



# Journal of Applied Sciences

ISSN 1812-5654

**science**  
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## Study on Fault Diagnosis of Turbine Using an Improved Cosine Similarity Measure for Vague Sets

L.L. Shi and J. Ye

Department of Electrical and Information Engineering, Shaoxing University, Shaoxing, China

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**Abstract:** Aiming at complexity and uncertainty of relation between vibration and fault types of turbine, a new method of fault diagnosis of turbine was proposed based on an improved vague cosine similarity measure. Compared with the previous cosine similarity measures for vague sets, the improved cosine similarity measure has more information to deal with vagueness and uncertainty problems by considering truth-membership functions, false-membership functions and hesitancy degree of vague sets and then it can overcome the undefined problem when the degree of membership and degree of non-membership are zero, respectively. Then, the cosine similarity measure was applied to fault diagnosis of turbine. For this fault diagnosis, the matter-element models of the turbine fault were built according to diagnostics derived from specialists' knowledge of practical experience and then through the vague cosine similarity measure between a fault-testing sample and fault knowledge samples, the vibration fault is determined according to the maximum cosine similarity measure value. The fault-diagnosis example of the turbine shows that the proposed method is simple and effective.

**Key words:** Vague sets, cosine similarity measure, fault diagnosis, turbine

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### INTRODUCTION

Vibration is the main phenomenon of steam turbine failure. Vibration signal is constant and small when the steam turbine operates normally. But when a fault occurs, the vibration will become stronger. Many faults may cause the vibration of turbine and the relationship between fault symptoms and causations is very complex. When the turbine vibration fault occurs, we need diagnose timely and accurately the cause of the trouble. The effective fault diagnosis can shorten the repair time of equipment and furthermore, it can avoid more accidents and large economic losses. So, it has very important practical value to diagnose the vibration fault of steam turbine.

In recent years, many turbine fault diagnosis methods have been proposed by researchers. The general methods of fault diagnosis are expert systems (Ye, 2009), neural networks (Yang *et al.*, 2012; Huang *et al.*, 2008), fuzzy approaches (Salahshoor *et al.*, 2011; Zhao *et al.*, 2012; Yang *et al.*, 2010; Kyriazis and Mathioudakis, 2009) and so on.

For turbine faults, the same symptom of the fault may have a variety of fault causations, so it is difficult to make precision and quantitative analysis. In view of this, fuzzy theory has been widely used in fault diagnosis and it has become a research hotspot.

Fuzzy sets theory proposed by Zadeh (1965) is suitable for handling uncertain information. Atanassov (1986) extended the fuzzy sets theory to Intuitionistic Fuzzy Sets (IFSs). Gau and Buehrer (1993) proposed vague sets and Bustince and Burillo (1996) pointed out that the notion of vague sets was the same as that of IFSs. The fuzzy method has successfully been applied in various fields.

A similarity measure is an important tool for determining the degree of similarity between two objects. Similarity measures between fuzzy sets, as an important content in fuzzy mathematics, have gained attention from researchers (Ye, 2011). Therefore, many similarity measures between fuzzy sets have been proposed by scholars in recent years. These similarity measures have their wide applications in various fields (Ye, 2011; Bustince *et al.*, 2006, 2007, 2008; Lee *et al.*, 2009).

The cosine similarity measure between two vectors was firstly proposed by Bhattacharya (1946). It is usually used in information retrieval as a classic measure.

When we express the fuzzy sets with vectors, the cosine similarity measure can be extended to determine the degree of similarity between two fuzzy sets. Based on the Bhattacharya's cosine similarity measure (Bhattacharya, 1946; Ye, 2011) presented a cosine similarity between two intuitionistic fuzzy sets and defined the cosine of the angle between two vectors

representations of the two IFSs. The cosine similarity measure was proposed by considering the degree of membership and degree of non-membership as a vector representation with the two elements. However, when the degree of membership and degree of non-membership are zero, respectively, the cosine similarity measure is without definition. In order to solve this problem, we extend the above cosine similarity measure proposed by Ye. In this study, we present a novel cosine similarity measure of Vague Sets (VSs) considering three aspects of information including truth-membership functions, false-membership functions and hesitancy degree. Finally, we applied it to fault diagnosis of turbine. The application of this new method to turbine sets demonstrates that the proposed method is simple and effective.

