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Impact of Cellular Environment on the Generalized Efficiency of Molecular Motor

T.F. Gao and R.Z. Huang

College of Physical Science and Technology, Shenyang Normal University, Shenyang, 110034, China

Abstract: Molecular motors, identified in a variety of conditions ranging from biological to synthetic systems, extract work from thermal fluctuations in out-of-equilibrium conditions. Many cellular processes require molecular motors to produce motion and forces. Based on a general model of molecular motors, the generalized efficiency of a thermal Brownian motor is calculated analytically. The results obtained here are of general significance. They can be used to analyze the performance characteristics of the Brownian motors operating in the zero heat leak case.

Key words: Molecular motor, irreversibility, heat leak, generalized efficiency

INTRODUCTION

Molecular motors are proteins that convert the chemical energy of ATP hydrolysis into mechanical motion (Holzbaur and Goldman, 2010). This motion is utilized for a wide variety of crucial functions *in vivo*, among the most significant of which is the transport of cargo within the cell. For example, kinesin and myosin V transport biological materials along microtubules and actin filaments, respectively. Myosin II is responsible for muscle contraction and is involved in cell locomotion. A more recent class of theoretical models describe how directed motion occurs, estimate bounds to their efficiency, provide typical force-velocity relations and investigate the role of fluctuations. The idea of a Brownian motor working due to nonuniform temperature first came up with the works of (Buttiker, 1987; Landauer, 1988). After analyzing the heat flow in a Brownian motor (Derenyi and Astumian, 1999) found that the efficiency can approach the Carnot engine in principle. Recently, (Hondou and Sekimoto, 2000) claimed that such a heat engine cannot attain the Carnot efficiency due to the irreversible heat flow. The study extends the previous works (Gomez-Marin and Sancho, 2006; Asfaw and Bekele, 2004; Sakaguchi, 1998) and establishes a more general model of molecular motors.

MATERIALS AND METHODS

The model of a thermal Brownian motor moved in a spatially asymmetric but periodic potential with an external load force f. The viscous medium is alternately in contact with the hot and cold heat reservoirs (Zhang *et al.*, 2006). The shape of a single sawtooth potential, $U_s(x)$, is described by:

$$U_s(x) = \begin{cases} U_0\left(\dfrac{x}{L_1}+1\right), & (-L_1 \leq x < 0) \\ U_0\left(\dfrac{-x}{L_2}+1\right), & (0 \leq x < L_2) \end{cases} \quad (1)$$

where, L_1 and L_2 are the lengths of the left and right sides of a sawtooth potential and U_0 is the height of the potential barrier. The temperature profile, $T(x)$, within the interval $-L_1 \leq x < L_2$ is described by:

$$T(x) = \begin{cases} T_h, & (-L_1 \leq x < 0) \\ T_c, & (0 \leq x < L_2) \end{cases} \quad (2)$$

where, T_h and T_c are the temperatures of the hot and cold heat reservoirs, respectively. Both $U_s(x)$ and $T(x)$ are taken to have the same period where, $L = L_1 + L_2$ is the period of the sawtooth potential.

Because the dynamic equation depends on the specific environment to which the particle is exposed (Fekade and Bekele, 2002; Dufty and Brey, 2005), one can find that the constant current at a steady state is given by Asfaw and Bekele (2004):

$$J = \frac{-F}{G_1 G_2 + (A_1 + A_2 + A_3)F} \quad (3)$$

Corresponding Author: Tianfu Gao, College of Physical Science and Technology, Shenyang Normal University, 110034, Shenyang, China Tel: 86-024-86593288 Fax: 86-024-86575015

When the thermodynamic forces in the Brownian motor:

$$X_1 = \frac{f}{T_c}$$

and:

$$X_2 = \frac{T_h - T_c}{T_c^2} = \frac{\Delta T}{T_c^2}$$

are very small, the constant current may be simplified into:

$$J = -C[X_1 - (U_0/L)X_2] \qquad (4)$$

Where:

$$C = \frac{T_c}{\gamma L} \left[\frac{\alpha}{\sinh(\alpha)} \right]^2$$

and $\alpha = U_0/(2T_c)$. By using Eq. 4, the particle's average drift velocity:

$$v = JL \qquad (5)$$

can be conveniently calculated.

