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Group Decision Making Approach Based on the Generalized Hybrid Harmonic Averaging Operators

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Abstract: Harmonic mean is suitable to aggregate negative indicators. In order to improve the flexibility of harmonic mean for modeling many different situations, this study provides a new class of operators called Generalized Ordered Weighted Harmonic Averaging (GOWHA) operators. Furthermore, the Generalized Hybrid Harmonic Averaging (GHHA) operators are developed based on the GOWHA operators which can reflect the importance degree of both the given arguments and the ordered position of the arguments. Then, an approach to multiple attribute group decision making is developed based on the GHHA operators. Finally, a numerical example shows that the developed approach is feasible. The research demonstrates the effectiveness of applying the developed approach to multiple attribute group decision making based on the GHHA operators.

Key words: Multiple attribute group decision making, harmonic mean, GOWHA operator, GHHA operator

INTRODUCTION

Aggregation operators, which can fuse several input arguments into a single representative output, have been disseminated throughout various fields including decision making (Beliakov *et al*., 2007; Chen *et al*., 2004, 2008; Jiang *et al*., 2010; Wang *et al*., 2013; Xu and Chen, 2007; Yager and Beliakov, 2010; Zhang *et al*., 2013), clustering (Jiang *et al*., 2010), combination forecasting (Chen *et al*., 2004), supply chain management and statistical regression (Yager and Beliakov, 2010). The Arithmetic Mean (AM), the Geometric Mean (GM) (Xu and Da, 2002) and the harmonic mean (HM) are three common aggregation operators.

It is worth mentioning that Yager introduced the Ordered Weighted Averaging (OWA) operator in 1988 (Yager, 1988; Torra and Narukawa, 2007) which assign the ith weight to the ith greatest input value. Since then, the OWA operator has been studied and applied by a lot of authors and various types of aggregation operators were proposed based on it. Induced Ordered Weighted Averaging (IOWA) operator was proposed based on the OWA operators in Yager and Filev (1999). Yager (2004a) developed the generalized OWA (GOWA) operator which combine the OWA operator with the generalized mean operator. Then, Merigo and Gil-Lafuente (2009) presented the Induced Generalized Ordered Weighted Averaging (IGOWA) operator which use order-inducing variables in the reordering process. In order to aggregate continuous interval value, the continuous OWA (C-OWA) operator (Yager, 2004b), the continuous ordered weighted geometric (C-OWG) operator (Yager and Xu, 2006) and the continuous ordered weighted harmonic (C-OWH) operator (Liu *et al*., 2013; Chen *et al*., 2008) were successively proposed. Zhou and Chen (2011) further extended the C-OWA operator and developed a new class of operators, called continuous generalized ordered weighted averaging (C-GOWA) operators. Moreover, the C-OWA operator, the C-OWG operator, the C-OWH operator and the continuous ordered weighted quadratic averaging (C-OWQ) operator are all the special cases of the C-GOWA operators. Considering the interrelationships among the input arguments, the Power operator and the Bonferroni mean operator were proposed (Zhou *et al*., 2012; Xu and Yager, 2011). Recently, Mesiar and Mesiarova-Zemankova (2011) introduced the Ordered Modular Averaging (OMA) operator (Chen *et al*., 2013) which is symmetric, idempotent and comonotone modular.

Compared with arithmetic mean operator and geometric mean operator, the main advantage of the Harmonic Mean (HM) aggregation operator is that it is suitable for directly aggregating negative indicators. Meanwhile, central tendency data can be aggregated by the HM aggregation operator (Jiang *et al*., 2010). Therefore, the induced ordered weighted harmonic averaging (IOWHA) operator was introduced by Chen *et al*. (2008) and then a combination forecasting method was developed based on the IOWHA operator. In

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order to aggregate fuzzy data, Wei (2011) proposed the fuzzy IOWHM (FIOWHM) operator and a method for fuzzy multi-attribute group decision making afterwards.

This study provides a generalization of the Ordered Weighted Harmonic Averaging (OWHA) operator by combining it with the generalized mean operator (Dyckhoff and Pedrycz, 1984). This combination leads to a class of operators which are denoted as the Generalized Ordered Weighted Harmonic Averaging (GOWHA) operators. It can be proved that the GOWHA operator is monotonic, commutative, idempotent and bounded. This study further consider the extension of the GOWHA operator to more general case in which the input argument values have different importance weights and introduce the Generalized Hybrid Harmonic Averaging (GHHA) operator. Then a new method based on the GHHA operator for multiple attribute group decision making is presented and a numerical example shows that the developed approach is feasible.

