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An Empirical Research on Dependence Mode of Style Assets in China Based on Vine Copula Model

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Abstract: In this study, we firstly describe the marginal distribution of style assets in Chinese stock market based on AR (1)-GJR (1, 1)-SkT (v, λ) model. These seven style assets in this analysis are including Large-cap Growth, Large-cap Value, Mid-cap Growth, Mid-cap Value, Small-cap Growth, Small-cap Value and Debt. And then, we introduced vine copula functions to analyze the dependence among these style assets. Finally, we make a comprehensive comparison between C vine copula and D vine copula model according to their good fitness in order to select the best vine copula. The result shows that there are structural differences among these styles assets dependence in Chinese stock market. These dependence among style assets show asymmetry and nonlinear character. The traditional method of linear correlation such as person correlation can not analyze this phenomenon. D vine copula model can best describe the dependencies structure among these styles assets. On the whole, the dependence between the same kinds of style assets is larger than that between different kinds of style assets. Among the same kind of style assets, the greater the gap between style assets size, the smaller dependence is. Unconditional dependence is larger than that of the conditional dependence among vine copula model. We give some advice to reduce the risk of style portfolio according to research findings.

Key words: Style asset in stock market, dependence structure, D-vine copula, C-vine copula, pair copula

INTRODUCTION

Sklar (1959) first proposed a joint distribution can be decomposed into k edge distribution and a copula function andthis copula function is to describe the dependencies between variables structure. Then more and more scholars have used copula model to analyze the dependency structure between the currency markets, stock markets and other markets (Patton, 2001; Kolev *et al*., 2006). Because copula technology can model the entire joint distribution and it is easily generalized to the case of conditional distributions which can closely match the real world. Therefore, copula has become a powerful tool to solve financial risk management, portfolio selection, asset pricing and other aspects. Existing research mostly only build two-dimensional copula model and less involved in high-dimensional copula model. Bivariate copula function can not build a joint distribution of high-dimensional portfolio and faces the "dimensionality curse" problem. At the same time, there is also difficulty in estimating high-dimensional copula model with the bivariate copula function.

Therefore, how to construct and estimate high-dimensional copula model has become a research focus and difficult problem.

In recent years, a method to model high-dimensional copula (vine copula) has been received attention. Vine copula is based on pair-copula construction (PCC) technology. Other scholars (Bedford and Cooke, 2001; Bedford and Cooke, 2002; Aas*et al*., 2009; Czado*et al*., 2012) have made great empirical research on PCC model. PCC model can decompose the high-dimensional joint distribution into series of pair-copula construction with strong modeling flexibility. PCC generally use the vine structure of graphical tool to decompose high-dimensional distribution. There are generally two kinds of typical vine structure: C vine copula and D vine copula. Existing research has focused on issues such as vine copula model parameter estimation methods, compare the goodness of fit, vine structure selection and dynamic risk measure of portfolio based on vine copula model (Fischer*et al*., 2009; Mendes*et al*., 2010).

Domestic scholars mainly make empirical research based on bivariate copula, while relatively little research is about vine copula model. Wei and Zhang (2007), Li and Shi (2007) and Yi (2012) have analyze the dependency structure of the stock market and use Monte Carlo simulation techniques to measure market risk VaR based on copula model. As for vine copula model, Wang *et al*. (2011) have proposed threshold mixed copula model to study the dependence between Chinese stock market and bond market. Cao and Chen (2011) and Gao (2013)and other scholars using C vine copula model to study the dependency structure and its VaR among exchange rate market, the stock market. However, these studies all used the same type of pair copula model to build C vine copula, ignoring the dependence difference between various yield. These scholars also do not make comparison of goodness of fit between C vine copula and D vine copula model.

In summary, most scholars make theoretical research and practical applications based binary copula model. Few scholars dedicated to analysis the dependence structure among high-dimension style assets in China stock market. However, investors often construct multi-class portfolio in practice. Therefore, how to accurately depict the dependence structure between various assets, construction joint distribution function which closely matches to the practice is an important issue. This can enhance asset allocation performance and the risk measurement capability of portfolio.