**PRELIMINARIES**

Here, we introduce some basic concepts and definitions related to fuzzy sets and VSs which will be needed in the following analysis.

**Definition 1:** Zadeh (1965) defined a Fuzzy Set A in the universe of discourse  $X = \{x_1, x_2, \dots, x_n\}$  as follows:

$$A = \{ \langle x, \mu_A(x) \rangle | x \in X \} \tag{1}$$

where  $\mu_A(x): X \rightarrow [0, 1]$ , the number  $\mu_A(x)$  indicates the membership degree of the element x to the set A.

Gau and Buehrer (1993) extended fuzzy sets to VSs and defined it as follows.

**Definition 2:** A vague set A in  $X = \{x_1, x_2, \dots, x_n\}$  is given by Gau and Buehrer (1993):

$$A = \{ \langle x, \mu_A(x), 1 - v_A(x) \rangle | x \in X \} \tag{2}$$

where,  $\mu_A(x): X \rightarrow [0, 1]$  and  $v_A(x): X \rightarrow [0, 1]$ , with the condition  $0 \leq \mu_A(x) + v_A(x) \leq 1$ , the number represent, respectively, a truth-membership function and a false-membership function of the element x to the set A.

Vss can be transformed into fuzzy sets if  $F_A(x_i) = v_A(x_i)$ .

For each VSs A in X, if  $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ ,  $x \in X$ , then  $\pi_A(x)$  is called vague degree of the element x to the vague set A. It indicates a hesitancy degree of x to A. It is obvious that  $0 \leq \pi_A(x) \leq 1$ ,  $x \in X$ . According to the above definition, a vector with the three-dimensional elements  $(\mu_A(x), v_A(x), \pi_A(x))$  can be used to characterize the degrees of the element x to the vague set A.

**COSINE SIMILARITY MEASURE FOR VAGUE SETS**

Here, we firstly introduce the definitions of cosine similarity measures between vectors, fuzzy sets and IFSs and then propose a new cosine similarity measure for VSs.

**Cosine similarity measures of two vectors:** Let  $X = (x_1, x_2, \dots, x_n)$  and  $Y = (y_1, y_2, \dots, y_n)$  be the two vectors of length n, according to Bhattacharya’s distance, cosine similarity measures are defined as the inner product of two vectors divided by the product of their lengths. The cosine formula (Bustince *et al.*, 2008) is as follows:

$$\text{Cos}(X, Y) = \frac{X \cdot Y}{\|X\|_2 \|Y\|_2} = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}} \tag{3}$$

It can express the cosine of the angle between the two vectors.

**Cosine similarity measures of two fuzzy sets:** The above cosine similarity measure can be extended to that of the fuzzy sets.

Assume that  $A = (\mu_A(x_1), \mu_A(x_2), \dots, \mu_A(x_n))$  and  $B = (\mu_B(x_1), \mu_B(x_2), \dots, \mu_B(x_n))$  are two fuzzy sets in the universe of discourse  $X = \{x_1, x_2, \dots, x_n\}$ ,  $x_i \in X$ . A cosine similarity measure for the fuzzy sets (Lee *et al.*, 2009) can be defined as follows:

$$C_{\text{Fuzzy}}(A, B) = \frac{\sum_{i=1}^n \mu_A(x_i) \mu_B(x_i)}{\sqrt{\sum_{i=1}^n \mu_A^2(x_i)} \sqrt{\sum_{i=1}^n \mu_B^2(x_i)}} \tag{4}$$

Obviously, the cosine similarity measure between the fuzzy sets is within the interval [0, 1].

**Cosine similarity measures of IFSs:** Based on the above extension of the cosine measure for fuzzy sets, a cosine similarity measure for IFSs was proposed by Ye (2011).

Assume that there are two IFSs A and B in  $X = \{x_1, x_2, \dots, x_n\}$ . A cosine similarity measure (Ye, 2011) between IFSs A and B is defined as follow:

$$C_{\text{IFSs}}(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i) \mu_B(x_i) + v_A(x_i) v_B(x_i)}{\sqrt{\mu_A^2(x_i) + v_A^2(x_i)} \sqrt{\mu_B^2(x_i) + v_B^2(x_i)}} \tag{5}$$

where, the numbers  $\mu_A(x_i)$ ,  $\mu_B(x_i)$ ,  $v_A(x_i)$  and  $v_B(x_i)$  represent, respectively, the membership degree and non-membership degree of the element x to the IFSs A and B.