THE GENERALIZED OWHA OPERATOR

In this section, the generalized OWHA (GOWHA) operators are provided based on the ordered weighted harmonic averaging operator. Moreover, some properties of the GOWHA operators are discussed.

The GOWHA operator adds to the OWHA operator an additional parameter controlling the power to which the argument values are raised. It can be defined as follows.

Definition 1: A GOWHA operator of dimension n is a mapping GOWHA: $(R^+)^n \rightarrow R^+$ defined by an associated weighting vector $w = (w_1, w_2, ..., w_n)^T$ such that $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$ and a parameter $\lambda \in (-\infty, +\infty)$, $\lambda \neq 0$, according to the following Eq. 1:

$$GOWHA_{w,\lambda}(a_1, a_2, \cdots, a_n) = \left(1 \Big/ \sum_{j=1}^{n} \frac{w_j}{b_j^\lambda}\right)^{1/\lambda} \quad (1)$$

where, b_j is the jth largest element in the collection $(a_1, a_2, ..., a_n)$.

The GOWHA operator is monotonic, commutative, idempotent and bounded. These properties can be proved in the following theorems.

Theorem 1: (Monotonicity). Let g be the GOWHA operator, if $a_i \le c_i$ for all I, then:

$$g(a_1, a_2, ..., a_n) \le g(c_1, c_2, ..., c_n)$$

Proof: Let:

$$g(a_1, a_2, \cdots, a_n) = \left(1 \Big/ \sum_{j=1}^{n} \frac{w_j}{b_j^\lambda}\right)^{1/\lambda}$$

and

$$g(c_1, c_2, \cdots, c_n) = \left(1 \Big/ \sum_{j=1}^{n} \frac{w_j}{d_j^\lambda}\right)^{1/\lambda}$$

The function $g = \left(1 \Big/ \sum_{j=1}^{n} \frac{w_j}{b_j^\lambda}\right)^{1/\lambda}$ is taken the natural log, then:

$$\log g = -\frac{1}{\lambda}\log\left(\sum_{j=1}^{n} \frac{w_j}{b_j^\lambda}\right)$$

Moreover, log g is taken the partial derivative with respect to b_j, then:

$$\frac{\partial \log g}{\partial b_j} = \frac{1}{\lambda} \times \frac{1}{\sum_{j=1}^{n}(w_j/b_j^\lambda)} \times \frac{w_j}{b_j^{2\lambda}} \times \log b_j \times b_j^\lambda \ge 0$$

Since $\partial \log g / \partial b_j \ge 0$, then log g is monotonic with respect to b_j, i.e., g is monotonic with respect to b_j. From $a_i \le c_i$ for all i, it is obvious that $b_j \le d_j$ for all j. Thus:

$$g(a_1, a_2, ..., a_n) \le g(c_1, c_2, ..., c_n)$$

Theorem 2: (Commutativity). Let g be the GOWHA operator, then:

$$g(a_1, a_2, ..., a_n) = g(a_{\sigma(1)}, a_{\sigma(2)}, ..., a_{\sigma(1)}) \quad (2)$$

where, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of (1, 2, ..., n).

Proof: Let:

$$g(a_1, a_2, \cdots, a_n) = \left(1 \Big/ \sum_{j=1}^{n} \frac{w_j}{b_j^\lambda}\right)^{1/\lambda}$$

and

$$g(a_{\sigma(1)}, a_{\sigma(2)}, \cdots, a_{\sigma(n)}) = \left(1 \Big/ \sum_{j=1}^{n} \frac{w_j}{d_j^\lambda}\right)^{1/\lambda}$$

Because $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of (1, 2, ..., n), it can be seen that $b_j = d_j$ for all j and $g(a_1, a_2, ..., a_n) = g(a_{\sigma(1)}, a_{\sigma(2)}, ..., a_{\sigma(1)})$.

This implies that the initial indexing of the input arguments does not matter.

