Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 13 (9): 1652-1656, 2013
ISSN 1812-5654 / DOI: 10.3923/jas.2013.1652.1656

Random Vibration Analysis of Semi-trailer Frame Based on Virtual Excitation Method

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Abstract: To determine the structure response of a vehicle frame under random road excitations. A mathematical model was been established using the Euler-Bernoulli beam elements and spring-damper elements for the Multiple-degree-of-freedom (MDOF) vibration of the vehicle frame with a finite element method and adopted a virtual excitation method to construct virtual road excitations based on four points, i.e., front and rear wheels, thus we offered a method for calculating the statistical characteristic of the random vibration response of the vehicle frame and realized characteristic analysis of the random vibration of the vehicle frame based on the virtual excitation method. Through example analysis, we obtained the structure response of the vehicle frame at different vehicle speeds.The results show that the main peak of structure resonance were primarily concentrated within 2 Hz. The response of the vibration on the frame is larger with vehicle velocity.

Key words: Virtual excitation method, random vibration, semi-trailer frame, response spectral

INTRODUCTION

The fatigue strength of the frame of transport vehicle under random road excitations is a main aspect requiring consideration in design (Pan *et al*., 2003) and has also been a difficult point and hot spot studied in academic circle and engineering world in recent years. In References (Zhang *et al*., 2006; Li *et al*., 2010a), a virtual excitation method was used to conduct random vibration analysis of the model for whole vehicle. The fatigue analysis and study of the main member, i.e., the vehicle frame, under random excitations have been constantly explored and improved and there has not been highly effective work done for the key sector, that is, stress spectrum calculation. As an innovative theoretical method with high efficiency and precision, the virtual excitation method provides a new and effective way for random vibration theory to be more widely used in engineering fields. Through near twenty years of development, the random vibration theory has been widely used in the engineering fields such as civil engineering, ocean engineering, earthquake engineering and vehicle engineering, etc. (Li *et al*., 2010b). In this study, we used Euler-Bernoulli beam elements and spring-damper elements to establish a mathematical model for the Multiple-Degree-Of-Freedom (MDOF) vibration of a vehicle frame with a finite element method and adopted a virtual excitation method to realize Random Vibration analysis of the frame and to provide a feasible way for fatigue analysis of the frame under random excitations.

VIBRATION MODEL OF VEHICLE FRAME

The study object in this study was XZ1210 transport vehicle, of which the frame is consists of 2 main girder , 19 cross girder, side girder and bracket etc. The main girder adopted variable cross-section gooseneck structure. The frame body was simulated with Euler-Bernoulli beam elements (Tang and Huang, 1999) With unsprung mass being neglected and with the stiffness and damping of tires and those of springs being jointly considered, we employed spring elements for simulation and selected every spring-damper in parallel as an element (Wang *et al*., 2010). Thus the complex vehicle frame structure could be simplified as vibration model as shown in Fig. 1. Then we can create the vibration equation of the whole frame:

$$\begin{gathered}[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K]\{X(t)\}=\{F(t)\}\\ \{F(t)\}=[C_q]\{\dot{q}(t)\}+[K_q]\{q(t)\}\end{gathered} \quad (1)$$

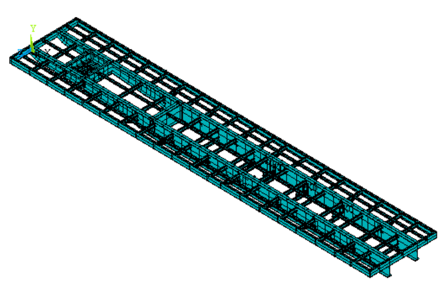

Fig. 1: Vibration model of vehicle frame

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were, [M], [C], [K] are the lumped mass matrix, damping matrix and, stiffness matrix {X(t)} is the node displacement {q(t)} is road excitations. $\lfloor K_q \rfloor$ and $\lfloor C_q \rfloor$ a are the spring stiffness matrix and damping matrix (Jiang and Li, 2011).