The cosine similarity measure is proposed by considering the degree of membership and degree of non-membership as a vector representation with the two elements. However, the above cosine similarity measure is without definition when  $\mu_A(x_i) = v_A(x_i) = 0$  or  $\mu_B(x_i) = v_B(x_i) = 0$ . In view of this, a new cosine similarity measure for VSs will be proposed according to the above cosine similarity measure between fuzzy sets in the following section.

**Cosine similarity measures for VSs:** Assume that  $A = \{ \langle x, \mu_A(x), 1-v_A(x) \rangle | x \in X \}$  and  $B = \{ \langle x, \mu_B(x), 1-v_B(x) \rangle | x \in X \}$  are two VSs in the universe of discourse  $X = \{x_1, x_2, \dots, x_n\}$ ,  $(\mu_A(x), v_A(x), \pi_A(x))$  and  $(\mu_B(x), v_B(x), \pi_B(x))$  can be used to characterize, respectively, truth membership functions, false membership functions and hesitancy degree in the vague set A and B. Then, the cosine similarity measure between VSs A and B can be defined as follows:

$$C_{vs_s}(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A(x_i)\mu_B(x_i) + v_A(x_i)v_B(x_i) + \pi_A(x_i)\pi_B(x_i)}{\sqrt{\mu_A^2(x_i) + v_A^2(x_i) + \pi_A^2(x_i)} \sqrt{\mu_B^2(x_i) + v_B^2(x_i) + \pi_B^2(x_i)}} \tag{6}$$

According to the above definition, the cosine similarity measures for VSs satisfy the following properties (Ye, 2011):

- P1 =  $0 \leq C_{vs_s}(A, B) \leq 1$
- P2 =  $C_{vs_s}(A, B) = C_{vs_s}(B, A)$
- P3 = If  $A = B$ , then there is  $C_{vs_s}(A, B) = 1$ .

**Proof:**

- P1 = According to the cosine value, it is obvious that  $0 \leq C_{vs_s}(A, B) \leq 1$
- P2 = It is obvious that the proposition is true
- P3 = When  $A = B$ , then  $\mu_A(x_i) = \mu_B(x_i)$ ,  $v_A(x_i) = v_B(x_i)$  and  $\pi_A(x_i) = \pi_B(x_i)$  ( $i = 1, 2, \dots, n$ )

Then:

$$C_{vs_s}(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{\mu_A^2(x_i) + v_A^2(x_i) + \pi_A^2(x_i)}{\mu_A^2(x_i) + v_A^2(x_i) + \pi_A^2(x_i)} = 1$$

So, there is  $C_{vs_s}(A, B) = 1$ .

Therefore, we have finished the proof.

Furthermore, if  $\mu_A(x_i) = v_A(x_i) = 0$ , then  $\pi_A(x_i) = 1$ , or if  $\mu_B(x_i) = v_B(x_i) = 0$ , then  $\pi_B(x) = 1$ ,  $x \in X$ .

In this case, the cosine similarity measure can be calculated by Eq. 6 and the values also satisfy the above properties (P1)-(P3).

Obviously, the new cosine similarity measure between two vague sets is based on considering three aspects of information including truth-membership functions, false-membership functions and hesitancy degree. Therefore, it has more information than the existing cosine similarity measures for VSs in dealing with vagueness and uncertainty.

**APPLICATION OF THE VAGUE COSINE SIMILARITY MEASURE TO FAULT DIAGNOSIS OF TURBINE**

**Fault-diagnosis method based on the vague cosine similarity measure:** We now develop a simple and straightforward fault diagnosis method for VSs based on the proposed cosine similarity measure.

The fault diagnosis method is described as follows:

- Step 1:** Establish the knowledge database of fault types referred to vague sets according to diagnostics derived from practical experience
- Step 2:** Give diagnosis-testing sample
- Step 3:** Calculate the vague cosine similarity measure between a fault-testing sample and fault knowledge samples by using Eq. 6
- Step 4:** Rank the vague cosine similarity measures and the fault type is determined according to the maximum value of the cosine similarity measures

**Application of fault diagnosis for turbine:** Here, two examples of the steam turbine-generator sets are used as the demonstration of the applications of the proposed cosine similarity of vague sets, as well as the effectiveness of the proposed fault-diagnosis method.