In this study, in order to capture series non-normal distribution characters such as autoregressive, conditional heteroscedasticity, biased, thick tail and spikes, we firstly describe the marginal distribution of style assets in Chinese stock market based on AR (1)-GJR (1, 1)-SkT (v, λ) model. And then, we introduced Vine copula functions to analyze the dependence among these style assets. Finally, we make a comprehensive comparison good fitness between C vine copula and D vine copula model according to their in order to select the best vine copula. Remainder of this study is organized as follows: second section introduces vine copula model and research methodology; third section makes empirical research on dependence among Chinese style assets in stock market. Finally, concluding section.

VINE COPULA MODEL AND RESEARCH METHODOLOGY

Vine copula model: Bedford and Cooke (2001) firstly introduced regular vine graphic to model high-dimensional copula model. Two special cases are C vine copula and D vine copula. Taking seven-dimensional case as an example, Fig. 1 and 2 show the structure of the two vines.

Figure 1 shows seven-dimensional exploded view of the C vine copula. Each tree has a master node, the master node is connected with other nodes, each edge thus formed and corresponds to a pair copula. Figure 2 shows the seven-dimensional exploded view of the structure of

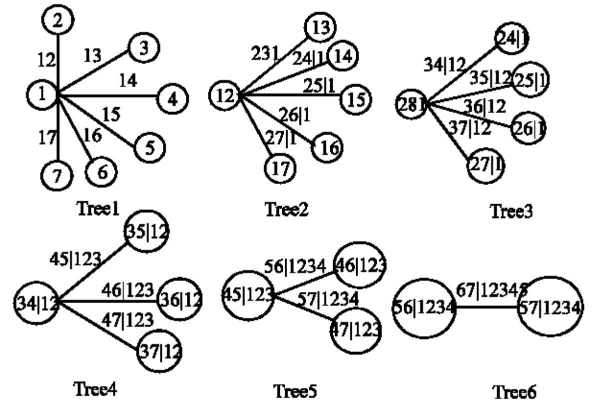

Fig. 1: C vine copula model

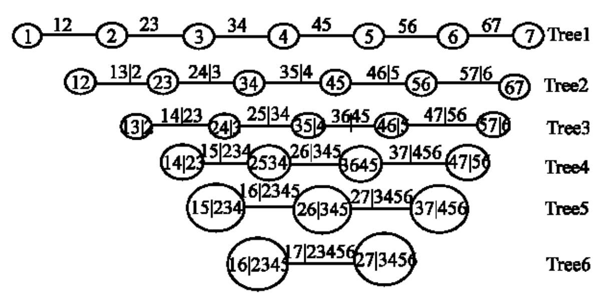

Fig. 2: D vine copula model

D vine copula. These vine copula has 6 trees, namely, tree (i = 1,..., 6), tree has 8-i nodes and 7-i edges. Each edge corresponds to a pair-copula.

Research methodology: Marginal distribution function: Considering the autoregression, conditional heteroscedasticity, skewed, fat-tailed, leptokurtic and other features of yield series of style assets, as well as the aggregation and asymmetry of fluctuation of return series, this study depicts the distribution feature of style assets by introducing AR (1)-GJR (1, 1) model:

$$R_{i,t} = c_0 + e_{i,t}; i=1,2,\cdots 7 \quad (1)$$

$$e_{i,t} = h_{i,t}\varepsilon_{i,t}, \varepsilon_{i,t} \sim SkT(v,\lambda) \quad (2)$$

$$h_{i,t} = \omega_{i,t} + \alpha e^2_{i,t-1} + \beta h_{i,t-1} + \gamma e^2_{i,t-1} I(e_{i,t-1} < 0) \quad (3)$$

Obviously, there are 7 parameters in each marginal distribution model. Equation 1 is mean equation including parameter c_0 and $e_{i,t}$. $e_{i,t}$ is the residual of style asset return and i = 1, 2,..., 7 represents different styles, i.e. Large-cap Growth, Large-cap Value, Mid-cap Growth, Mid-cap

