Journal of Applied Sciences

ISSN 1812-5654

science alert

ANSInet
an open access publisher
http://ansinet.com

Journal of Applied Sciences 13 (11): 1948-1953, 2013
ISSN 1812-5654 / DOI: 10.3923/jas.2013.1948.1953

Sliding Mode Control for Uncertain Discrete Large-scale Systems with Delays and Unmatched

Hejun Yao, Yingying Lian, Fushun Yuan, Xvling Hou
School of Mathematics and Statistics, Anyang Normal University, 455002, Anyang, Henan, China

Abstract: The problem of sliding mode control for uncertain discrete-time large-scale systems with delays is considered in this stduy. The systems have time varying and unmatched uncertainty needed to satisfy the norm bounded condition. The sliding-mode surface is designed by linear matrix inequality approach. Then the sliding mode controller satisfied the arriving condition is designed. The conservative feature is overcome in the traditional sliding mode control approach which needs matched uncertainty. A detailed example is presented to illustrate the proposed methodology.

Key words: Sliding mode control, discrete-time systems, large-scale system, delay, linear matrix inequality

INTRODUCTION

Time delay is frequently encountered in various engineering, communication and biological systems. It is well known that time delays can degrade a system's performance and even cause system instability. Therefore, the study of delay systems has received much attention and various analysis and synthesis methods have been developed over the past years (Shyu *et al*., 2005; Liu, 2003; Xu and Lam, 2005). The problem of robust H∞ control for uncertain discrete-time systems with time-varying delays has been considered by Xu and Chen (2004). Exponential stability output feedback controllers are designed in terms of LMI. A sliding mode control of delay systems is obtained by LMI in (Gouaisbaut, 2002). Park considered the problems of robust non-fragile control for uncertain discrete-delay large-scale system under state feedback gain variations. Based on the Lyapunov stability theory, the state feedback control design for robust stability is given in terms of linear matrix inequality (Park, 2004).

The sliding mode control approach, based on using of discontinuous control laws, is known to be an efficient alternative way to tackle many challenging problems of robust stabilization (Chen, 2006; Kau *et al*., 2005). For instance, an appropriate sliding mode strategy can achieve stabilization by "dominating" nonlinear terms and additive disturbances, provided some appropriate "matching conditions" hold. However, the combination of delay phenomenon with relay actuators makes the situation much more complex: designing a sliding controller without taking delays into account may lead to unstable or chaotic behaviors or, at least, results in highly chattering behaviors. Recently, the problems of sliding mode control for uncertain systems with delays have been studied (Janardhanan *et al*., 2004; Mi and Jing, 2006; Mi *et al*., 2006). But on the sliding mode control for time-delay large-scale systems, a few results have been present. In, Zhang *et al*. (2002), Shyu considered the design method of large-scale time-delayed system with dead-zone input via sliding mode control. For the discrete time-delay large-scale systems, we have not found any results on sliding mode control. In this stduy, we consider the problem of sliding mode control for uncertain discrete-time large-scale system with delays and unmatched uncertainty. The sliding-mode surface is designed by LMI approach. Then the sliding mode controller satisfied the arriving condition is given.

PROBLEM FORMULATION

Consider the following uncertain discrete-delay large-scale systems composed of N interconnected subsystems described by:

$$\begin{aligned} s_i:\ & (i=1,2,\cdots,N) \\ & x_i(k+1)=(A_i+\Delta A_i(k))x_i(k)+B_iu_i(k) \\ & \qquad +\sum_{j=1}^{N}(A_{ij}+\Delta A_{ij}(k))x_j(k-h_{ij}), \\ & x_i(k)=\psi_i(k) \qquad -h\le k\le 0 \end{aligned} \tag{1}$$

where, $x_i(k)\in R^{ni}$ is the state vector, $u_i(k)\in R^{mi}$ the control input, h_{ij} positive integers representing the delays of the system, $h=\max_{i,j=1,2,\cdots,N}\{h_{ij}\}$, $\Psi_i(k)$ real-valued initial function