If a steam turbine-generator set is operating properly, vibration conditions are usually small and constant but the vibration signature will change when some faults generate. Hence, diagnostic information can be supplied by the spectrum of the vibration signal. However, the same vibration signs may be caused by a variety of causations, such as mechanical structure, load, vacuum degree, hot inflation of cylinder body and rotor, fluctuation of network load, temperature of lubricant oil and ground. So it is important to establish the knowledge database of fault types referred to vague sets according to diagnostics derived from practical experience.

In vibration fault diagnosis of the generator sets, Ye established the relationship between fault symptoms and causations of the turbine, shown in Table 1 (Ye, 2009).

**Table 1: Knowledge of system fault**

| Fault samples                   | Frequency range (f: operating frequency) |             |             |             |             |             |             |                |              |
|---------------------------------|--|-------------|-------------|-------------|-------------|-------------|-------------|----------------|--------------|
|                                 | 0.01-0.39 f                              | 0.40-0.49 f | 0.50 f      | 0.51-0.99 f | f           | 2 f         | 3-5 f       | Odd times of f | High         |
| frequency >5 f unbalance        | [0.00,0.00]                              | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.85,1.00] | [0.04,0.06] | [0.04,0.07] | [0.00,0.00]    | [0.00,0.00]  |
| Pneumatic force couple          | [0.00,0.00]                              | [0.28,0.31] | [0.09,0.12] | [0.55,0.70] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00, 0.00]   | [0.08, 0.13] |
| Offset center                   | [0.00,0.00]                              | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.30,0.58] | [0.40,0.62] | [0.08,0.13] | [0.00, 0.00]   | [0.00, 0.00] |
| Oil-membrane oscillation        | [0.09,0.11]                              | [0.78,0.82] | [0.00,0.00] | [0.08,0.11] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00, 0.00]   | [0.00, 0.00] |
| Radial impact friction of rotor | [0.09,0.12]                              | [0.09,0.11] | [0.08,0.12] | [0.09,0.12] | [0.18,0.21] | [0.08,0.13] | [0.08,0.13] | [0.08, 0.12]   | [0.08, 0.12] |
| Symbiosis loose fault           | [0.00,0.00]                              | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.18,0.22] | [0.12,0.17] | [0.37,0.45] | [0.00, 0.00]   | [0.22, 0.28] |
| Damage of antithrust bearing    | [0.00,0.00]                              | [0.00,0.00] | [0.08,0.12] | [0.86,0.93] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00, 0.00]   | [0.00, 0.00] |
| Surge                           | [0.00,0.00]                              | [0.27,0.32] | [0.08,0.12] | [0.54,0.62] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00, 0.00]   | [0.00, 0.00] |
| Looseness of bearing block      | [0.85,0.93]                              | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.08, 0.12]   | [0.00, 0.00] |
| Non-uniform bearing stiffness   | [0.00,0.00]                              | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.00,0.00] | [0.77,0.83] | [0.19,0.23] | [0.00, 0.00]   | [0.00, 0.00] |

There exist ten fault patterns used as the knowledge of fault samples which are represented by a vague set  $A_i$  ( $i = 1, 2, \dots, 10$ ). The power spectrum of the vibration signals referred to VSs is divided into nine ranges of different frequencies.

Now, we investigate the fault-diagnosis problems by means of the cosine similarity of vague sets. The new cosine similarity measure between two vague sets is based on three aspects of information including truth-membership functions, false-membership functions and hesitancy degree. So, a vague set can be considered as a vector representation with the three elements  $(\mu_A(x), \nu_A(x), \pi_A(x))$ . According to the ten faults knowledge samples shown in Table 1, we can express the vague sets as follows:

