}_0)^2+(\sin\beta)^2\text{-}2N \sin\hat{\omega}_0 \sin\beta \cos(\beta\text{-}\hat{\omega}_0)^2$
$m_7 = N \sin\hat{\omega}_0+\sin\beta \sin(\beta\text{-}\hat{\omega}_0)$
$m_8 = (N \sin\hat{\omega}_0)^2+(\sin\beta)^2+2N \sin\hat{\omega}_0 \sin\beta \cos(\beta\text{-}\hat{\omega}_0)$
$\beta = N\hat{\omega}_0$

Equation 12 is the new formula for phase delay calculation.

When $\omega_0\text{-}\hat{\omega}_0$, the whole derivation is similar to what was mentioned earlier but with no approximation. The expressions of θ_1 and θ_2 are different from Eq. 9 and 10, but the formula for phase delay calculation is just the same as Eq. 12.

To sum up, the steps of the proposed estimator are as follows:

Step 1: Estimate the signal frequency by discrete spectrum correction, denoted by $\hat{\omega}_0$
Step 2: Calculate the DTFTs of $s_1(n)$ and $s_2(n)$ at $\hat{\omega}_0$, denoted by $S_1(\hat{\omega}_0)$ and $S_2(\hat{\omega}_0)$
Step 3: Calculate ϕ_1 and tan ϕ2

$$\tan\phi_1 = \frac{\mathrm{Im}[S_1(\hat{\omega}_0)]}{\mathrm{Re}[S_1(\hat{\omega}_0)]}$$

$$\tan\phi_2 = \frac{\mathrm{Im}[S_2(\hat{\omega}_0)]}{\mathrm{Re}[S_2(\hat{\omega}_0)]}$$

Step 4: Substitute $\hat{\omega}_0$, N, $\tan\phi_1$ and $\tan\phi_2$ into Eq. 12 and calculate the phase delay estimate

Another contribution of the proposed method is that we can obtain the exact values of initial phases by Eq. 9 or 10 if only the signal frequency is known. In contrast, the conventional parameter estimation methods based on discrete spectrum correction always bring about significant errors in phase estimation when the sinusoidal frequency is quite low or close to the Nyquist frequency, because the negative frequency contribution is neglected in the algorithms.

Note that the proposed estimator is proved to be particularly effective when the sinusoidal frequency is quite low or close to the Nyquist frequency, or when there are not enough sampled data available for DTFT calculation. However, when the signal frequency moves away from the zero and Nyquist frequency, the traditional DTFT-based estimator is preferred because the negative frequency contribution is negligible.

The modified DTFT-based delay estimator of So (2005) is unbiased and it assumes known frequency. If the signal frequency is unknown, it is suggested that the problem might be solved by using the maximum likelihood frequency estimator (Kenefic and Nuttall, 1987; So, 2005). However, it seems to be unfeasible. The noisy real tones is converted into analytic forms by combining the in-phase and quadrature-phase components (So, 2005):

$$r_1(kT) = x_1(kT) + jx_1((k-\Delta)T) \quad (13)$$

where, Δ is a positive integer such that $\pi/(2\hat{\omega}_0) = \Delta\cdot T$. To achieve that, the sampling period T should be properly chosen. However, it is so difficult to achieve due to the error of frequency estimation and the relative fixity of the sampling period in practice. In contrast, the proposed estimator in this section has no requirement of choosing the sampling period and is obviously more practical than the method of So (2005).

PERFORMANCE ANALYSIS

Under noisy conditions, the Mean Square Error (MSE) of phase delay estimate is computed by:

$$\mathrm{mse}[\Delta\hat{\theta}] = \beta^2 + \sigma^2_{\Delta\theta} = (\overline{\Delta\theta} - \Delta\theta)^2 + \sigma^2_{\Delta\theta} \tag{14}$$

where, $\Delta\hat{\theta}$ is phase delay estimate, β is bias of phase delay estimate, $\beta = \overline{\Delta\theta} - \Delta\theta$, $\overline{\Delta\theta}$ is mean of phase delay estimate, $\Delta\theta$ is theoretic value of phase delay, $\sigma^2_{\Delta\theta}$ is variance of phase delay estimate.