Corresponding Author: Hejun Yao, School of Mathematics and Statistics, Anyang Normal University, 455002, Anyang, Henan, China

on [-h, 0], A_i, B_i and A_{ij} known real constant matrices with appropriate dimensions, $\Delta A_i(k)$, $\Delta A_{ij}(k)$ unknown matrices representing time-varying parameter uncertainties:

- Assumption 1 B_i is full-rank
- Assumption 2 The matrix pair (A_i, B_i) is controllable

For the systems (1), there are nonsingular transformations matrices $T_i\Delta A_i(k)$, $T_i\Delta A_{ij}(k)$ which make the systems (1) be equivalent to:

$$\begin{aligned} z_i(k+1) &= \begin{bmatrix} z_{i1}(k+1) \\ z_{i2}(k+1) \end{bmatrix} = T_i x_i(k+1) \\ &= T_i(A_i + \Delta A_i(k))x_i(k) + T_i B_i u_i(k) \\ &\quad + \sum_{j=1}^{N}(T_i A_{ij} + T_i \Delta A_{ij}(k))x_j(k-h_{ij}) \\ &= T_i A_i T_i^{-1} z_i(k) + T_i \Delta A_i(k) T_i^{-1} z_i(k) \\ &\quad + T_i B_i u_i(k) + \sum_{j=1}^{N}(T_i A_{ij} T_i^{-1} \\ &\quad + T_i \Delta A_{ij}(k) T_i^{-1}) z_j(k-h_{ij}) \end{aligned} \tag{2}$$

Where:

$$T_i A_i T_i^{-1} = \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} \tag{3}$$

$$T_i \Delta A_i(k) T_i^{-1} = \begin{bmatrix} 0 \\ \Delta A_{i2}(k) \end{bmatrix} \tag{4}$$

$$T_i B_i = \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} \tag{5}$$

$$T_i A_{ij} T_i^{-1} = \begin{bmatrix} A_{ij11} & A_{ij12} \\ A_{ij21} & A_{ij22} \end{bmatrix} \tag{6}$$

$$T_i \Delta A_{ij}(k) T_i^{-1} = \begin{bmatrix} 0 \\ \Delta A_{ij2}(k) \end{bmatrix} \tag{7}$$

According to Eq. 2-7, we obtain:

$$\begin{aligned} z_i(k+1) &= \begin{bmatrix} A_{i11} & A_{i12} \\ A_{i21} & A_{i22} \end{bmatrix} z_i(k) + \begin{bmatrix} 0 \\ \Delta A_{i2}(k) \end{bmatrix} z_i(k) \\ &\quad + \begin{bmatrix} 0 \\ B_{i2} \end{bmatrix} u_i(k) + \sum_{j=1}^{N} \{ \begin{bmatrix} A_{ij11} & A_{ij12} \\ A_{ij21} & A_{ij22} \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ \Delta A_{ij2}(k) \end{bmatrix} \} z_j(k-h_{ij}) \end{aligned} \tag{8}$$

With the feature of B_i, we can obtain that the $\Delta A_{i2}(k)$, $\Delta A_{ij2}(k)$ satisfy the traditional matched conditions $\Delta A_{i2}(k) = B_{i2}\Delta L_{i2}(k)$, $\Delta A_{ij2}(k) = B_{i2}\,\Delta M_{ij2}(k)$, where $\Delta l_{i2}(k)$, $\Delta M_{ij2}(k)$ satisfying:

$$\|\Delta L_{i2}(k)\| \le L_{i2}, \quad \|\Delta M_{ij2}(k)\| \le M_{ij2} \tag{9}$$

where, L_{i2}, M_{ij2} are constants.