VIRTUAL EXCITATION METHOD

Basic principle: According to the vibration theory (Lin and Zhang, 2004), the frequency response function of a linear system with constant coefficient under zero initial conditions was the complex amplitude ratio of the input harmonic quantity to the output harmonic quantity, that is:

$$H(\omega)=\frac{X_\omega(t)}{Y_\omega(t)} \tag{2}$$

where, H (ω) is the frequency response function; $X_\omega(t)$ is the Fourier transform of response X (t) and $X_\omega(t)$ is the Fourier transform of excitation y(t).

When a linear system was subjected to the action of a single-point stable random excitation source F(t) with an auto power spectral density of S_{FF} (ω), the auto power spectra of its response X(t) were:

$$S_{XX}(\omega) = |H|^2 S_{FF}(\omega) \tag{3}$$

where, H is the frequency response function, of which the meaning is that when the random excitation is replaced by unit excitation $e^{i\omega t}$, the corresponding simple harmonic response should be $X(t) = He^{i\omega t}$. If the excitation $e^{i\omega t}$ was pre-multiplied by a constant $\sqrt{S_{FF}}$, that is, a virtual excitation $\tilde{F}(t)=\sqrt{S_{FF}}e^{i\omega t}$ was constructed, then the corresponding virtual response would be:

$$\tilde{X}(t)=\sqrt{S_{FF}}H(\omega)e^{i\omega t} \tag{4}$$

Thus, the calculation equations for the auto power spectral density and cross power spectral density of the actual response were as follows:

$$\tilde{X}^*\tilde{X}=\left|\tilde{X}\right|^2=|H|^2 S_{FF}=S_{XX} \tag{5}$$

$$\tilde{F}^*\tilde{X}=\sqrt{S_{FF}}e^{-i\omega t}\sqrt{S_{FF}}He^{i\omega t}=S_{FF}H=S_{FX} \tag{6}$$

$$\tilde{X}^*\tilde{F}=\sqrt{S_{FF}}H^*e^{-i\omega t}\sqrt{S_{FF}}He^{i\omega t}=H^*S_{FF}=S_{XF} \tag{7}$$

where, S_{XX} is the auto power spectral density of the actual response; S_{FX} is the cross power spectral density of the actual excitation with the actual response and S_{XX} is the cross power spectral density of the actual response with the actual excitation. If there were multiple response variables, the following calculation equation for the power spectrum matrix could be gained from Eq. 2-3:

$$S_{FF}=\tilde{X}^*\tilde{X}^T,\ S_{FX}=\tilde{F}^*\tilde{X}^T,\ S_{XF}=\tilde{X}^*\tilde{F}^T$$

where, "*" means complex conjugation and "T" means transposition.

Basic equations in virtual excitation method for multi-point excitation: If it was assumed that a linear structure with n degrees of freedom was subjected to the action of an m-point partial-coherence stable random excitation {x(t)} and its auto power spectral density $[S_{XX}(\omega)]$ was known (Sun and Wang, 2006), then $[S_{XX}(\omega)]$ could be decomposed into:

$$S_{xx}(\omega)=\sum_{j}^{m}\lambda_j\{\psi\}_j\{\psi\}_j^{*T} \tag{8}$$

where, λ_j and $\{\Psi\}_j$ are the characteristic value and characteristic vector of $[S_{XX}(\omega)]$, respectively and * is the conjugate of the function.

Every characteristic value and its corresponding characteristic vector were employed to construct a virtual excitation corresponding to the excitation {x(t)}:

$$\{\tilde{x}(t)\}_j=\{\psi\}_j^*\sqrt{\lambda_j}e^{i\omega t} \tag{9}$$

Where:

$$\{\tilde{x}(t)\}_j=\{\tilde{x}_1,\tilde{x}_2,\cdots,\tilde{x}_m\}^T$$

The virtual response under the action of the virtual excitation $\{\tilde{x}(t)\}_j$ could be constructed from Eq. 1:

$$\{\tilde{y}(t)\}_j=[H(\omega)]\{\tilde{x}(t)\}_j \tag{10}$$

The power spectral density of the actual response was:

$$S_{yy}(\omega)=\sum_{j=1}^{m}\{\tilde{y}(t)\}_j^*\{\tilde{y}(t)\}_j^T \tag{11}$$