Theorem 3: (Idempotency). Let g be the GOWHA operator and $a_i = a$ (i = 1, 2, ..., n), then:

$$g(a_1, a_2, ..., a_n) = a \quad (3)$$

Proof: Let:

$$g(a_1, a_2, \cdots, a_n) = (1 \Big/ \sum_{j=1}^{n} \frac{w_j}{b_j^{\lambda}})^{1/\lambda}$$

If $a_i = a$ (i = 1, 2, ..., n), then:

$$g(a_1, a_2, \cdots, a_n) = (1 \Big/ \sum_{j=1}^{n} \frac{w_j}{a^{\lambda}})^{1/\lambda} = a$$

Theorem 4: (Boundedness). Let g be the GOWHA operator, $a^* = \max_i\{a_i\}$ and $a_* = \min_i\{a_i\}$. Then:

$$0 < a_* \leq g(a_1, a_2, ..., a_n) \leq a^*$$

Proof: Since $a^* = \max_i\{a_i\}$ and $a_* = \min_i\{a_i\}$, so $a_* \leq a^i \leq a^*$ for all i. From the monotonicity of the GOWHA operator, then:

$$g(a_*, a_*, \cdots, a_*) \leq g(a_1, a_2, \cdots, a_n) \leq g(a^*, a^*, \cdots, a^*) \quad (4)$$

According to theorem 3, then:

$$a_* \leq g(a_1, a_2, \cdots, a_n) \leq a^*$$

The satisfaction of these properties, commutativity, boundedness and monotonicity implies that the GOWHA operators are mean operators for any choice of λ and weighting vector $w = (w_1, w_2, ..., w_n)^T$ such that $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$.

THE GENERALIZED HYBRID HARMONIC AVERAGING OPERATOR

By combining the GOWHA operator and the hybrid weighted aggregation operator, this study develops the generalized hybrid harmonic averaging (GHHA) operator which can be defined as follows.

Definition 2: An GHHA operator of dimension n is a mapping GHHA: $(R^+)^n \rightarrow R^+$, according to the following formula:

$$GHHA_{w,\omega,\lambda}(a_1, a_2, \cdots, a_n) = (1 \Big/ \sum_{j=1}^{n} \frac{w_j}{H(a_{\sigma(j)})})^{1/\lambda} \quad (5)$$

where, $w = (w_1, w_2, ..., w_n)^T$ is an associated reciprocal weighting vector such that $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. σ: {1, 2, ..., n} being a permutation such that:

$$H(a_{\sigma(1)}) \geq H(a_{\sigma(2)}) \geq \cdots \geq H(a_{\sigma(n)}) \quad (6)$$

$H(a_{\sigma(j)})$ is the jth largest value in $\{a_1^{\lambda}/n\omega_1, a_2^{\lambda}/n\omega_2, \cdots, a_n^{\lambda}/n\omega_n\}$ and $\omega = (\omega_1, \omega_2, ..., \omega_n)$ is the weighting vector obtained by arguments a_j(j = 1, 2, ..., n), satisfying $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$ and n is called the balanced factor.

The GHHA operator is a generalization of the GWHA and GOWHA operators. Especially:

- If $w = (1/n, 1/n, ..., 1/n)^T$, then:

$$GHHA_{w,\omega}(a_1, a_2, \cdots, a_n) = (1 \Big/ \sum_{i=1}^{n} \frac{\omega_j}{a_j^{\lambda}})^{1/\lambda}$$

$$= GWHA_{\omega}(a_1, a_2, ..., a_n)$$

- If $\omega = (1/n, 1/n, ..., 1/n)^T$, then:

$$GHHA_{w,\omega}(a_1, a_2, \cdots, a_n) = GOWHA_w(a_1, a_2, \cdots, a_n)$$

Consequently, the GHHA operator can reflect the importance degrees of both the given arguments and the ordered position of the arguments.

AN APPROACH BASED ON THE GHHA OPERATOR TO GROUP DECISION MAKING

The GOWHA operator and the GHHA operator can be applied in a wide range of areas such as statistics, economics, the selection of financial products, engineering, soft computing and decision theory.