And the unmatched uncertainties $\Delta A_{i1}(k)$, $\Delta A_{ij1}(k)$ are assumed to be of the forms:

$$\Delta A_{i1}(k) = D_i F(k) E_i, \quad \Delta A_{ij1}(k) = D_{ijd} F(k) E_{ijd} \tag{10}$$

where, D_i, E_i, D_{ijd}, E_{ijd} are known real constant matrices of appropriate dimensions and E_i, E_{ijd} satisfying the following conditions:

$$E_i = E_{ia} c_i, \quad E_{ijd} = E_{ijad} c_j \tag{11}$$

F (k) is unknown time-varying matrix function satisfying:

$$F^T(k)\, F(k) \le I$$

For the uncertain discrete time-delay large-scale system (2), we choose the following sliding mode surface:

$$s_i(k) = c_i z_i(k) = \begin{bmatrix} c_{i1} & I \end{bmatrix} z_i(k) \qquad i = 1, 2, \cdots, N$$

where, c_{i1} is of appropriate dimensions. With equation $S_i(k) = 0$, we obtain:

$$z_{i2}(k) = -c_{i1} z_{i1}(k) \tag{12}$$

Then the arriving condition is given by[14]:

$$s_i^T(k)\Delta s_i(k) < 0, \quad s_i(k) \ne 0 \quad i=1,2,\cdots,N$$

MAIN RESULTS

In this section, we are going to determine a sliding mode surface and a sliding mode controller which makes the state trajectories of each subsystem move to the sliding surface in finite time.

Inserting Eq. 12 into 8, we obtain the sliding mode equation:

$$\begin{aligned} z_{i1}(k+1) &= A_{i11} z_{i1}(k) + A_{i12} z_{i2}(k) \\ &\quad + \sum_{j=1}^{N}(A_{ij11} z_{j1}(k-h_{ij}) + A_{ij12}) z_{j2}(k-h_{ij}) \\ &= (A_{i11} - A_{i12} c_{i1}) z_{i1}(k) + \sum_{j=1}^{N}(A_{ij11} \\ &\quad - A_{ij12} c_{j1}) z_{j1}(k-h_{ij}) \\ &= \bar{A}_i z_{i1}(k) + \sum_{j=1}^{N} \bar{A}_{ij} z_{j1}(k-h_{ij}) \end{aligned} \tag{13}$$

Where:

$$\overline{A}_i = A_{i11} - A_{i12}c_{i1}$$
$$\overline{A}_{ij} = A_{ij11} - A_{ij12}c_{j1} \qquad i, j = 1, 2, \cdots, N$$

Lemma1[2]: The LMI:

$$\begin{bmatrix} Y(x) & W(x) \\ W^T(x) & R(x) \end{bmatrix} > 0$$

is equivalent to:

$$R(x)>0,\ Y(x)-W(x)R^{-1}(x)W^T(x)>0$$

where, $Y(x) = Y^T(x)$, $R(x) = R^T(x)$ and $W(x)$ depend on x.

Lemma 2[12]: For any $x, y \in R^n$ and matrix function F (k) with

$$F^T(k)F(k) \le 1$$

The inequality:

$$2x^T F(k)y \le x^T x + y^T y$$

is always satisfied.

Theorem 1: The sliding mode Eq. 13 is asymptotically stable, if there exist matrices $Y_i \in R^{m_i \times (n_i - m_i)}$ $(n_i\text{-}m_i)\times(n_i\text{-}m_i)$, positive definite matrices $Q_i, T_i \in R^{(n_i-m_i)\times(n_i-m_i)}$, such that the following matrix inequalities hold:

$$\begin{bmatrix} -Q_i & \Psi_{i0} & \Psi_{i1} & \cdots & \Psi_{iN} \\ * & -Q_i + \delta_i T_i & 0 & \cdots & 0 \\ * & * & -\delta(A_{i1})T_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & -\delta(A_{iN})T_N \end{bmatrix} < 0 \qquad (14)$$

Where:

$$\Psi_{i0} = A_{i11}Q_i - A_{i12}Y_i$$
$$\Psi_{in} = A_{in11}Q_n + L_{in12}Y_n \qquad (n = 1, 2, \cdots, N)$$

where, δ(.) s a real value function satisfying:

$$\delta(0) = 0;\ \forall E \ne 0, \delta(E) = 1;\ \delta_i = \sum_{j=1}^{N} \delta(A_{ji})$$

sliding mode surface is designed as follows:

$$s_i = c_i z_i(k) = [c_{i1} \quad I] z_i(k) = \left[Y_i Q_i^{-1} \quad I\right] z_i(k)$$

- **Proof:** Choosing the following Lyapunov for sliding the sliding mode Eq. 13:

$$V(k) = \sum_{i=1}^{N} [z_{i1}^T(k)P_i z_{i1}(k)$$
$$+\sum_{j=1}^{N}\sum_{m=1}^{h_{ij}} z_{j1}^T(k-m)R_j\delta(A_{ij})z_{j1}(k-m)]$$

where, P_t, R_t are positive definite matrices in Theorem1.