ROAD EXCITATION

Power spectra of road excitation: If the road roughness was assumed as a spatial stable random process and its statistical characteristic could be expressed with power spectral density (Xu *et al*., 2012), the spatial frequency spectral density Gq(n) of the road roughness was:

$$G_q(n) = G_q(n_0)\left(\frac{n}{n_0}\right)^{-W}$$

where, n is the spatial frequency; n_0 = 0.1 m^{-1} is the reference spatial frequency; Gq (n_0) is the power spectrum of the road roughness at the reference spatial frequency and w is the frequency index which determines the frequency structure of the road surface spectra.

In addition to the road roughness, travel speed should also be considered for the excitations undergone by vehicle. If the displacement spectral density of road excitation within temporal frequency f was assumed as G_q(f) and the travel speed of vehicle was assumed as v, then the relationship between the displacement spectral density and the temporal frequency spectral density was:

$$G_q(f) = \frac{1}{v}G_q(n)$$

In China national standard GB/T7031-2005 Mechanical Vibration-Reporting of Road Surface Spectrum Measurement Data, the road roughness was classified into 8 levels based on the power spectral density of road surface and the range and geometric mean of the power spectrum $G_q(n_0)$ of every level of road roughness were given with frequency index specified as w = 2.

Virtual road excitation for whole vehicle: When a vehicle traveled at a constant speed, the road roughness excitations could be regarded as a stable random process and the input spectral density from the road to four wheels was:

$$[S_q(f)] = \begin{bmatrix} 1 & e^{-j\omega\tau} & \gamma & \gamma e^{-j\omega\tau} \\ e^{j\omega\tau} & 1 & \gamma e^{j\omega\tau} & \gamma \\ \gamma & \gamma e^{-j\omega\tau} & 1 & e^{-j\omega\tau} \\ \gamma e^{j\omega\tau} & \gamma & e^{j\omega\tau} & 1 \end{bmatrix} G_q(f)$$

where, $G_q(f)$ is the power spectral density of the vertical displacement of road roughness excitation; γ is the coherence function between the paths of the left and right wheels; τ = L/u is the time difference between the front and rear wheels; L is the wheelbase and μ is the travel speed of the vehicle at a constant travel speed. The non-zero characteristic value was obtained through decomposition of the above equation:

$$\lambda_1 = 2\left[1 - \mathrm{coh}(f)\right]G_q(f)\,\lambda_2 = 2\left[1 + \mathrm{coh}(f)\right]G_q(f)$$

And the corresponding characteristic vector was:

$$\{\psi\}_1 = \frac{1}{2}\{-e^{j\omega\tau}, -1, e^{-j\omega\tau}, 1\}^T \{\psi\}_2 = \frac{1}{2}\{e^{j\omega\tau}, 1, e^{j\omega\tau}, 1\}^T$$

The virtual road excitations based on 4 wheels were constructed from Eq. 9:

$$\{q(t)\}_s = \{\psi\}_s^* \sqrt{\lambda_s} e^{j\omega\tau} (s = 1, 2) \tag{12}$$

Statistical characteristic of vibration response: Based on Eq. 2, the relationship between the virtual road excitation and the virtual response was (Xu *et al*., 2009):

$$\{\tilde{X}(t)\} = H(\omega)\{\tilde{q}(t)\} \tag{13}$$

where, H (ω) is the frequency response function of the system and ω = 2πf.

Eq. 13 was differentiated twice to obtain:

$$\{\dot{\tilde{X}}(t)\} = i\omega H(\omega)\{\tilde{q}(t)\} \tag{14}$$

$$\{\ddot{\tilde{X}}(t)\} = -\omega^2 H(\omega)\{\tilde{q}(t)\} \tag{15}$$

Eq. 13, 14 and 15 were substituted into Eq. 1 and the frequency response function could be gained:

$$H(\omega) = \frac{i\omega C_q + K_q}{-\omega^2 M + i\omega C + K} \tag{16}$$

With the road excitations and vehicle parameters being determined, the virtual response X(t) could be obtained based on Eq. 10 and further the displacement,velocity and acceleration spectra at any position could be calculated.