In general, the negative frequency interference is negligible when the signal frequency moves away from the zero and Nyquist frequency. So the DTFT-based estimator in Section 2 is generally regarded as an unbiased one and the MSE equals the variance approximately. Following the derivations by Zhang *et al.* (2007), the variance of $\Delta\hat{\theta}$ attained by the DTFT-based estimator for large N and SNR is shown to be:

$$\mathrm{var}(\Delta\hat{\theta}) \approx \frac{2}{N\cdot \mathrm{SNR}} \tag{15}$$

where, SNR is SNR of the signals.

Following the derivations in So (2005), the Cramer-Rao lower bound (CRLB) for phase delay between two noisy real-valued sinusoids has the expression:

$$\mathrm{CRLB} = \frac{2}{N\cdot \mathrm{SNR}} \tag{16}$$

Comparing Eq. 15 and 16, we observe that the variance of $\Delta\hat{\theta}$ attained by the DTFT-based estimator equals the CRLB for sufficiently large N and SNR, which indicates the optimality of the DTFT-based estimator under these conditions.

However, when the signal frequency is quite low or close to the Nyquist frequency, the DTFT-based estimator brings about significant biases in phase delay estimation. That is to say, it becomes a biased estimator. As a result, the MSE of $\Delta\hat{\theta}$ considerably exceeds the CRLB.

The proposed estimator with negative frequency contribution considered, is expected to remove the bias of the DTFT-based one and can attain optimum performance even when the sinusoidal frequency is quite low or close to the Nyquist frequency. The derivation of the theoretical variance of the proposed estimator is just similar to Zhang *et al.* (2007) and the variances of two estimators are almost identical.

SIMULATION RESULTS

Computer simulations have been carried out to validate the performance of the proposed estimator for real sinusoids. To draw a comparison, the phase delay estimates are given by the proposed estimator and the DTFT-based estimator, respectively.

Under noiseless conditions, the relative errors of phase delay versus the signal frequency are shown in Fig. 1. In simulations, the phase delay equals 3.6°, the number of sampled points equals 1024, the sampling frequency equals 1024 Hz and the frequency resolution equals 1 Hz. The sinusoidal frequency varies from 0.5-2.5 Hz in Fig. 1a and from 509.5 -511.5 Hz in Fig. 1b, both with a step length of 0.05 Hz. The relative errors of phase delay are computed by comparing the estimated values with the theoretic value of phase delay. As shown in Fig. 1, the DTFT-based estimator brings about significant errors, while the accuracy of the proposed estimator is so desirable that it almost approaches the lower limit of double precision arithmetic. It is observed that the accuracy of the DTFT-based estimator zooms quickly and equals that of the proposed estimator when the signal frequency equals an integer or half an integer, i.e., f_0 = 0.5 Hz, 1 Hz, 1.5 Hz, …., 511.5 Hz. That is because the side lobe of negative frequency components in the spectrum is just equal to zero at the spectrum peak of positive frequency components when f_0 is a multiple of half a frequency resolution. In other words, the contribution of negative frequency is void at this very moment. The curves in Fig. 1b are similar to those in Fig. 1a, owing to the mirror image feature of signal spectrum (De Vegte, 2002). Further simulations also show that the estimation accuracy of signal frequency directly affects the performance of the proposed estimator under noiseless conditions. The more exactly the signal frequency is estimated, the more exact the phase delay estimate is.

To exhibit the performance of the proposed estimator for a larger range of signal frequencies, simulations are carried out with the sinusoidal frequency varying from 0.05 Hz to 499.95 Hz and a step length equal to 0.5 Hz. The sampling frequency is changed to be 1000 Hz and the frequency resolution equals about 0.9766 Hz. As shown in Fig. 2, the proposed estimator is always superior to the DTFT-based estimator under noiseless conditions.