In this section, the study presents an approach to multiple attribute group decision making based on the GHHA operator and the WHA operator proposed in this study. For a multi-attribute group decision making problem, it is assumed that $X = \{x_1, x_2, ..., x_n\}$ is the feasible alternative set, $U = \{u_1, u_2, ..., u_m\}$ is a finite set of attributes with $\omega = (\omega_1, \omega_2, ..., \omega_n)$ being its weighting vector which is used to represent the importance weights of different attributes, where $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$. Let $D = \{d_1, d_2, ..., d_t\}$ be the set of decision makers, $\eta = (\eta_1, \eta_2, ..., \eta_t)^T$ be the weighting vector of decision makers which is used to represent the importance weights of decision makers, where $0 \leq \eta \leq 1$ and $\sum_{j=1}^{t} \eta_j = 1$. In group decision making procedure, the scoring value $a_{ij}^{(k)}$ is presented for alternative $x_i \in X$ with respect to the attribute $u_j \in U$ by the decision maker $d_k \in D$. Thus, the decision matrix $A^{(k)} = (a_{ij}^{(k)})_{n \times m}$ can be obtained by the decision maker $d_k \in D$.

The procedure of group decision making: To get the best alternative in the group decision making, the following steps are involved:

Step 1: Let U_1 be a subscript set of cost attributes and U_2 be a subscript set of benefit attributes in the decision making problems. In order to transform positive indicator to negative indicator, the approach of normalizing decision matrix is proposed with the purpose of measuring all attributes in dimensionless units and facilitating inter-attribute comparisons, as follows:

$$r_{ij}^{(k)} = a_{ij}^{(k)} \Big/ \sqrt{\sum_{i=1}^{n} \left(a_{ij}^{(k)}\right)^2}, \text{ if } i \in \{1, 2, \cdots, n\}, j \in U_1$$

$$r_{ij}^{(k)} = \left(\frac{1}{a_{ij}^{(k)}}\right) \Big/ \sqrt{\sum_{i=1}^{n} (1/a_{ij}^{(k)})^2}, \text{ if } i \in \{1, 2, \cdots, n\}, j \in U_2$$

Therefore, every decision making matrix $A^{(k)} = (a_{ij}^{(k)})_{n\times m}$ is converted into a normalized decision making matrix $R^{(k)} = (r_{ij}^{(k)})_{n\times m}$

Step 2: Utilize the GHHA aggregation operator:

$$z_{ij} = GHHA_{w,\eta,\lambda}(r_{ij}^{(1)}, r_{ij}^{(2)}, \cdots, r_{ij}^{(t)})$$

$$= \left(1 \Big/ \sum_{k=1}^{t} \frac{w_j}{H(r_{ij}^{\sigma(k)})}\right)^{1/\lambda} \quad i = 1, 2, \cdots, n, \; j = 1, 2, \cdots, m \tag{7}$$

to aggregate preference information of all decision makers into a collective decision matrix $Z = (z_{ij})_{n\times m}$, where $H(r_{ij}^{\sigma(k)})$ is the kth largest value in $\{(r_{ij}^{(1)})^\lambda / t\eta_1, (r_{ij}^{(2)})^\lambda / t\eta_2, \cdots, (r_{ij}^{(t)})^\lambda / t\eta_t\}$ and w is the weight vector which can be obtained by the fuzzy linguistic quantifiers

Step 3: Utilize the WHA aggregation operator to obtain the comprehensive attribute value of alternative $x_i \in X$ from collective decision matrix $Z = (z_{ij})_{n\times m}$, as follows:

$$z_i(\omega) = 1 \Big/ \sum_{j=1}^{m} \frac{\omega_j}{z_{ij}}, \; i = 1, 2, \cdots, n \tag{8}$$

Step 4: Rank the comprehensive attribute value $z_i(\omega)$ in increasing order. The smaller the comprehensive attribute value of the alternative is, the better the alternative is. Then the alternatives $x_i \in X$ can be ranked according to $z_i(\omega)$, i = 1, 2, ..., n

Step 5: The end

Numerical example: Now, this study presents an illustrative example of the new approach applied in a group decision-making problem. A petroleum group board of directors wants to choose an investment project from four alternatives. The main evaluation indexes of these investment projects are as follows:

- Investment recovery period (u_1)
- Fixed assets investment (u_2)
- Comprehensive energy consumption (u_3)
- Investment risk loss value (u_4)
- Management cost (u_5)