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and hesitancy degree. So, a vague set can be considered as a vector representation with the three elements  $(\mu_A(x), \nu_A(x), \pi_A(x))$ . According to the ten faults knowledge samples shown in Table 1, we can express the vague sets as follows:

$$A_1 = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.00, 0.00, 1.00}{x_2} + \frac{0.00, 1.00, 0.00}{x_3} + \frac{0.00, 1.00, 0.00}{x_4} + \frac{0.85, 0.00, 0.15}{x_5} + \frac{0.04, 0.94, 0.02}{x_6} + \frac{0.04, 0.93, 0.03}{x_7} + \frac{0.00, 1.00, 0.00}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

$$A_2 = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.28, 0.69, 0.03}{x_2} + \frac{0.09, 0.88, 0.03}{x_3} + \frac{0.55, 0.30, 0.15}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.00, 1.00, 0.00}{x_6} + \frac{0.00, 1.00, 0.00}{x_7} + \frac{0.00, 1.00, 0.00}{x_8} + \frac{0.08, 0.87, 0.05}{x_9}$$

$$A_3 = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.00, 1.00, 0.00}{x_2} + \frac{0.00, 1.00, 0.00}{x_3} + \frac{0.00, 1.00, 0.00}{x_4} + \frac{0.30, 0.42, 0.28}{x_5} + \frac{0.40, 0.38, 0.22}{x_6} + \frac{0.08, 0.87, 0.05}{x_7} + \frac{0.00, 1.00, 0.00}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

$$A_4 = \frac{0.09, 0.89, 0.02}{x_1} + \frac{0.78, 0.18, 0.04}{x_2} + \frac{0.00, 1.00, 0.00}{x_3} + \frac{0.08, 0.89, 0.03}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.00, 1.00, 0.00}{x_6} + \frac{0.00, 1.00, 0.00}{x_7} + \frac{0.00, 1.00, 0.00}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

$$A_5 = \frac{0.09, 0.88, 0.03}{x_1} + \frac{0.09, 0.89, 0.02}{x_2} + \frac{0.08, 0.88, 0.04}{x_3} + \frac{0.09, 0.88, 0.03}{x_4} + \frac{0.18, 0.79, 0.03}{x_5} + \frac{0.08, 0.87, 0.05}{x_6} + \frac{0.08, 0.87, 0.05}{x_7} + \frac{0.08, 0.88, 0.04}{x_8} + \frac{0.08, 0.88, 0.04}{x_9}$$

$$A_6 = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.00, 1.00, 0.00}{x_2} + \frac{0.00, 1.00, 0.00}{x_3} + \frac{0.00, 1.00, 0.00}{x_4} + \frac{0.18, 0.78, 0.04}{x_5} + \frac{0.12, 0.83, 0.05}{x_6} + \frac{0.37, 0.55, 0.08}{x_7} + \frac{0.00, 0.10, 0.00}{x_8} + \frac{0.22, 0.72, 0.06}{x_9}$$

$$A_7 = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.00, 1.00, 0.00}{x_2} + \frac{0.08, 0.88, 0.04}{x_3} + \frac{0.86, 0.07, 0.09}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.00, 1.00, 0.00}{x_6} + \frac{0.00, 1.00, 0.00}{x_7} + \frac{0.00, 0.10, 0.00}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

$$A_8 = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.27, 0.68, 0.05}{x_2} + \frac{0.08, 0.88, 0.04}{x_3} + \frac{0.00, 0.86, 0.07}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.00, 1.00, 0.00}{x_6} + \frac{0.00, 1.00, 0.00}{x_7} + \frac{0.00, 0.10, 0.00}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

$$A_9 = \frac{0.85, 0.07, 0.08}{x_1} + \frac{0.00, 1.00, 0.00}{x_2} + \frac{0.00, 1.00, 0.00}{x_3} + \frac{0.00, 1.00, 0.00}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.00, 1.00, 0.00}{x_6} + \frac{0.00, 1.00, 0.00}{x_7} + \frac{0.08, 0.88, 0.04}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

$$A_{10} = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.00, 1.00, 0.00}{x_2} + \frac{0.00, 1.00, 0.00}{x_3} + \frac{0.00, 1.00, 0.00}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.77, 0.17, 0.07}{x_6} + \frac{0.19, 0.77, 0.04}{x_7} + \frac{0.00, 1.00, 0.00}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