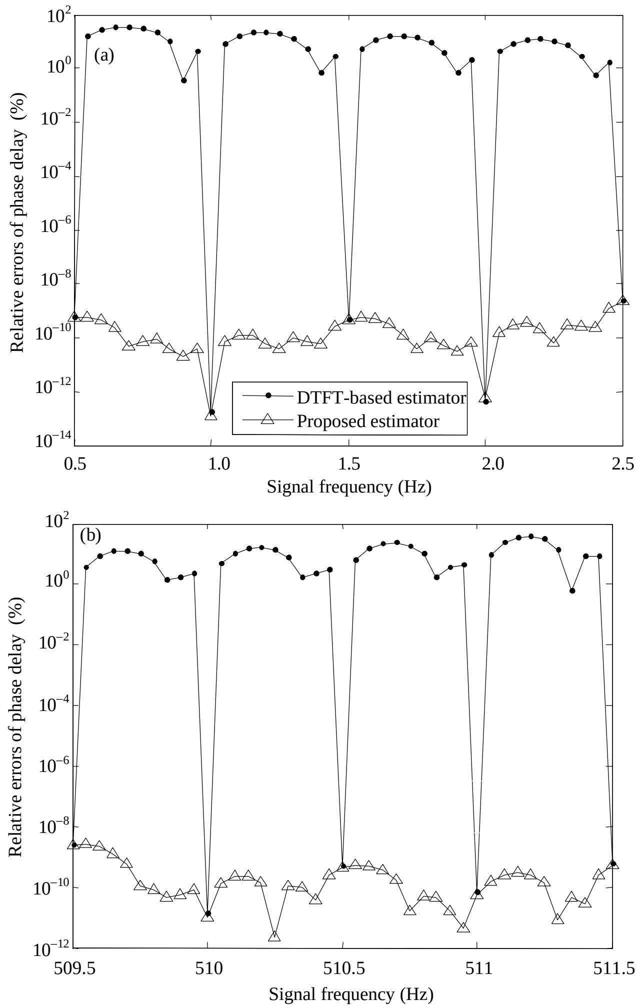

Fig. 1(a-b): Relative errors of phase delay under noiseless conditions when the signal frequency is quite low and/or close to the Nyquist frequency, (a) Signal frequency varying from 0.5 Hz to 2.5 Hz and (b) Signal frequency varying from 509.5 Hz to 511.5 Hz

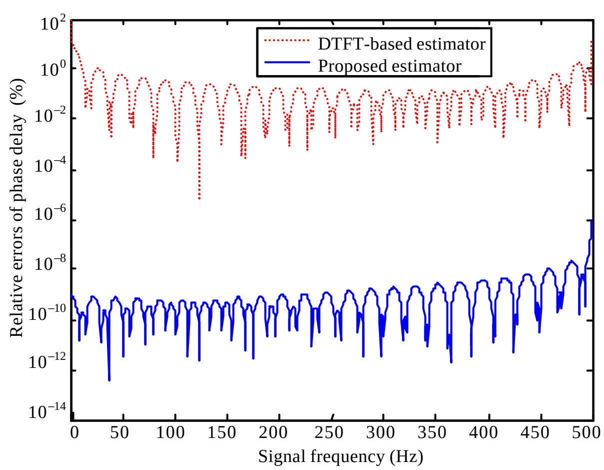

Fig. 2: Relative errors of phase delay vs. a large range of signal frequencies under noiseless conditions

For real-valued sinusoids in the presence of white Gaussian noise, extensive computer simulations have been conducted to evaluate the performance of the proposed estimator. Comparisons are also made with the DTFT-based estimator as well as the CRLB. The sampling frequency equals 1024 Hz and the white Gaussian noises superimposed on two signals are not correlative. All simulation results provided are averages of 200 independent runs.

Figure 3-5 show the biases, variances and MSEs of the proposed estimator and the DTFT-based estimator

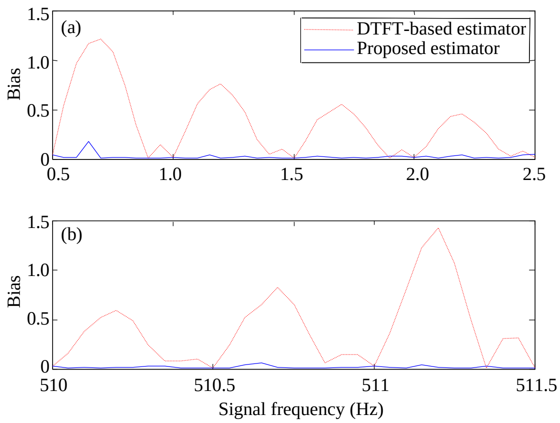

Fig. 3(a-b): Biases of phase delay at SNR=20 dB when the signal frequency is quite low and/or close to the Nyquist frequency, (a) Signal frequency varying from 0.5-2.5 Hz and (b) Signal frequency varying from 510-511.5 Hz