Suppose the weighting vector of five attributes is ω = (0.25, 0.15, 0.25, 0.20, 0.15)^T and a set of four alternatives $\{x_1, x_2, x_3, x_4\}$. These alternatives are evaluated and scored under the above five attributes by three experts whose weighting vector is η = (0.4, 0.3, 0.3) and the expert set is $\{d_1, d_2, d_3\}$. It is obvious that evaluation indexes are all cost attributes which means that the higher of the score the lager of cost price. Then the decision matrices $R^{(k)} = (r_{ij}^{(k)})_{4\times 5}$ (k = 1, 2, 3) can be obtained which are listed in Table 1-3:

Step 1: Since the attributes are all cost indexes and all of them are measured with the same dimension units by scores ranging from 0 to 100. Hence, the decision matrixes need not to be normalized. Assume that the controlling parameter λ = 2. The associated weighting vector w = (0.067, 0.0666, 0.267)^T can be obtained by the fuzzy linguistic quantifier "Most" with the pair (α, β). From Eq. 7, then:

Table 1: The decision matrix $R^{(1)}$ given by expert d_1 according to five evaluation indexes

Alternatives	u_1	u_2	u_3	u_4	u_5
x_1	85	76	85	78	90
x_2	72	77	85	80	96
x_3	81	75	80	85	75
x_4	90	72	78	79	88

u_1, u_2, u_3, u_4 and u_5 are five evaluation indexes, respectively. x_1, x_2, x_3 and x_4 are four alternatives, respectively

Table 2: The decision matrix $R^{(2)}$ given by expert d_2 according to five evaluation indexes

Alternatives	u_1	u_2	u_3	u_4	u_5
x_1	84	86	84	77	90
x_2	80	94	95	80	88
x_3	76	85	90	77	75
x_4	84	84	85	85	85

u_1, u_2, u_3, u_4 and u_5 are five evaluation indexes, respectively. x_1, x_2, x_3 and x_4 are four alternatives, respectively

Table 3: The decision matrix $R^{(3)}$ given by expert d_3 according to five evaluation indexes

Alternatives	u_1	u_2	u_3	u_4	u_5
x_1	96	79	85	90	83
x_2	76	88	87	97	78
x_3	85	75	80	93	96
x_4	85	90	80	81	89

u_1, u_2, u_3, u_4 and u_5 are five evaluation indexes, respectively. x_1, x_2, x_3 and x_4 are four alternatives, respectively

$z_{11} = 85.8004$; $z_{12} = 79.1395$; $z_{13} = 85.2366$
$z_{14} = 78.7613$; $z_{15} = 86.3842$; $z_{21} = 75.5925$
$z_{22} = 85.0438$; $z_{23} = 87.6321$; $z_{24} = 81.6150$
$z_{25} = 86.3860$; $z_{31} = 78.8222$; $z_{32} = 76.2790$
$z_{33} = 81.3349$; $z_{34} = 81.0004$; $z_{35} = 76.6823$
$z_{41} = 86.7598$; $z_{42} = 80.5303$; $z_{43} = 80.4089$
$z_{44} = 81.3675$; $z_{45} = 87.0558$

u_1, u_2, u_3, u_4 and u_5 are five evaluation indexes, respectively. x_1, x_2, x_3 and x_4 are four alternatives, respectively

Step 2: Utilize the WHA aggregation operator to derive the comprehensive attribute value of each alternative:

$$z_1(\omega) = 83.2093;\ z_2(\omega) = 82.5712$$
$$z_3(\omega) = 79.1320;\ z_4(\omega) = 83.0958$$

Step 3: Rank the comprehensive attribute value $z_i(\omega)$ $(i = 1, 2, 3, 4)$:

$$z_3(\omega)<z_2(\omega)<z_4(\omega)<z_1(\omega)$$

Then, the alternatives $x_i \in X$ could be ranked according to $z_i(\omega)$, I = 1, 2, ..., n

$$x_3 \succ x_2 \succ x_4 \succ x_1$$

Thus, it is concluded that the optimal one is x_3.

