Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 13 (10): 1734-1739, 2013
ISSN 1812-5654 / DOI: 10.3923/jas.2013.1734.1739

Locating-pricing Game Model of Duopoly Enterprises Within a Circular Plane City

[1]Rong Lin, [1]Shouzhong Wang, [2]Shuyang Wang and [3]Yunyan Wang
[1]Henan University, Kaifeng, 475001, China
[2]China University of Geosciences, Beijing, 100083, China
[3]China Agricultural University, Beijing, 100083, China

Abstract: Based on the Hotelling basic model, this study expanded linear city to a circular flat urban area and analyzed the pricing and location behavior of two companies which provide homogeneous products on a two-dimensional plane. The pricing game models were separately established in the condition of fixed and not fixed site and they were solved combined with the plane geometry characteristics of the hyperbolic.

Key words: Pricing and location, hotelling model, hyperbolic, game

INTRODUCTION

In Duopoly market, if two competing companies located in different position, companies not only have price competition, but also compete for the location. Hotelling (1929) was one of the first models to study the problem of price competition in the case of product differentiation, it proposed location differences based on the linear space to indicate the product differentiation and established a location and pricing two stage game models, it also open the study of the theory of spatial competition. Since, then, many scholars expand this model. Economides (1986) assume consumers transportation cost function be the distance of a Squared based on the original model of Hotelling. Klemperer (1987) assume that manufacturers cost is symmetrical and study the two stage incomplete information competitive model in the presence of metastases cost. Chawla *et al.* (2006) established a location game model that two participants have sequential turn in the market that the customer demand is a non-uniform distribution. Li *et al.* (2008) Consider the economies of network scale and brand loyal effect in a competitive market, establish a two-stage game model5 that conclude suppliers and service process outsourcing.

Domestic scholars also do many related research. Gu *et al.* (1999, 2002) constructed two-stage game model considering the consumer choice and the uneven distribution of consumers for research the effect of gathering of the demand for enterprise product differentiation strategy. Guo and Guo (2003) established the continuous space model with price discrimination. Chen *et al.* (2003) analysis the second degree price discrimination and its restrictions under the assumption of the demand function is linearity. Cao and Gu (2002) analysis two-stage location-pricing model according to the different values of the total consumer surplus When the duopoly manufacturers take discriminatory policy. Xu and Zhu (2007) introduce the network externalities to linear transportation costs under the Hotelling model. Yu and Zhang (2008) constructed a Hotelling model that transportation costs can pay selectively by the merchant and commodity price cut constrained by the guide price.

This study analyzes the duopoly enterprises' competitive behavior of location and pricing in the two-dimensional plane city, established a two-stage complete information dynamic game model of under the linear costs about two companies. In this model, we assumed the corporate can located at any pointing the circular flat city and finally obtained the equilibrium solution with the plane geometry characteristics of hyperbolic.

THE FIXED POSITION MODEL

Hotelling model: Assuming a linear city's length is 1, consumers are evenly distributed in the interval [0, 1] and the distribution's density is 1. Suppose that there are two competing shops located at both ends of the city, the shop 1 at point a, shop 2 at point 1-b and a = 1-b. Two shops sell the homogeneous products and unit product cost c. Consumers' travel cost for purchase goods is proportional to the distance to the shop and unit distance cost t. assumed that the consumer has a unit demand that is, consumption is a unit or zero. And consumer surplus is S.

Corresponding Author: Shouzhong Wang, Department of E-commerce, Collage of Business Administration, Henan University, Kaifeng, Henan, China

Considering the Nash equilibrium of price competition of two shops Suppose that the two shops choose their own sales price at the same time. For simplicity, assume that S is sufficiently large relative to the total purchase cost (sales price plus travel expenses) so that all consumers can buy a product unit. p_i indicates the sales price of shop i, $D_i(p_1, p_2)$ demand function, (i = 1, 2). If the payment of a consumer living in the point X to purchase one unit product have no difference between two shops, then, consumers living in the left of point X will purchase at the shop 1 and consumers lives in the right of point X will purchase at the shop 2, the demand were $D_1 = x$, $D_2 = 1\text{-}x$_[p_1+t(x-a) = p_2+t(1-b-x)] (__)/()/D_Dd__. The demand function of two shops:

$$D_1(p_1, p_2) = x = \frac{p_2 - p_1}{2t} + \frac{1-b+a}{2}$$

$$D_2(p_1, p_2) = 1 - x = \frac{p_1 - p_2}{2t} + \frac{1+b-a}{2}$$

Profit function:

$$\pi_1(p_1, p_2) = (p_1 - c)D_1(p_1, p_2) = (p_1 - c)\left(\frac{p_2 - p_1}{2t} + \frac{1-b+a}{2}\right)$$

$$\pi_2(p_1, p_2) = (p_2 - c)D_2(p_1, p_2) = (p_2 - c)\left(\frac{p_1 - p_2}{2t} + \frac{1+b-a}{2}\right)$$

So the equilibrium prices of two shops are as follows.