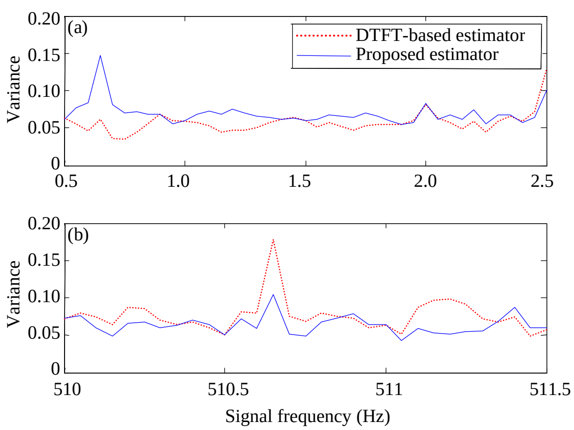

Fig. 4(a-b): Variances of phase delay at SNR =20 dB when the signal frequency is quite low and/or close to the Nyquist frequency, (a) Signal frequency varying from 0.5-2.5 Hz and (b) Signal frequency varying from 510-511.5 Hz

versus the signal frequency at N = 1024 and SNR = 20dB, respectively. It is observed from Fig. 3 that the DTFT-based estimator brings about significant biases when the signal frequency is quite low or close to the Nyquist frequency, while the proposed estimator is almost unbiased. Furthermore, the biases of the DTFT-based estimator vary in the manner of decaying oscillation. As shown in Fig. 4, the variances of the two estimators are almost identical. Figure 5 shows the MSEs of the two estimators as well as the CRLB. It is seen that the proposed estimator approaches the CRLB and gives the optimum performance, while the DTFT-based estimator considerably exceeds the CRLB owing to the biasedness of the algorithm. So the proposed estimator is proved to be superior to the DTFT-based estimator.

Similar simulations are carried out to exhibit the performance of the two estimators for a larger range of signal frequencies under noisy conditions. The sinusoidal frequency varies from 0.5-0 Hz and from 502-511.5 Hz. As shown in Fig. 6-8, the biases and MSEs of the DTFT-based estimator approach those of the proposed estimator asymptotically when the signal frequency moves away from the zero and Nyquist frequency.

To exhibit the performance of the proposed estimator under different SNRs, simulations are carried out with the SNR varying from 0 dB-30 dB. Figure 9 shows the MSEs of the proposed estimator and the DTFT-based estimator as well as the CRLB versus SNR at N = 1024 and f_0 = 1.2Hz. It is seen that the DTFT-based estimator gives good performance for SNR<5dB, but for higher SNRs, its performance gradually degrades from the proposed estimator which indicates the biasedness of the algorithm.

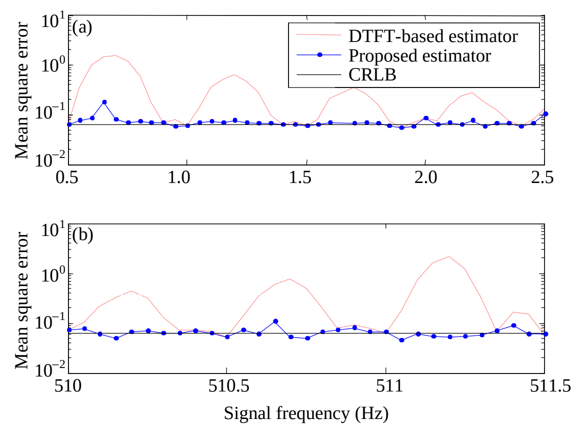

Fig. 5(a-b): MSEs of phase delay at SNR=20 dB when the signal frequency is quite low and/or close to the Nyquist frequency (a) Signal frequency varying from 0.5-2.5 Hz and (b) Signal frequency varying from 510-511.5 Hz

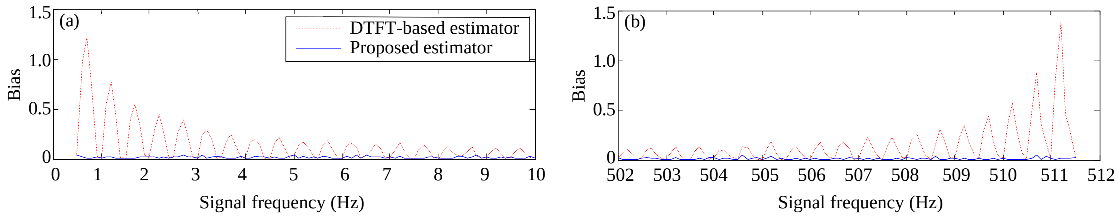

Fig. 6(a-b): Biases of phase delay vs. a large range of signal frequencies at SNR = 20 dB, (a) Signal frequency varying from 0.5-10 Hz and (b) Signal frequency varying from 502-511.5 Hz