The study compares the GHHA operators with the Weighted Harmonic Averaging (WHA) operator (Beliakov *et al*., 2007), the Quasi Ordered Weighted Averaging (QOWA) operators with $f(x) = e^x$ (Liu *et al*., 2013), the Generalized ordered weighted averaging (GOWA) operators for $\lambda = 3$ (Yager, 2004a) and the Bonferroni mean (BM) operators (Xu and Yager, 2011). Table 4 presents different results obtained by using the five different types of aggregation operators.

According to Table 4, an ordering of the alternatives can be established. The ranking results are shown in Table 5.

It can be seen that the ordering of the alternatives may be different when different types of aggregation operators are used. But these results may lead to the same decisions.

Furthermore, it is possible to analyze how the different parameter λ plays a role in the aggregation results. It is necessary to consider different values of λ: -100, -93, -86, ..., -2, 5, 12, ..., 89, 96 which are provided by the decision maker. The aggregation results of comprehensive attribute value $z_i(\omega)$ are shown in Fig. 1.

It is observed from Fig. 1 that the aggregation results decrease as λ increases when $\lambda<0$ and also decrease as λ increases when $\lambda>o$. From Fig. 1, It can be seen that the ranking of the alternatives is different, thus leading to different decisions. However, it seems that x_4 is the best choice when $\lambda \le -14$ and x_4 is the best choice when $\lambda \ge 1$. Due to the fact that each particular family of GHHA operator may give different results, the decision maker will select for his decision the one that is closest to his interest.

Table 4: Comprehensive attribute values of four alternatives obtained by five different types of aggregation operators

Alternatives	WHA	QOWA	GOWA	BM	GHHA
x_1	83.1333	82.0343	85.9012	82.1283	83.2075
x_2	81.3796	83.4792	81.8645	81.4792	82.5696
x_3	79.9324	81.1287	78.8798	78.4841	79.1298
x_4	83.5322	83.9384	83.1020	79.6354	83.0946

WHA: Weighted harmonic averaging operator, QOWA: Quasi ordered weighted averaging operator, GOWA: Generalized ordered weighted averaging operator, BM: Bonferroni mean operator, GHHA: Generalized hybrid harmonic averaging operator

Table 5: Ranking of the alternatives based on the comprehensive attribute values obtained by five different operators

Operators	Ranking
WHA	$x_3 \succ x_2 \succ x_1 \succ x_4$
QOWA	$x_3 \succ x_1 \succ x_2 \succ x_4$
GOWA	$x_3 \succ x_2 \succ x_4 \succ x_1$
BM	$x_3 \succ x_4 \succ x_2 \succ x_1$
GHHA	$x_3 \succ x_2 \succ x_4 \succ x_1$

WHA: Weighted harmonic averaging operator, QOWA: Quasi ordered weighted averaging operator, GOWA: Generalized ordered weighted averaging operator, BM: Bonferroni mean operator, GHHA: Generalized hybrid harmonic averaging operator

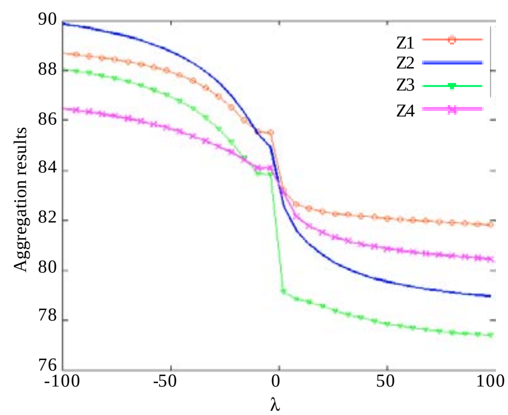

Fig. 1: Variations of the aggregation results with parameter λ

CONCLUSION

This study has introduced the generalized ordered weighted harmonic averaging (GOWHA) operator. It provides a very general formulation that includes a wide range of aggregation operators and it was proved to be monotonic, commutative, idempotent and bounded. Moreover, some monotone properties with respect to the reciprocal weighting vector of the GOWHA operator and the OWHA operator have been studied. The GOWHA operator was extended to the situation which not only focuses on the degree of importance with input arguments, but also considers the serial positions of the input arguments. Then the GHHA operator based on the GOWHA operator were proposed. These operators were applied to multiple attribute group decision making. The numerical example is illustrated to show the feasibility and effectiveness of applying these operators to the problem of group decision making. The application of the developed operators in other fields is a promising direction for future research.