Value, Small-cap Growth, Small-cap Value and Debt respectively. Equation 2 is skewed student-t distribution function including parameter ν and λ. The skewed student-t distribution function can simultaneously depict some non-normal distribution characteristics such as spikes, thick tail and skew. Hansen (1994) proposed a deflection concept about skewed student-t distribution and give its density function. Symbols λ represent the degrees of freedom parameter and asymmetry parameters. Equation 3 is variance equation which includes 4 parameters ω; α, β, γ; $(e_{i,t-1}<0)$ is an indicative index. If $e_{i,t-1}<0$, it is 1, otherwise it is 0. It indicates that the return series fluctuation facing negative impact is stronger than the in the scenario of positive impact. For GRJ (1, 1) Model, Equation 3 is also subject to the following constraint:

$$\alpha+2\,\beta<2,\ \alpha>-\gamma,\ \beta\epsilon(0,\ 1) \tag{4}$$

EMPIRICAL ANALYSIS

Data selection and analysis: In our analysis, the style index issued by S&P/CITIC (Large-cap Value, Large-cap Growth, Mid-cap Value, Mid-cap Growth, Small-cap Value, Small-cap Growth and Debt) is chose. All sample data is daily closing restoration prices after ex-right in the period from 27 February, 2004 to 11 September, 2012. A total number of 2081 sample data is available. There are 2080 yield data each of the series, which are calculated as follows:

$$R_{i,t} = 100 * \mathrm{Ln}(P_{i,t} / P_{i,t-1}) \tag{5}$$

In the above equation, R_{it} represents the logarithmic yield of index i during period t, while P_{it} represents the closing price of index i at the end of period t; P_{it} represents the closing price of index I at the end of period t-1. R_i (i = 1, 2.., 7 represents the return series of style assets such as Large-cap Growth (LG), Large-cap Value (LV), Mid-cap Growth (MG), Mid-cap Value (MV), Small-cap Growth (SG), Small-cap Value (SV) and Bond Index (Debt).

Table 1 shows that there are higher person correlations among these style assets. All style assets demonstrate left skewness and leptokurticness. The small-cap index shows the greatest left skewness, followed by mid-cap index and large-cap index. According to the result of JB test, all style assets reject the assumption of normal distribution at 1% confidence level. We carry out stationary test on the sample data. As shown by the ADF statistics and P value (with constant parameters but no time trend parameters included), all style assets reject the null assumption of normal distribution at 99% significance level. No unit root is found in the sequence, which means that all style return series are all stationary time sequences with left skewness and leptokurticness features.

Edge distribution parameter estimation: According to features of all style assets return, we use GJR (1, 1)-SkT (ν, λ) to create the marginal distribution functions for all

Table 1: Descriptive statistics of style asset return (2004-2012)

Style asset	LG	LV	MG	MV	SG	SV	Debt
Mean	0.04	0.03	0.04	0.04	0.05	0.05	0.01
Max	9.23	9.27	9.26	9.19	9.35	9.34	0.90
Min	-9.79	-9.43	-9.89	-9.61	-9.10	-9.27	-0.71
SD	1.96	1.93	2.07	2.04	2.12	2.12	0.08
Skewness	-0.23	-0.40	-0.42	-0.53	-0.51	-0.68	-0.16
Kurtosis	5.41	6.02	5.13	5.57	5.46	5.62	20.60
Jarque-Bera	519.00	846.55	453.96	671.41	613.09	754.58	26864.93
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LM (2)	31.26	48.3	49.47	62.6	67.52	76.34	91.85
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LM (4)	34.05	40.66	37.73	39.81	48.08	46.46	48.44
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
LM (10)	22.56	23.35	21.86	22.74	26.21	24.26	25.35
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Pearson correlation							
LG	1	0.88***	0.92**	0.88**	0.91**	0.86**	0.031
LV		1	0.88**	0.93**	0.88**	0.88**	0.055**
MG			1	0.93**	0.96**	0.92**	0.032
MV				1	0.94**	0.96**	0.042
SG					1	0.95**	0.038
SV			.			1	0.04
Debt							1

Note: ***,**Mean that the statistical parameter is significantly at the 1 and 5% confidence level, respectively, JB statistics and LM statistic p-value are in parentheses. Data source is JuYuan database