Suppose that the VSs of fault-testing samples are as follow:

$$B_1 = \frac{0.00, 1.00, 0.00}{x_1} + \frac{0.00, 1.00, 0.00}{x_2} + \frac{0.10, 0.90, 0.00}{x_3} + \frac{0.90, 0.10, 0.00}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.00, 1.00, 0.00}{x_6} + \frac{0.00, 1.00, 0.00}{x_7} + \frac{0.00, 1.00, 0.00}{x_8} + \frac{0.00, 1.00, 0.00}{x_9}$$

$$B_2 = \frac{0.39, 0.61, 0.00}{x_1} + \frac{0.07, 0.93, 0.00}{x_2} + \frac{0.00, 1.00, 0.00}{x_3} + \frac{0.06, 0.94, 0.00}{x_4} + \frac{0.00, 1.00, 0.00}{x_5} + \frac{0.13, 0.87, 0.00}{x_6} + \frac{0.00, 1.00, 0.00}{x_7} + \frac{0.00, 1.00, 0.00}{x_8} + \frac{0.35, 0.65, 0.00}{x_9}$$

The cosine similarity measure values of between  $B_1$  and vague sets  $A_i$  ( $i = 1, 2, \dots, 10$ ) are calculated by Eq. 6 as follows:

$$C_{VSS}(B_1, A_1) = 0.7891, C_{VSS}(B_1, A_2) = 0.9799$$

$$C_{VSS}(B_1, A_3) = 0.8282, C_{VSS}(B_1, A_4) = 0.8236$$

$$C_{VSS}(B_1, A_5) = 0.9057, C_{VSS}(B_1, A_6) = 0.8714$$

$$C_{VSS}(B_1, A_7) = 0.9995, C_{VSS}(B_1, A_8) = 0.9774$$

$$C_{VSS}(B_1, A_9) = 0.7979, C_{VSS}(B_1, A_{10}) = 0.8099$$

The vibration fault is determined according to the maximum value of cosine similarity measures. Obviously, the fault-diagnosis order of the fault-testing sample  $B_1$  is as follows:

$$A_7 \rightarrow A_2 \rightarrow A_8 \rightarrow A_5 \rightarrow A_6 \rightarrow A_3 \rightarrow A_4 \rightarrow A_{10} \rightarrow A_9 \rightarrow A_1$$

The cosine similarity measure values between  $B_2$  and vague sets  $A_i$  ( $i = 1, 2, \dots, 10$ ) are calculated by Eq. 6 as follows:

$$C_{VSS}(B_2, A_1) = 0.8563, C_{VSS}(B_2, A_2) = 0.9128$$

$$C_{VSS}(B_2, A_3) = 0.9066, C_{VSS}(B_2, A_4) = 0.8953$$

$$C_{VSS}(B_2, A_5) = 0.9738, C_{VSS}(B_2, A_6) = 0.9567$$

$$C_{VSS}(B_2, A_7) = 0.8720, C_{VSS}(B_2, A_8) = 0.9201$$

$$C_{VSS}(B_2, A_9) = 0.9403, C_{VSS}(B_2, A_{10}) = 0.8938$$

Therefore, for the fault-testing sample  $B_1$ , we can think that the vibration of the turbine is firstly resulted from damage of antithrust bearing and then pneumatic force couple. This result is the same as in the study by Ye (2009).

Similarly, the fault-diagnosis result of the fault-testing sample  $B_2$  is as follows:

$$A_5 \rightarrow A_6 \rightarrow A_9 \rightarrow A_8 \rightarrow A_2 \rightarrow A_3 \rightarrow A_4 \rightarrow A_{10} \rightarrow A_7 \rightarrow A_1$$

According to the above diagnosis order of the fault-testing sample  $B_2$ , we can think that the vibration of the turbine is resulted from radial impact friction of rotor. By actual checking, the vibration of the turbine is mainly caused by the friction between the rotor and cylinder body in the turbine (Ye, 2009). So, the diagnosis results consistent with the actual situation.

### CONCLUSION

Aiming at complexity and uncertainty of relation between vibration and fault types of turbine, a new

method of fault diagnosis of turbine was proposed on the basis of the vague cosine similarity measure. In the study, we have provided an improved cosine similarity measure for vague sets, with considering truth-membership functions, false-membership functions and hesitancy degree of vague sets. The new cosine similarity for vague sets has more information than the existing cosine similarity measures of VSs in dealing with vagueness and uncertainty problems, furthermore, it can overcome the undefined problem when the degree of membership and degree of non-membership are zero, respectively. In the study, we also used two examples of the steam turbine-generator sets to illustrate the application of the proposed cosine similarity of vague sets. The diagnosis results prove that the new cosine similarity measure of vague sets is simple and effective and the method can be used for fault diagnosis of turbine fault-diagnosis.

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