The above-mentioned virtual excitation method was very convenient to use and included uniform equations for calculation of auto power spectral density and cross power spectral density. As long as the power spectral density of actual excitation was known, the power spectral density corresponding to actual response and actual excitation could be obtained from the virtual excitation and virtual response.

EXAMPLE AND RESULT ANALYSIS

The parameters of a vehicle frame were known as follows: ct1 = 4,700 N sec m^{-1}, ct2 =5, 800 N sec m^{-1}, E = 20,700 MPa, side member I1 = 3.68×10-4 m^4, cross member I2 = 9.85×10-6 m^4, kt1 = 792,000 N m^{-1} and kt2 = 688,000 N m^{-1}. Level C road surface $G_q(n_0)$ = 256×10^{-6} m^2 m^{-1}, γ = 0.5; the vehicle speed v was 40, 60, 80 km h^{-1}, respectively and the frequency range was 0.2-20 Hz. Matlab simulation program was prepared and the power spectral densities of vertical displacement and acceleration of the random response of a node were obtained, as shown in Fig. 2-5.

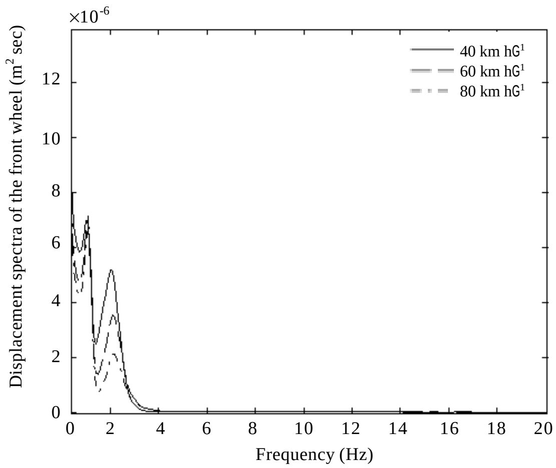

Fig. 2: Displacement spectra of the front wheel

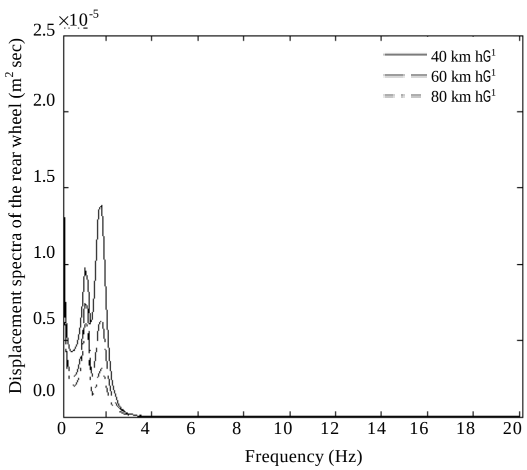

Fig. 3: Displacement spectra of the rear wheel

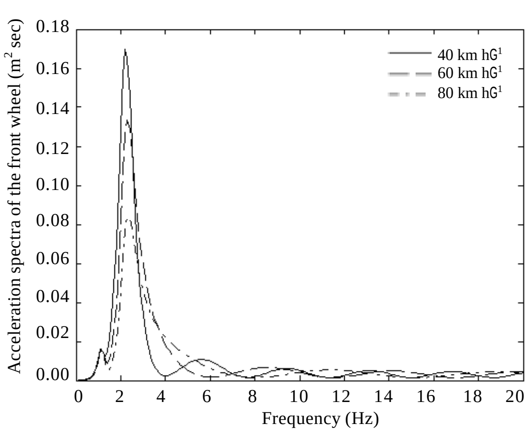

Fig. 4: Acceleration spectra of the front wheel

The analysis results show that the main peak values of structure response at different vehicle speeds were