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REFERENCES

Beliakov, G., A. Pradera and T. Calvo, 2007. Aggregation Functions: A Guide for Practitioners. Vol. 221, Springer-Verlag, Berlin, Germannnny, ISBN-13: 9783540737216, Pages: 361.

Chen, H.Y., C.L. Liu and Z.H. Sheng, 2004. Induced ordered weighted harmonic averaging (IOWHA) operator and its application to combination forecasting method. Chin. J. Manage. Sci., 12: 35-40.

Chen, H.Y., J.P. Liu and H. Wang, 2008. A class of continuous ordered weighted harmonic(C-OWH) averaging operators for interval argument and its applications Syst. Eng. Theor. Pract., 28: 86-92.

Chen, L., Z. Xu and X. Yu, 2013. The ordered multiplicative modular geometric operator. Knowl. Based Syst., 39: 144-150.

Dyckhoff, H. and W. Pedrycz, 1984. Generalized means as model of compensative connectives. Fuzzy Sets Syst., 14: 143-154.

Jiang, H., S. Yi, J. Li, F. Yang and X. Hu, 2010. Ant clustering algorithm with *K*-harmonic means clustering. Expert Syst. Appl., 37: 8679-8684.

Liu, J.P., S. Lin, H.Y. Chen and Q. Xu, 2013. Penalty-based continuous aggregation operators and their application to group decision making. Knowl. Based Syst., 37: 490-501.

Merigo, J.M. and A.M. Gil-Lafuente, 2009. The induced generalized OWA operator. Inform. Sci., 179: 729-741.

Mesiar, R. and A. Mesiarova-Zemankova, 2011. The ordered modular averages. IEEE Trans. Fuzzy Syst., 19: 42-50.

Torra, V. and Y. Narukawa, 2007. A view of averaging aggregation operators. IEEE Trans. Fuzzy Syst., 15: 1063-1067.

Wang, H.J., X.F. Zhao, R. Lin and G.W. Wei, 2013. Some extended harmonic aggregation operators with 2-tuple linguistic information and their application to potential evaluation of emerging technology commercialization. JDCTA, 7: 668-676.

Wei, G.W., 2011. FIOWHM operator and its application to multiple attribute group decision making. Expert Syst. Appl., 38: 2984-2989.

Xu, Z.S. and Q.L. Da, 2002. The ordered weighted geometric averaging operators. Int. J. Intell. Syst., 17: 709-716.

Xu, Z.S. and J. Chen, 2007. An interactive method for fuzzy multiple attribute group decision making. Inform. Sci., 177: 248-263.

Xu, Z. and R.R. Yager, 2011. Intuitionistic fuzzy bonferroni means. IEEE Trans. Syst. Man Cybern. B, 41: 568-578.

Yager, R.R., 1988. On ordered weighted averaging aggregation operators in multicriteria decisionmaking. IEEE. Trans. Syst. Man Cybernet., 18: 183-190.

Yager, R.R. and D.P. Filev, 1999. Induced ordered weighted averaging operators. IEEE Trans. Syst. Man Cybernet. Part B: Cybernet., 29: 141-150.

Yager, R.R., 2004a. Generalized OWA aggregation operators. Fuzzy Optim. Decis. Mak., 3: 93-107.

Yager, R.R., 2004b. OWA aggregation over a continuous interval argument with applications to decision making. Trans. Syst. Man Cyber., 34: 1952-1963.

Yager, R.R. and Z. Xu, 2006. The continuous ordered weighted geometric operator and its application to decision making. Fuzzy Sets Syst., 157: 1393-1402.

Yager, R.R. and G. Beliakov, 2010. OWA operators in regression problems. Trans. Fuzzy Syst., 18: 106-113.

Zhang, Q.S., F.C. Liu and Y.R. Huang, 2013. Early warning index weight assignment and decision-making mechanism for city significant emergency in uncertain environment. Int. J. Dig. Cont. Technol. Appl., 7: 327-335.

Zhou, L., H. Chen and J. Liu, 2012. Generalized power aggregation operators and their applications in group decision making. Comput. Indust. Eng., 62: 989-999.

Zhou, L.G. and H.Y. Chen, 2011. Continuous generalized OWA operator and its application to decision making. Fuzzy Sets Syst., 168: 18-34.