With the forward difference of V (k) along the solution of system (13), we obtain:

$$\begin{aligned} \Delta V(k) &= V(k+1) - V(k) \\ &= \sum_{i=1}^{N} \{[z_{i1}^T(k+1)P_i z_{i1}(k+1) - z_{i1}^T(k)P_i z_{i1}(k)] \\ &\quad + \sum_{j=1}^{N} [z_{j1}^T(k)R_j\delta(A_{ij})z_{j1}(k) \\ &\quad - z_{j1}^T(k-h_{ij})R_j\delta(A_{ij})z_{j1}(k-h_{ij})]\} \\ &= \sum_{i=1}^{N} \{z_{i1}^T(k)[\overline{A}_i^T P_i \overline{A}_i - P_i + \delta_i R_i]z_{i1}(k) \\ &\quad - \sum_{j=1}^{N} z_{j1}^T(k-h_{ij})\delta(A_{ij})R_j z_{j1}(k-h_{ij}) \\ &\quad + 2\sum_{j=1}^{N} z_{i1}^T(k)\overline{A}_i^T P_i \overline{A}_{ij} z_{j1}(k-h_{ij}) \\ &\quad + \sum_{j=1}^{N}\sum_{l=1}^{N} z_{j1}^T(k-h_{ij})\overline{A}_{ij}^T P_i \overline{A}_{il} z_{l1}(k-h_{il})\} \\ &= \sum_{i=1}^{N} \xi^T \Sigma \xi \end{aligned}$$

where, $\xi = [z_{i1}(k)\ \ z_{11}(k\text{-}h_{i1}) \ldots x_{i1}(k\text{-}h)_{iN}]^T$:

$$\Sigma = \begin{bmatrix} J_{i0} & \overline{A}_i^T P_i \overline{A}_{i1} & \overline{A}_i^T P_i \overline{A}_{i2} & \cdots & \overline{A}_i^T P_i \overline{A}_{iN} \\ * & J_{i1} & \overline{A}_{i1}^T P_i \overline{A}_{i2} & \cdots & \overline{A}_{i1}^T P_i \overline{A}_{iN} \\ * & * & J_{i2} & \cdots & \overline{A}_{i2}^T P_i \overline{A}_{iN} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & J_{iN} \end{bmatrix} < 0$$

$$J_{i0} = \overline{A}_i^T P_i \overline{A}_i - P_i + \delta_i R_i$$
$$J_{in} = \overline{A}_{in}^T P_i \overline{A}_{in} - \delta(A_{in})R_n \qquad (n = 1, 2, \cdots, N)$$

The matrix inequality Σ<0 Eq. 15 can been rewritten as:

$$\Sigma = \begin{bmatrix} -P_i + \delta_i R_i & 0 & \cdots & 0 \\ 0 & -\delta(A_{i1})R_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -\delta(A_{iN})R_N \end{bmatrix} + \begin{bmatrix} \overline{A}_i^T \\ \overline{A}_{i1}^T \\ \vdots \\ \overline{A}_{iN}^T \end{bmatrix} P_i \left[\overline{A}_i \quad \overline{A}_{i1} \quad \cdots \quad \overline{A}_{iN}\right] < 0$$

a With lemma1, the inequality Eq. 15 is equivalent to:

$$\begin{bmatrix} -P_i^{-1} & \overline{A}_i & \overline{A}_{i1} & \cdots & \overline{A}_{iN} \\ * & -P_i+\delta_i R_i & 0 & \cdots & 0 \\ * & * & -\delta(A_{i1})R_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \cdots & -\delta(A_{iN})R_N \end{bmatrix} < 0 \quad (16)$$