Table 2: Marginal distribution parameter estimation result

	c_0	ω	α	β	γ	ν	λ	Log-likelihood value	K-S statistics
LG	0.0246	0.0298**	0.0497***	0.9376***	0.0122	7.1267***	-0.0317	-4105.632	0.015(0.67)
	(0.027)	(0.016)	(0.012)	(0.015)	(0.019)	(1.069)	(0.027)		
LV	-0.0151	0.0198	0.0585	0.9383	0.0000	6.3573	-0.0550	-4017.833	0.017(0.65)
	(15.984)	(0.413)	(11.041)	(3.273)	(31.967)	(469.709)	(7.663)		
MG	-0.0107	0.0523***	0.0661***	0.9178***	0.0114	8.4869***	-0.1465***	-4231.530	0.013(0.72)
	(0.060)	(0.025)	(0.021)	(0.019)	(0.050)	(1.563)	(0.029)		
MV	-0.0165	0.0436	0.0717	0.9204	0.0000	6.9465	-0.1805	-4151.354	0.012(0.75)
	(0.863)	(0.561)	(1.282)	(0.680)	(0.0000)	(41.980)	(0.264)		
SG	0.0260	0.0822***	0.0699***	0.8988***	0.0265	8.4887***	-0.1966***	-4246.633	0.014(0.69)
	(0.027)	(0.031)	(0.015)	(0.020)	(0.021)	(1.484)	(0.030)		
SV	0.0077	0.0698**	0.0800***	0.9072***	0.0000	7.0240***	-0.2590***	-4228.437	0.015(0.68)
	(0.021)	(0.026)	(0.015)	(0.017)	(0.015)	(1.080)	(0.028)		
Debt	0.0106**	0.0003	0.3508	0.7160***	0.0000	3.1134***	-0.0637***	2920.795	0.024
	(0.002)	(0.000)	(0.220)	(0.159)	(0.029)	(0.251)	(0.035)		(0.54)

Values set out in brackets are relevant standard deviations. **Mean being significant at the level of 5% and ***Means being significant at the level of 1%

Table 3: Alternative Copula Model Summary

Number	1	2	3	4	5	6	7
Copula type	Gaussian	Student t	Clayton	Gumbel	Frank	Joe	BB1
Number	8	9	10	11	12	13	14
Copula type	BB6	BB7	BB8	Rotated clayton (180°)	Rotated gumbel (180°)	Rotated clayton (90°)	Rotated gumbel (90°)

In Table 3, BB1 is Clayton-Gumbel Copula, BB6 is Joe-Gumbel Copula, BB7 is Joe-Clayton Copula, BB8 is Joe-Frank Copula

style assets. In the following step, marginal distribution parameters are estimated by Matlab (2011b) software. The result is detailed in Table 2 below. The K-S statistics and K-S probability value are obtained by following a few steps. The first is probability integral transformation of the original sequence using the conditional marginal distribution based on estimation, followed by testing whether the transformed sequences follow uniform distribution (0, 1) with K-S test approach. The K-S statistics and K-S probability value as set out in Table 2 show that both of the two sequences accept null assumption that the transformed sequence follows uniform distribution (0, 1). Autocorrelation test is also conducted to each of the transformed sequences and the shows that they are not auto-correlated. Therefore, it can be concluded that the two transformed sequences are independent to each other. Based on the above analysis, we can draw the conclusion that GJR (1, 1)-SkT (v, λ) model fits well with the conditional marginal distribution of each sequence and is sufficient for describing the conditional marginal distribution of each yield.

The GJR (1, 1)-SkT (v, λ) model based on estimation can be used to determine the conditional distribution of style assets. According to the above conditional distribution and after probability integral transformation of the original sequences, we can get two new sequences $\{u_t\}$ and $\{v_t\}$, both following uniform distribution (0, 1). The result can serve as the basis for subsequent related empirical analysis.

Pair copula type selection in vine copula model: There are many types of copulas in academia, Table 3 listed these more representative copula model and give their number.