Then, we can get the location strategy police of two shops through calculate the partial derivative of the equilibrium profit about a, b, respectively.

THE FIXED POSITION MODEL CONSTRUCTION

It is assumed that the market is linear In 2.1, in this section the competitive market to expand to a two-dimensional plane city. We will construct a hotelling model in a two-dimensional plane city to analysis price-competitive behavior of the duopoly enterprise products. Firstly, we assumed the two enterprises' location is fixed.

Assuming there is a circular flat city as a unit circle, provided that the center of the circle is in point (0, 0), consumers are uniformly distributed in the area of a circle and the distribution density is 1. Assuming only two enterprises in this area to provide service products, enterprise 1 located in the point (-1, 0) and enterprise 2 located in the point (1, 0) of the circular city, as shown in Fig. 1. The two enterprises 'products are equal in quality. The unit cost of the product is c and the travel cost of the

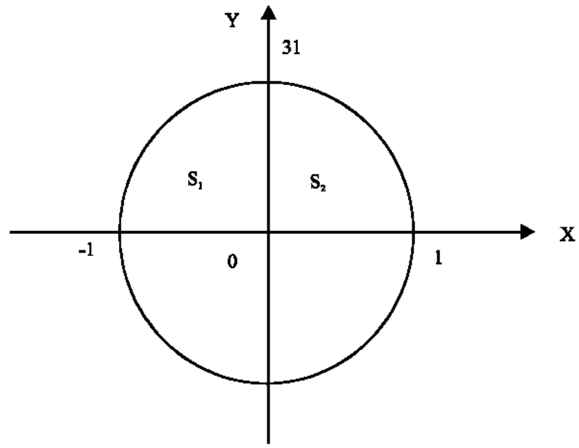

Fig. 1: Fixed circular flat city

consumers to purchase goods is proportional to the distance from the consumer to enterprise, one unit of distance cost is t that is, a consumer whose Distance is x from enterprise 1 should spend t×x travel costs. Assuming that all consumers have a unit demand and the consumer surplus is s.

Suppose two enterprises will ultimately achieving a balance through competitive behavior, then enterprises 1 and 2 will get the respective regions, the region has its boundaries, any consumer in this border will have balanced: $p_1+t.d_1 = p_2+t.d_2$, where p_1, p_2 are the prices of product for enterprises 1 and 2, d_1, d_2 are the distance of consumers to enterprises 1 and 2. And make:

$$d = |d_1 - d_2| = \frac{|p_1 - p_2|}{t}$$

If p_1, p_2 are a constant, d can be seen as a fixed value that is, the absolute value of the distance difference of consumers located in the border to the two enterprises. So we can get a hyperbolic equation:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1,\ \frac{\square}{\frac{\sqrt{|p_1 - p_2|}}{2t}},\ b = \sqrt{c^2 - a^2},\ c = 1.14$$

The focus of this hyperbola is (-1, 0), (1, 0), as shown in Fig. 2.

If $p_1|p_2$, then the market boundaries of two enterprises will be the left branch of the hyperbola when equalized. This is because enterprise 2 has a lower price. Consumers who purchase at enterprise 2 can withstand the more travel costs. but it was not until some consumers Distance to enterprise 2 is too far to regards to the distance difference equal to d, for these consumers, there is a

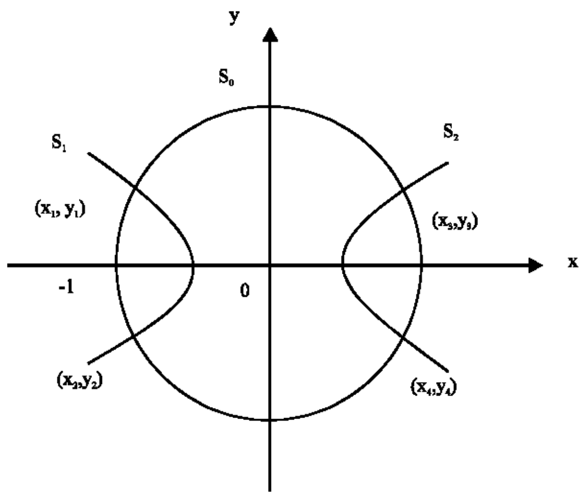

Fig. 2: Intersection when fixed

equation: $p_1+t.d_1 = p_2+t.d_2$. That is, these consumers more far away from the enterprise 2 and would pay more travel costs, so that the enterprise 2's low price lost its attractiveness t these consumers who distributed left branch of the hyperbolic.