Where:

$$\overline{A}_i = A_{i11} - A_{i12}c_{i1}$$
$$\overline{A}_{ij} = A_{ij11} - A_{ij12}c_{j1} \qquad i, j = 1, 2, \cdots, N$$

Pre-and post-multiplying the inequality Eq. 16 by block-diagonal matrix $\{I, P_i^{-1}, P_1^{-1}, P_2^{-1}, \cdots, P_N^{-1}\}$, By letting $Q_i = P_i^{-1}, Y_j = c_{i1}Q_j, T_i = Q_i R_i Q_i$ we can obtain the inequality Eq. 15 is equivalent to Eq. 14. With inequality Eq. 15 or 14, we know $\Delta V<0$, the sliding mode equation is asymptotically stable.

Theorem 2: For the uncertain discrete time-delay large-scale systems Eq. 1, the states will hit the sliding mode surface Eq. 13 in finite time if we choose the following controller:

$$\begin{aligned} u_i(k) = -B_{i2}^{-1}[&c_{i1}(A_{i11}z_{i1}(k) + A_{i12}z_{i2}(k) \\ &+ \sum_{j=1}^{N}(A_{ij11}z_{j1}(k-h_{ij}) + A_{ij12}z_{j2}(k-h_{ij})) \\ &+ A_{i21}z_{i1}(k) + A_{i22}z_{i2}(k) \\ &+ \sum_{j=1}^{N}(A_{ij21}z_{j1}(k-h_{ij}) + A_{ij22}z_{j2}(k-h_{ij}) \\ &- c_{i1}z_{i1}(k) - z_{i2}(k) + \frac{1}{2}s_i^T(k)c_{i1}D_iD_i^Tc_{i1}^Ts_i(k) \\ &+ \frac{1}{2}s_i^T(k)E_{ia}^TE_{ia}^Ts_i(k) \\ &+ \sum_{j=1}^{N}\frac{1}{2}s_i^T(k)c_{i1}D_{id}D_{id}^Tc_{i1}^Ts_i(k) \\ &+ \frac{1}{2}\frac{s_i}{\|s_i\|^2}\sum_{j=1}^{N}s_j^T(k-h_{ij})E_{ijad}^TE_{ijad}s_j(k-h_{ij}) \\ &+ \frac{s_i}{\|s_i\|^2}\|s_i^T(k)B_{i2}\|L_{i2} \\ &+ \frac{s_i}{\|s_i\|^2}\sum_{j=1}^{N}\|s_i^T(k)B_{i2}\|M_{ij}\|z_j(k-h_{ij})\| \\ &+ (k_{i1}\,\mathrm{sgn}\, s_i(k) + k_{i2}s_i(k))] \end{aligned} \quad (17)$$

- **Proof:** With the forward difference of s_i (k) along the solution of system (2), we obtain:

$$\begin{aligned} s_i^T(k)\Delta s_i(k) &= s_i^T(k)[c_iz_i(k+1) - c_iz_i(k)] \\ &= s_i^T(k)[c_{i1}(A_{i11}z_{i1}(k) + A_{i12}z_{i2}(k) \\ &+ \sum_{j=1}^{N}(A_{ij11}z_{j1}(k-h_{ij}) + A_{ij12}z_{j2}(k-h_{ij}))) \\ &+ A_{i21}z_{i1}(k) + A_{i22}z_{i2}(k) + B_{i2}u_i(k) \\ &+ \sum_{j=1}^{N}(A_{ij21}z_{j1}(k-h_{ij}) + A_{ij22}z_{j2}(k-h_{ij}) \\ &- c_{i1}z_{i1}(k) - z_{i2}(k) + \Pi_i(k)] \end{aligned} \quad (18)$$

Where:

$$\begin{aligned} \Pi_i(k) = &c_{i1}D_iF(k)E_iz_i(k) \\ &+ \sum_{j=1}^{N}(c_{i1}D_{ijd}F(k)E_{ijd}z_j(k-h_{ij})) \\ &+ B_{i2}\Delta L_{i2}(k) + \sum_{j=1}^{N}(B_{i2}\Delta M_{ij2}(k)z_j(k-h_{ij})) \end{aligned} \quad (19)$$