In the system, there are two heat flows. One is the heat leak between the hot and the cold regions. For a thermal Brownian particle, the heat leak between the regions is $k\Delta T$ where, k is the coefficient of heat leak. The other is the temperature difference that makes the particles move from the high-to the low-temperature reservoir. $(U_0 + fL_1) + \gamma vL_1$ is the energy needed by a particle to climb up the equivalent potential barrier and overcome the viscous drag force. On the other hand, the particle will recross the boundary between the two regions. The kinetic energy change due to the Brownian motor is equal to $\Delta T/2$ (Zhang *et al.*, 2006). Thus, during the time $t = L/v$, the total heat transferred from the hot reservoir to the system is given by:

$$Q_h = U_0 + (\gamma v + f)L_1 + \Delta T/2 + tk\Delta T \qquad (6)$$

Similarly, the total heat transferred from the system to the cold reservoir is given by:

$$Q_c = U_0 - (\gamma v + f)L_2 + \Delta T/2 + tk\Delta T \qquad (7)$$

From Eq. 6 and 7, one can find the net work, W, done by the Brownian motor in one cycle is:

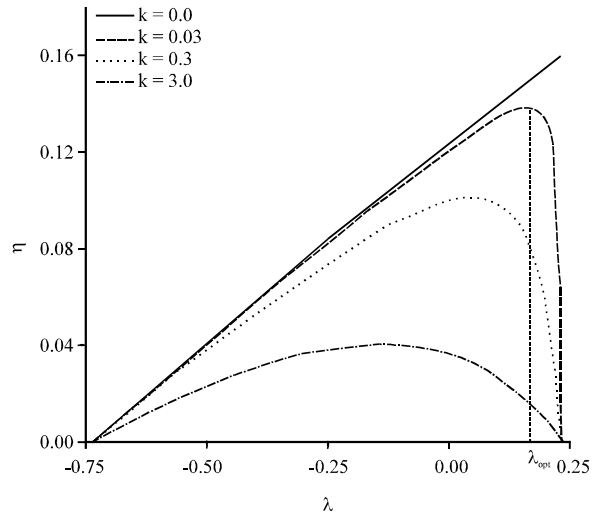$$W = (\gamma v + f)L \qquad (8)$$

Fig. 1: The curves of generalized efficiency η varying with scaled load λ for different coefficients of heat leak k

Based on Eq. 6 and 8, the generalized efficiency of Brownian motor may be defined as:

$$\eta = \frac{W}{Q_h} = \frac{(\gamma v + f)L}{U_0 + (\gamma v + f)L_1 + \frac{\Delta T}{2} + k(L/v)\Delta T} \qquad (9)$$

In order to explore how the generalized efficiency of the system depends on some quantities characterizing the cellular environment, some new parameters:

$$\beta = \frac{T_h}{T_c}$$

and:

$$\lambda = f\frac{L_1}{T_c}$$

is introduced. Using Eq. 3, 5 and 9, the characteristic curves of Brownian motor can be plotted.

Figure 1 indicates the influence of scaled load λ on the generalized efficiency for some given values of the parameters, $\beta = 1.2$, $1 = 0.6$ and $u = 2$. It shows that when $k = 0$, the efficiency increases linearly with the increase of λ. When $k>0$, the efficiency is not a monotonic function of λ. Therefore, only for an appropriate match between λ and k, can the higher efficiency appear. With the increase of k, the maximum efficiency and the corresponding optimized λ will decrease. When λ is equal to λ_{min} and λ_{max}, which is called the stall force (Kostur *et al.*, 2006), to the current is equal to zero and consequently, the efficiency
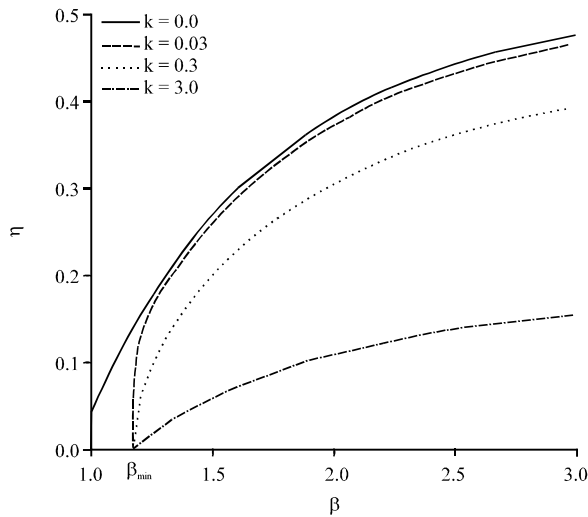
Fig. 2: The curves of generalized efficiency η versus temperature ratio of two reservoirs β for different coefficients of heat leak k

is always equal zero. It implies that for a Brownian motor, the range of λ is subjected to $\lambda_{min}<\lambda<\lambda_{max}$.