When $p_1|p_2$, due to the symmetry of the model, we know that the market boundaries of the two enterprises is the right branch of the hyperbola.

Model solution: There is $|p_1-p_2|$ in the above analysis and the model has symmetry, therefore we can only simply analysis the situation of $p_1 \le p_2$. The D1, D2, respectively are the demand of enterprise 1 and 2. First, we calculate the intersection coordinates of the circular and the right branch of hyperbolic. The calculation results are:

$$\left(-a\sqrt{1+b^2},\ b^2\right), \left(-a\sqrt{1+b^2},\ -b^2\right), \left(a\sqrt{1+b^2},\ b^2\right), \left(a\sqrt{1+b^2},\ -b^2\right)$$

$$s_1 = \int_{-b^2}^{b^2}\left(\sqrt{1-y^2} - \frac{a\sqrt{y^2+b^2}}{b}\right)$$
$$dy = \arcsin\,(1-a^2) - a\sqrt{1-a^2}\ \ln\left(\sqrt{2-a^2}+\sqrt{1-a^2}\right)$$

When $p_1 \le p_2$, the Demand functions of firms 1 and 2 are:

$$D_2(p_1, p_2) = \pi - s_1$$

And the profit functions of two enterprises are:

$$\pi_2(p_1, p_2) = (p_2-c)\times(\pi-D_1)$$

Make:

$$\begin{cases} \dfrac{\partial \pi_1(p_1, p_2)}{\partial p_1} = 0 \\ \dfrac{\partial \pi_2(p_1, p_2)}{\partial p_2} = 0 \end{cases}$$

We can calculate:

$$p_1 = p_2 = c + \frac{t\pi}{\sqrt{2}+\ln(\sqrt{2}+1)}$$

and:

$$\pi_1(p_1, p_2) = \pi_2(p_1, p_2) = \frac{t\pi_2}{2\left[\sqrt{2}+\ln\left(\sqrt{2}+1\right)\right]}$$

By the above calculation, the price equilibrium solution of the Enterprise 1 and is:

$$p_1 = p_2 = c + \frac{t\pi}{\sqrt{2}+\ln\left(\sqrt{2}+1\right)}$$

the profit of two enterprises is:

$$\pi_1(p_1, p_2) = \pi_2(p_1, p_2) = \frac{t\pi^2}{2\left[\sqrt{2}+\ln\left(\sqrt{2}+1\right)\right]}$$

Now firms 1 and 2, respectively occupy half area of the circular city, the boundaries of the two regions should be the y-axis, as shown in Fig. 1. Any one consumer in the y-axis and the two manufacturers constitute a triangle, the triangle in the y-axis is vertical, so d_1 = d_2, it is indicated that the required payment is no difference to purchase at enterprise 1 or enterprise 2 for consumers that located on the boundaries for consumers Located in the area s1, there will be d_1 <d_2, the consumers purchase products at enterprise 1 will pay less than enterprise 2 so consumers located in the region s_1 will be purchased at enterprise 1,.similarly, consumers located in the area s2 will choose enterprise 2.

THE ARBITRARY LOCATION MODEL

The arbitrary location model construction: The basic assumptions are similar with 2.1, but in this section, two Enterprise can be located at anywhere in the circular city, as shown in Fig. 3. Assume that firms 1 and 2 are located at points A and point B (A, B can be chosen arbitrarily at the circular city). Since, the city is a circular flat city, in