Table 4: Type selection of pair copula and parameter estimation results based on C vine copula and D vine copula

	AIC	BIC	Likelihood value
C vine copula	-24089.6	-23880.9	12081.81
D vine copula	-24106.4	-23903.3	12089.19

In this study, we choose the type of pair-copula model according to the rule with maximum-value of likelihood function and minimum value of AIC and BIC. We using R 3.0 software to finish the analysis and referent the algorithm proposed by Aas *et al*. (2009). Sample data involves seven kinds of style assets, so the dimensions are 7. According to pair-copula decomposition principle, there is 21 type of pair copula need to be determined which are corresponds to 21 dependences. According to the result in Table 4, D vine copula is more suitable to describe the asymmetrical and nonlinear dependence among the seven style asset in Chinese stock market because it has the max value of likelihood function and mix value of AIC and BIC.

Dependence among the style asset based on D Vine copula model: Figure 3 gives the dependence among seven style asset in China stock market according to result from Table 4.

In Fig. 3, these nodes in tree.1 represent these style assets. Each connection between two nodes represents the dependence. The dependence parameter and suitable type of pair copula are show under the connection. Seen from tree.1 level, there are relative higher unconditional correlations among the seven style asset but the type of pair copula varies with different style assets. For example, Student t copula model is suitable to describe the dependence between Large-cap Growth

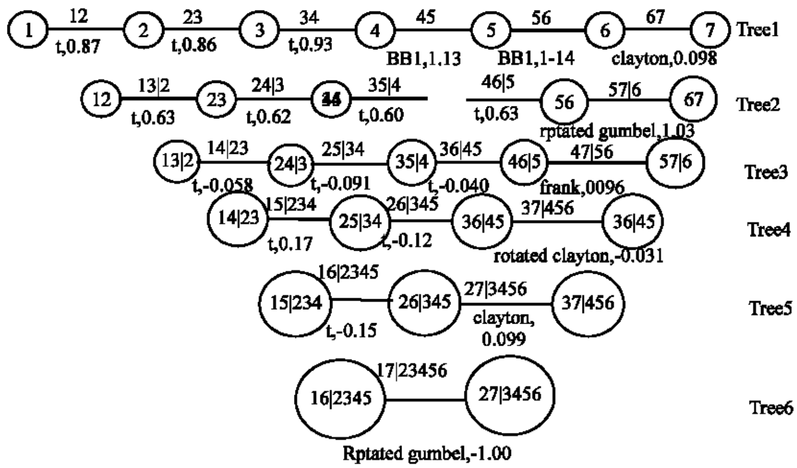

Fig. 3: Dependence based on D vine copula model

Table 5: Good fitness of different vine copula model

Internal pair-copula type in vine copula model	C vine copula model AIC	C vine copula model BIC	C vine copula model Likelihood value	D vine copula model AIC	D vine copula model BIC	D vine copula model Likelihood value
C (1,1…1)	-23104.64	-22986.2	11573.32	-23107.00	-22988.56	11574.5
C (2,2…2)	-24084.53	-23847.64	12084.26	-24089.33	-23852.44	12086.66
C (5,5…5)	-22181.69	-22063.24	11111.84	-21997.56	-21879.11	11019.78
C (6,6…6)	-16274.66	-16101.36	8158.33	-16889.93	-16771.48	8465.963
C (7,7…7)	-23926.66	-23689.78	12005.33	-23801.25	-23564.36	11942.62
C (8,8…8)	-21132.55	-20895.66	10608.27	-21501.7	-21264.82	10792.85
C(9,9…9)	-23079.4	-22842.52	11581.70	-22947.34	-22710.45	11515.67
C(10,10…10)	-21241.56	-21004.67	10662.78	-20482.37	-20245.49	10283.19
C(11,11…11)	-16693.28	-16574.84	8367.64	-17144.22	-17025.78	8593.111
C(2,2,2,2,2,3,2,2,2,2, 3,2,2,7,13,7,2,1,2,1,2)	-24089.62	-23880.93	12081.81			
C(2,2,2,7,7,3,2,2,2,2, 14,2,2,2,5,2,2,13,2,3,14)				-24106.39	-23903.34	12089.19