Figure 2 shows the plots of the generalized efficiency versus a temperature ratio of two reservoirs for $l = 0.6$, $u = 2$ and $\lambda = 0.2$. The results show that the generalized efficiency of thermal Brownian motor increases as the temperature ratio β is increased. When $k>0$, there exists a minimum of β at which the generalized efficiency is equal to zero. It can be clearly observed that when the temperature ratio β is small ($\beta<\beta_{min}$), the negative efficiency is observed and the Brownian heat engine can not perform a cycle.

RESULTS AND DISCUSSION

When the heat leak in the system is negligible, $k = 0$ and Eq. 9 may be simplified into:

$$\eta = \frac{(\gamma v + f)L}{U_0 + (\gamma v + f)L_1 + \frac{\Delta T}{2}} \qquad (10)$$

Equation 10 shows clearly that the efficiency of thermal Brownian motor is always lower due to the kinetic energy change.

When the kinetic energy change due to the particle motion is further ignored, Eq. 10 may be written as:

$$\eta = \frac{(\gamma v + f)L}{U_0 + (\gamma v + f)L_1} \qquad (11)$$

It can be clearly obtained that thermal Brownian motor can not attain the Carnot efficiency with the consideration of the heat leak and the kinetic energy change. Furthermore, the model adopted in previous work (Asfaw and Bekele, 2004) is only a special case of the present model so that all the results in the work (Asfaw and Bekele, 2004) can be directly derived from the present paper.

CONCLUSION

On the basis of the general model of Brownian motors restricted in a periodic asymmetric piecewise-linear potential and contacted alternately with the hot and cold heat reservoirs along the space coordinate, the influence of the cellular environment, such as the external load and external heat reservoirs on the performance of the system have been systemically analyzed. General expressions for the generalized efficiency of the system have been analytically derived. It is expected that these general results may be put to use in self-propelling micromachines or pumps and mixers for microfluidics (Mayor *et al.*, 2005).

REFERENCES

Asfaw, M. and M. Bekele, 2004. Current, maximum power and optimized efficiency of a Brownian heat engine. Eur. Phys. J. B: Condens. Matter Complex Syst., 38: 457-461.

Buttiker, M., 1987. Transport as a consequence of state-dependent diffusion. Zeitschrift fur Physik B Condensed Matter, 68: 161-167.

Derenyi, I. and R.D. Astumian, 1999. Efficiency of Brownian heat engines. Phys. Rev. E, 59: R6219-R6222.

Dufty, J.W. and J.J. Brey, 2005. Brownian motion in a granular fluid. New J. Phys.,7: 20.

Fekade, S. and M. Bekele, 2002. Model-dependent nature of blowtorch effect on the escape and equilibration rates. Eur. Phys. J. B: Condens. Matter Complex Syst., 26: 369-374.

Gomez-Marin, A. and J.M. Sancho, 2006. Tight coupling in thermal Brownian motors. Phys. Rev. E, 74: 062102.

Holzbaur, E.L.F. and Y.E. Goldman, 2010. Coordination of molecular motors: From *in vitro* assays to intracellular dynamics. Curr. Opin. Cell Biol., 22: 4-13.

Hondou, T. and K. Sekimoto, 2000. Unattainability of Carnot efficiency in the Brownian heat engine. Phys. Rev. E, 62: 6021-6025.

Kostur, M., L. Machura, P. Hanggi, J. Luczka and P. Talkner, 2006. Forcing inertial Brownian motors: Efficiency and negative differential mobility. Phys. A: Stat. Mech. Appl., 371: 20-24.

Landauer, R., 1988. Motion out of noisy states. J. Stat. Phys., 53: 233-248.

Mayor, P., G. D'Anna, A. Barrat and V. Loreto, 2005. Observing Brownian motion and measuring temperatures in vibration-fluidized granular matter. New J. Phys., 7: 28.

Sakaguchi, H., 1998. A Langevin simulation for the Feynman Ratchet model. J. Phys. Soc. Jpn., 67: 709-712.

Zhang, Y., B.H. Lin and J.C. Chen, 2006. Performance characteristics of an irreversible thermally driven Brownian microscopic heat engine. Eur. Phys. J. B: Condens. Matter Complex Syst., 53: 481-485.