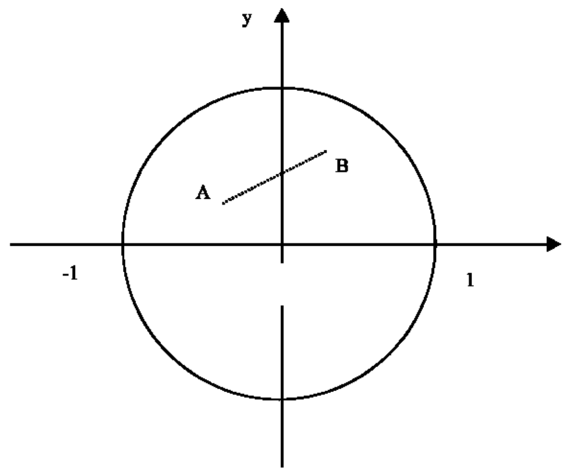

Fig. 3: Not fixed circular flat city

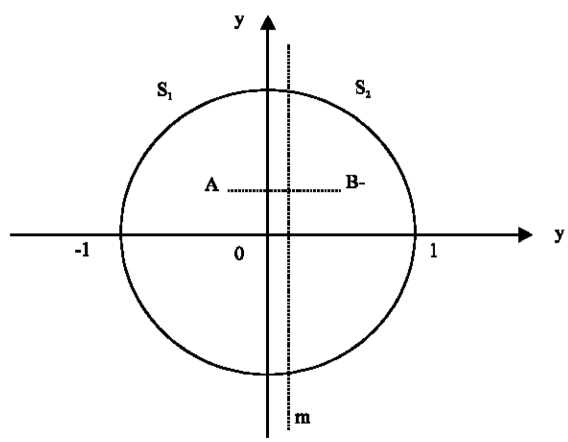

Fig. 4: Axis rotated

order to facilitate the calculation, we can rotate the axis, make the line AB runs parallel to the x-axis as Fig. 4 shown. So for firms 1 and 2 at arbitrary position, we can assume that they two enterprises have the same y-coordinate value and enterprise 1 is located at the point A (m1, n), enterprise 2 is located at point B (m2, n), m1<=m2 in Fig. 4 the line m is the midperpendicular of AB and the midperpendicular m intersect line AB at the point (m, n). The midperpendicular m's equation is:

$$x_m = m = \frac{m_1 + m_2}{2}$$

Then we translational coordinate systems as Fig. 5,thus line AB is coincide with the x-axis,and line m is coincide with the y-axis. Now we can know A is located in the point:

$$-\frac{m_2 - m_1}{2}, 0$$

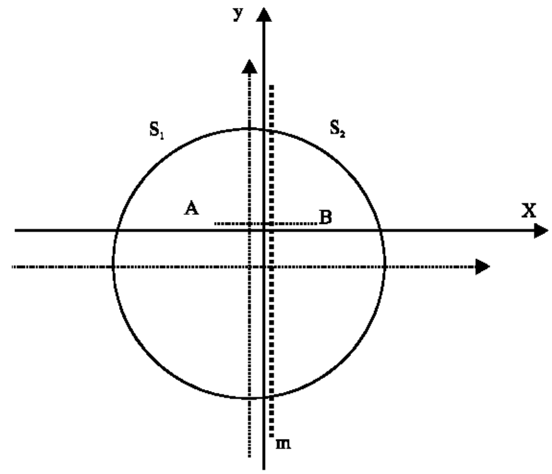

Fig. 5: Translational coordinate systems

B located in the point:

$$-\frac{m_2 - m_1}{2}, 0$$

the center coordinates of the circular city is $(x+m)^2+(y+n)^2 = 1$

The same as supra Analysis, assuming firms 1 and 2 eventually reach equilibrium through a competitive behavior: p_1+t.d_1 = p_2+t.d_2. Similarly we can get a hyperbola equation::

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

and:

$$a = \frac{|p_1 - p_2|}{2t},\ b = \sqrt{c^2 - a^2},\ c = \frac{m_2 - m_1}{2}$$

The focus point of This hyperbolic equation is:

$$A = -\frac{m_2 - m_1}{2}, 0, B = \frac{m_2 - m_1}{2}, 0$$

the left branch, right branch of this hyperbola intersect with the circle and divided it into three parts, namely s_1, s_2, s_0, as shown in Fig. 6. And the equations of two asymptotes of the hyperbolic were:

$$p: y = \frac{b}{a}x, \quad q: y = -\frac{b}{a}x$$

If $p_1 \geq p_2$, when equalized , the boundaries line of market region that each enterprises owned would be the left branch of the hyperbola. The analysis is similar with