Note: Table 5, C (1,1…1) mean that the vine copula model is made up of all same type pair copula of Gaussian-copula

and Large-cap Value, the same is true for Large-cap Value and Mid-cap Growth, Mid-cap Growth and Mid-cap Value. Their dependence levels are closer, 0.87, 0.86 and 0.93, respectively. This shows that symmetry and fat tail distribution among above style asset. Meanwhile, the dependence between Mid-cap Value and Small-cap Growth is 1.13 which is suitable for the Clayton-Gumbel copula model. The dependence between Mid-cap Growth and Mid-cap Value is 1.14, it is also suitable for the Clayton-Gumbel copula model which reflect on the upper tail or lower tail of asymmetry distribution. The dependence mode between Small-cap Value and Debt is suitable for Clayton copula model which show the lower tail distribution character.

Seen from tree.2-tree.5, dependences between all style asset are decreased, even negative correlation. For example, in tree.2, the dependence between Large-cap Growth and Mid-cap Growth on condition of Large-cap Value is reduced to 0.63 compared with their unconditional dependence of 0.93. The same to these dependence of Large-cap Value and Mid-cap Value, Mid-cap Growth and Small-cap Growth, Mid-cap Value and Small-cap Value. On the whole, the dependence between the same kinds of style assets is larger than that between different kinds of style assets. Among the same kind of style assets, the greater the gap between style assets size, the smaller dependence is. Unconditional dependence is larger than that of the conditional dependence among Vine copula model. According to the results, there are two ways to improve the efficiency of asset allocation According to the results, there are two ways to improve the efficiency of asset allocation. One way is to balanced the asset allocation ratio, another way is to increase the size gap of same type of style assets in allocation assets. These practices can reduce internal correlation in the portfolios and reduce its overall risk finally.

Robustness test: In order to highlight the good fitness of D vine copula model comparing with other vine model, this study also calculate the good fitness of C vine copula model and D vine copula model with same type of pair copula. The result is show in table 5. According to value of likelihood function, AIC and BIC of different Vine copula model, we can see the D vine copula model with composite structure of C (2, 2, 2, 7, 7, 3, 2, 2, 2, 2, 14, 2, 2, 2, 5, 2, 2, 13, 2, 3, 14) is most suitable to analysis the asymmetric, nonlinear dependence structure of style assets in China stock market.

The number is from the Table 3. Each bracket has 21 copula type, ellipsis indicates the same number, the other is so on. These pair copula models (whose number are 3, 4, 13, 14) do not exit in the vine copula model which is made up of all same pair copula, because theses copula model is not suitable for negative correlation coefficient in data.

CONCLUSION

This study taken seven style assets in China stock market as the study sample. The kind of style asset in this analysis is including Large-cap Growth, Large-cap Value, Mid-cap Growth, Mid-cap Value, Small-cap Growth, Small-cap Value and Debt. The aims is to analysis the complex dependence among these style assets and build joint distribution function which is more closely matches the real world. As style asset series show some non-normal distribution characters such as autocorrelation, time-varying, biased, volatility clustering, thick tail spikes, we firstly describe the marginal distribution of style assets based on AR (1)-GJR (1, 1)-SkT (v, λ) model. And then, we introduced vine copula functions to analyze the dependence among these style assets. Finally, we make a comprehensive comparison between C-vine copula and D-vine copula model according to their good fittness in order to select the best vine copula. The result shows that there are structural differences among these styles assets dependence in Chinese stock market. These dependence among style assets show asymmetry and nonlinear character. The traditional method of linear correlation such as person can not analyze this phenomenon. Single type of copula model can not describe this complex dependency patterns. D-vine copula model can best describe the dependencies structure among these styles assets. On the whole, the dependence between the same kinds of style assets is larger than that between different kinds of style assets. Among the same kind of style assets, the greater the gap between style assets size, the smaller dependence is. Unconditional dependence is larger than that of the conditional dependence among vine copula model. The conclusion also shows some inspiration to portfolio construction. there are two ways to improve the efficiency of asset allocation According to the results, there are two ways to improve the efficiency of asset allocation. One way is to balance the asset allocation ratio, another way is to increase the size gap of same type of style assets in allocation assets. These practices can reduce internal correlation in the portfolios and reduce its overall risk finally.

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