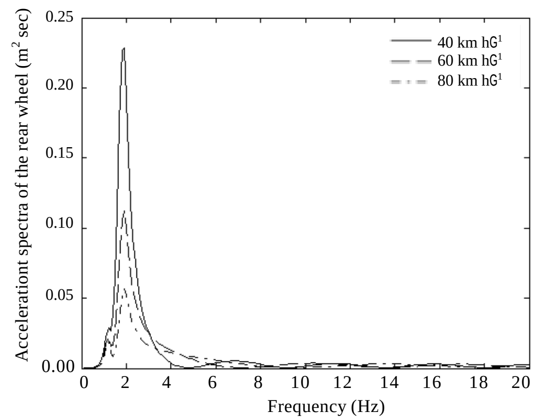

Fig. 5: Acceleration spectra of the rear wheel

concentrated near the frequency of 2.5 Hz and that the influence of the vehicle speed on the vibration was large as the amplitude was obviously increased with the increase in the vehicle speed.

It can be seen from Fig. 2-5 that there were resonances occurring at both low and high frequencies frame was a low frequency resonance. The influence of the low frequency resonance on the frame was far larger than that of a high frequency resonance and the dynamic stress of the low frequency resonance had serious sudden change, so the low frequency resonance was one of the main causes for fatigue damage of the frame.

CONCLUSION

A kinetic model has been established for a vehicle frame with a finite element method and used the virtual excitation method theory to construct virtual road excitations based on four points, i.e., front and rear wheels; thus we offered a method for calculating the statistical characteristic of random vibration response of the vehicle frame based on the virtual excitation method which not only maintained the theoretical precision but also greatly simplified the calculation. Through example analysis, we obtained the structure response of the vehicle frame at different vehicle speeds and provided a feasible way for stress spectrum calculation in fatigue life analysis of the vehicle frame under random excitations.

REFERENCES

Jiang, H.Q. and S.C. Li, 2011. Mode analysis and optimization design for semi-trailer frame. Hoisting Conv. Machin., 5: 27-30.

Li, J., Y.Y. Qin and Q. Zhao, 2010a. Application of multi-point virtual excitation method in random vibration analysis of whole vehicle. Automobile Eng., 32: 254-257.

Li, J., Y.Y. Qin and Q. Zhao, 2010b. Modeling and simulation of random vibration of automobiles. J. Jilin Univ. Eng. Technol. Edn., 40: 316-319.

Lin, J.H. and Y.H. Zhang, 2004. Virtual Excitation Method for Random Vibration. 1st Edn., Science Press, China.

Pan, W.M., M.J. Yang and Y. Xu, 2003. Computation of stress spectral density of structural members in a random vibration system. J. Southwest Jiaotong Univ., 38: 12-16.

Sun, Z.Y. and H. Wang, 2006. Equivalent excitation method for random structural vibration analysis. Eng. Mechan., 27: 20-22.

Tang, F. and S.L. Huang, 1999. The frame model of IVECO40-10 vehicle and its dynamic analysis. Chin. J. Applied Mechan., 16: 136-139.

Wang, W.D., L.N. Sun and Z.J. Du, 2010. Vibration analysis of mobile manipulator based on virtual excitation method. J. Jilin Univ. Eng. Technol. Edn., 40: 1082-1085.

Xu, W.T., J.H. Lin, Y.H. Zhang, D. Kennedy and F.W. Williams, 2009. 2D moving element method for random vibration analysis of vehicles on Kirchhoff plate with Kelvin foundation. Latin Am. J. Solids Struct., 6: 169-183.

Xu, H.L., Z.X. Han, J.Q. Cui and L.P. Wei, 2012. Analysis of random vibration and fatigue strength of 7140 bus frame. J. Mach. Des., 29: 7-9.

Zhang, L.L., J.S. Tang and L.B. Li, 2006. Vibration analysis of automotive suspension based on virtual excitation method. Vibr. Impact, 25: 167-169.