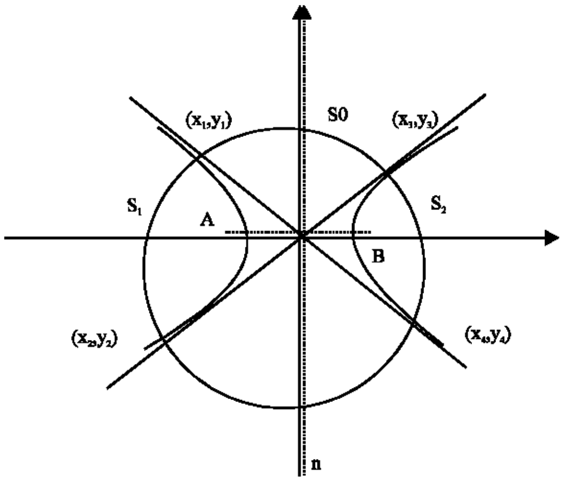

Fig. 6: Asymptotic line analysis

2.2. enterprise 2 has a lower price, Consumers who purchase at enterprise 2 can withstand the more travel costs. but it was not until some consumers Distance to enterprise 2 is too far to regards to the distance difference equal to d, for these consumers, there is a equation: p_1+t.d = p_2+t.d. That is, these consumers more far away from the enterprise 2 and would pay more travel costs, so that the enterprise 2's low price lost its attractiveness t these consumers who distributed left branch of the hyperbolic.

When $p_1 \le p_2$, the market boundaries of the two enterprises is the right branch of the hyperbola.

Due to the arbitrariness of choosing A and B, so the supra analysis is right for two companies located at anywhere within the circular plane.

MODEL SOLUTION

Pricing stage: First, calculating the four intersections of the hyperbola and circle, they are (x_1, y_1) (x_2, y_2) (x_3, y_3) (x_4, y_4), as shown in Fig. 6.

When $p_1 \le p_2$, the boundaries of enterprises 1 and 2 is the left branch of the hyperbola that is, the market area of enterprises 1 is s_1. so the product demand of enterprises 1 and 2 are:

$$D_1 = 1 \times s_1 = s_1,\ D_2 = 1 \times (\pi - s_1) = \pi - s_1$$

And:

$$s_1 = \int_{y_2}^{y_1}\left[\left(\sqrt{1-(y+n)^2}+m\right)-\frac{\sqrt[a]{y^2+b^2}}{b}\right]dy,$$

$$s_2 = \int_{y_2}^{y_2}\left[\left(\sqrt{1-(y+n)^2}-m\right)-\frac{\sqrt[a]{y^2+b^2}}{b}\right]dy,$$

$$s_0 = \pi - s_1 - s_2$$

Similarly, when $p_1 \ge p_2$, the situation, the boundaries is the right branch of the hyperbola. Now the market area of enterprises 2 is s_2. So the product demand of enterprises 1 and 2 are $D_1 = 1 \times (\pi - s_2) = \pi - s_2$, $D_2 = 1 \times s_2 = s_2$.

Therefore, when $p_1 \ge p_2$, the demand function of enterprises 1 and 2 are: $D_1(p_1, p_2) = s_1$; $D_2(p_1, p_2) = \pi - s_1$. The profit functions of two enterprises are:

$$\pi_1(p_1, p_2) = (p_1 - c_0) \times D_1$$

$$\pi_2(p_1, p_2) = (p_1 - c_0) \times (\pi - D_1)$$

Make:

$$\begin{cases} \dfrac{\partial \pi_1(p_1, p_2)}{\partial p_1} = 0 \\ \dfrac{\partial \pi_2(p_1, p_2)}{\partial p_2} = 0 \end{cases}$$

we can get the equilibrium solution as follows:

$$p_1 = p_2 = c_0 + 2t\frac{\frac{\pi}{2}}{\frac{b}{2}\ln\left|\frac{1-n}{-1-n}\right|} = c_0 + \frac{2t\pi}{b\ln\left|\frac{1-n}{-1-n}\right|}$$

$$\pi_1(p_1, p_2) = \pi_2(p_1, p_2) = (p_1 - c_0) \times D_1 = \frac{2t\pi}{b\ln\left|\frac{1-n}{-1-n}\right|} \times \frac{\pi}{2} = \frac{\frac{\pi}{2}}{\frac{b}{2}\ln\left|\frac{1-n}{-1-n}\right|}$$

Locating stage: In 3.2.1, we obtained Equilibrium solution of the profits of the two enterprises. Now we solve the first derivative of π_1, π_2 about m_1, m_2 and n to analysis the impact of the enterprise position on corporate profits, then we can determined the optimal location of enterprise relatively.