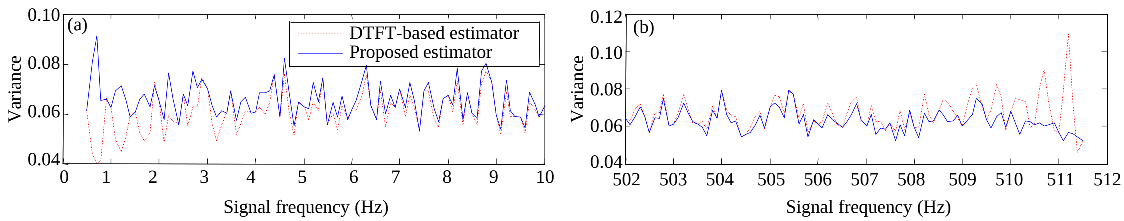

Fig. 7(a-b): Variances of phase delay vs. a large range of signal frequencies at SNR = 20 dB, (a) Signal frequency varying from 0.5-10 Hz and (b) Signal frequency varying from 502-511.5 Hz

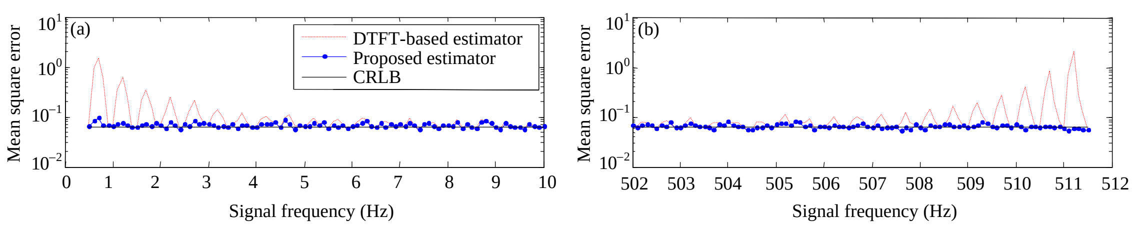

Fig. 8(a-b): MSEs of phase delay vs. a large range of signal frequencies at SNR = 20 dB, (a) Signal frequency varying from 0.5-10 Hz and (b) Signal frequency varying from 502-511.5 Hz