There is:

$$\frac{\partial \pi_1}{\partial n} = \frac{\partial \pi_2}{\partial n} = \frac{2t\pi^2}{m_1 - m_2} \times \frac{\frac{2}{1-n^2}}{\ln\left|\frac{-n+1}{-n-1}\right|} \le 0$$

So, we can know that the profit function of enterprise 1 and 2 is a decreasing function about n.

When $0 \ge n \ge -1$, the:

$$\frac{\partial \pi_1}{\partial m_1} = \frac{2t\pi^2}{(m_1 - m_2)^2 \ln\left|\frac{-n+1}{-n-1}\right|} \le 0$$

so the profit function of enterprise 1 is a decreasing function about m_1. And the:

$$\frac{\partial \pi_2}{\partial m_2} = \frac{2t\pi^2}{(m_1 - m_2)^2 \ln\left|\frac{-n+1}{-n-1}\right|} \le 0$$

so the profit function of enterprise 2 is a increasing function about m_2. In this study, we assume that the position of enterprise 1 and 2 have the same n. so for a given value of n, the equilibrium value of location of enterprises 1 and 2 are respectively $(-\sqrt{1-4n^2-m}, n)$, $(\sqrt{1-4n^2-m}, n)$.

When $1>n\ge 0$:

$$\frac{\partial \pi_1}{\partial m_1} = \frac{-2t\pi^2}{(m_1 - m_2)^2 \ln\left|\frac{-n+1}{-n-1}\right|} \le 0$$

And the:

$$\frac{\partial \pi_2}{\partial m_2} = \frac{-2t\pi^2}{(m_1 - m_2)^2 \ln\left|\frac{-n+1}{-n-1}\right|} \le 0$$

so the profit function of enterprise 2 is a decreasing function about m_2. so for a given value of n, the equilibrium value of location of enterprises 1 and 2 are respectively $(-\sqrt{1-4n^2-m}, n)$, $(\sqrt{1-4n^2-m}, n)$.

CONCLUSION

In this study, based on the hotelling model, we establish a duopoly location and pricing competition model who supply homogeneous products within a unit circular flat-city. By solving the model combined with the plane geometry characteristics of the hyperbolic, we can get the following conclusions: 1)when the fixed position of the two companies are point (-1,0) and (1,0) in the circular flat-city , the equilibrium utility lines is the y-axis, the equilibrium solution is that the two companies will selected a same price and get half of the market region; 2) when the position of the two companies can be selected arbitrarily in the units circular flat-city, the equilibrium price of the two companies are equal and each enterprise get the half of the market region. And the two companies follow the principle of maximum difference when they selected location, they always try to as far away as possible from each other.

REFERENCES

Cao, Y.J. and X.Y. Gu, 2002. Horizontal differentiation model with network externality. J. Manege. Sci. China, 5: 59-64.

Chawla, S., U. Rajan, R. Ravi and A. Sinha, 2006. Min-Max payoffs in a two-player location game. Oper. Res. Lett., 34: 499-507.

Chen, S.G., X.W. Tang and S.R. Zhao, 2003. Nash equilibrium of second degree price discrimination under two manufacturers. Syst. Eng. Theory Methodol. Appl., 12: 303-306.

Economides, N., 1986. Minimal and maximal product differentiation in Hotelling's duopoly. Econ. Lett., 21: 67-71.

Gu, F., G. Xue and P. Huang, 1999. Hotelling's competition with consumer's choice. Syst. Eng. Theory Methodol. Appl., 4: 32-37.

Gu, F., P.Q. Huang and D.S. Zhou, 2002. Strategy of minimum product differentiation under heterogeneity. J. Syst. Eng., 17: 467-471.

Guo, C.X. and Y.H. Guo, 2003. The price space discrimination in oligarchic competition. Oper. Res. Manage. Sci., 12: 28-32.

Hotelling, H., 1929. Stability in competition. Econ. J., 39: 41-57.

Klemperer, P., 1987. The competitiveness of markets with switching costs. RAND J. Econ., 18: 138-150.

Li, H., J. Chang and M. Dong, 2008. Location selection of BPO Vendors based on a game theory model. Human Syst. Manage., 27: 313-320.

Xu, B. and D.L. Zhu, 2007. Extended hotelling model with network externality. J. Manage. Sci. China, 10: 9-17.

Yu, Y. and W. Zhang, 2008. Evolutionary game analysis on hotelling model based promotional strategies adoption. J. Syst. Manage., 17: 665-674.