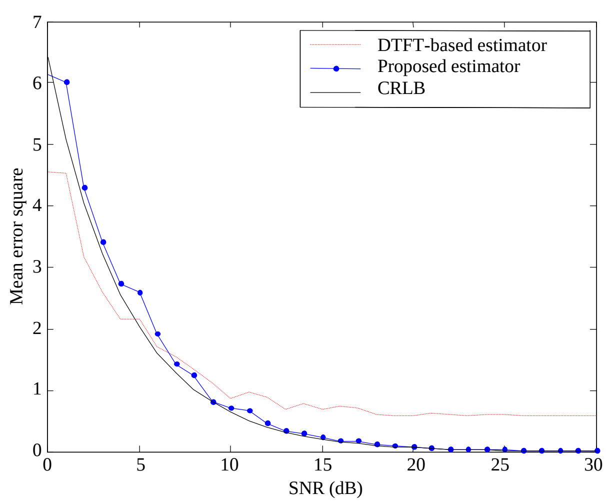

Fig. 9: MSEs of phase delay vs. SNR at f_0 =1.2 Hz

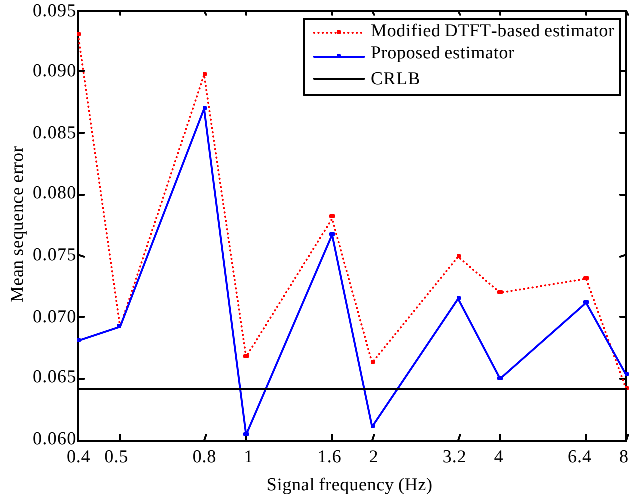

Fig. 10: MSEs of phase delay versus signal frequency at SNR = 20 dB

The performance of the proposed estimator is also compared with (So, 2005) for the case of known frequency. Figure 10 shows the MSEs of the proposed estimator and the modified DTFT-based delay estimator of (So, 2005) as well as the CRLB versus the signal frequency at N = 1024 and f_0 = 20dB. The sampling frequency equals 1024 Hz. In order to make Δ equal an integer, the signal frequency is properly chosen to be 0.4, 0.5, 0.8, 1, 1.6, 2, 3.2, 4, 6.4 and 8 Hz. We see that both estimators are unbiased and the proposed estimator is even more superior to the modified DTFT-based estimator.

CONCLUSION

The traditional DTFT-based phase delay estimator for real sinusoids with unknown frequency is biased when the sinusoidal frequency is quite low or close to the Nyquist frequency. A simple unbiased phase delay estimator with negative frequency contribution considered, has been developed for real signals with unknown frequency. Simulation results show that the proposed estimator has removed the bias of the DTFT-based one and can attain optimum performance even when the sinusoidal frequency is quite low or close to the Nyquist frequency. Furthermore, the proposed estimator is proved to be particularly effective when there are not enough sampled data available for DTFT calculation.

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REFERENCES

Carter, G.C., 1993. Coherence and Time Delay Estimation: An Applied Tutorial for Research, Development, Test and Evaluation Engineers. IEEE Press, New York, USA., ISBN-13: 9780780310063, Pages: 506.

Chen, K., S. Zhang and X. Guo, 2004. Spectrum rectifying with negative frequency contribution eliminating. Chin. J. Mech. Strength, 26: 25-28.

De Vegte, J.V., 2002. Fundamentals of Digital Signal Processing. Prentice-Hall, Englewood Cliffs, NJ., USA., ISBN-13: 9780130160775, Pages: 810.

Etter, D.M. and S.D. Stearns, 1981. Adaptive estimation of time delays in sampled data systems. IEEE Trans. Acoust. Speech Signal Process., 29: 582-587.

Hertz, D. and M. Azaria, 1985. Time delay estimation between two phase shifted signals via generalized cross-correlation methods. Signal Process., 8: 235-257.

Kang, D., X. Ming and Z. Xiaofei, 2000. Phase difference correction method for phase and frequency in spectral analysis. Mech. Syst. Signal Process., 14: 835-843.

Kenefic, R.J. and A.H. Nuttall, 1987. Maximum likelihood estimation of the parameters of a tone using real discrete data. IEEE J. Oceanic Eng., 12: 279-280.

Knapp, C.H. and G.C. Carter, 1976. The generalized correlation method for estimation of time delay. IEEE Trans. Acoust. Speech Signal Process., 24: 320-327.

Maskell, D.L. and G.S. Woods, 2002. The discrete-time quadrature subsample estimation of delay. IEEE Trans. Instrum. Meas., 51: 133-137.

Qi, G. and X. Jia, 2001. High accuracy frequency and phase estimation of single-tone based on phase of DFT. Acta Electronica Sinica, 29: 1164-1167.

So, H.C., 2001. Time-delay estimation for sinusoidal signals. IEE Proc. Radar Sonar Navigation, 148: 318-324.

So, H.C., 2005. A comparative study of two discrete-time phase delay estimators. IEEE Trans. Instrum. Meas., 54: 2501-2504.

Xie, M., K. Ding and K. Mo, 1998. Spectrum line interaction and distinguish method of the spectrum interpolation correction. Chin. J. Vib. Eng., 11: 52-57.

Zhang, F., Z. Geng and W. Yuan, 2001. The algorithm of interpolating windowed FFT for harmonic analysis of electric power system. IEEE Trans. Power Delivery, 16: 160-164.

Zhang, H., Y. Tu and P. Niu, 2007. Error analysis of phase difference measurement using FFT-based method and DTFT-based method. Chin. J. Electron. Meas. Instrum., 21: 61-65.