Combing the Eq. 9-11 and 19, with the lemma 2, we obtain:

$$\begin{aligned} s_i^T(k)\Pi_i(k) = &s_i^Tc_{i1}D_iF(k)E_{ia}s_i(k) \\ &+ \sum_{j=1}^{N}s_i^T(k)c_{i1}D_{ijd}F(k)E_{ijad}s_j(k-h_{ij}) \\ &+ s_i^T(k)B_{i2}\Delta L_{i2}(k) \\ &+ \sum_{j=1}^{N}s_i^T(k)B_{i2}\Delta M_{ij2}(k)z_j(k-h_{ij}) \end{aligned} \quad (20)$$

$$\begin{aligned} \le &\frac{1}{2}s_i^T(k)c_{i1}D_iD_i^Tc_{i1}^Ts_i(k) \\ &+ \frac{1}{2}s_i^T(k)E_{ia}^TE_{ia}^Ts_i(k) \\ &+ \sum_{j=1}^{N}(\frac{1}{2}s_i^T(k)c_{i1}D_{ijd}D_{ijd}^Tc_{i1}^Ts_i(k) \\ &+ \frac{1}{2}s_j^T(k-h_{ij})E_{ijad}^TE_{ijad}s_j(k-h_{ij})) \\ &+ \|s_i^T(k)B_{i2}\|L_{i2} \\ &+ \sum_{j=1}^{N}\|s_i^T(k)B_{i2}\|M_{ij}\|z_j(k-h_{ij})\| \end{aligned}$$

With the controller Eq. 17 and 18-20, we obtain:

$$s_i^T(k)\Delta s_i(k) \le -(k_{i1}\|s_i(k)\| + k_{i2}s_i^2(k)) < 0 \quad s_i(k) \ne 0$$

where, k_{i1}, k_{i2} are constants satisfying $k_{i1}>0$, $k_{i2}>0$ (i = 1, 2, ..., N)

NUMERICAL EXAMPLE

Consider a uncertain time-delay large-scale systems of Eq. 8 which is composed of the following three interconnected subsystems:

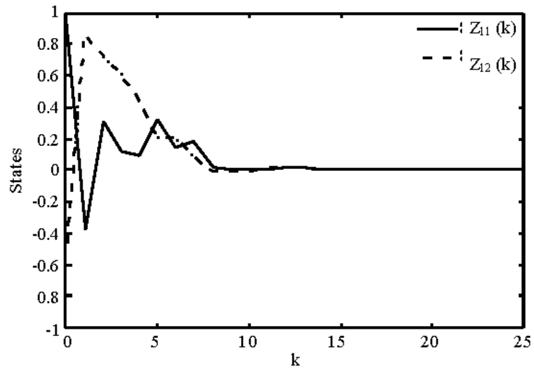

Fig. 1: State responses of subsystem 1

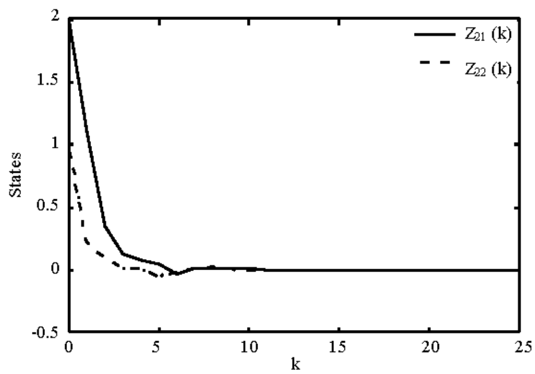

Fig. 2: State responses of subsystem 2

$$\bar{A}_1 = \begin{bmatrix} 0 & 0.5 \\ 0 & 0.1 \end{bmatrix},\ \Delta\bar{A}_1(k) = \begin{bmatrix} 0 & 0.4\cos(k) \\ 0.2\sin(k) & 0 \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \bar{A}_{12} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix}, \Delta\bar{A}_{12}(k) = \begin{bmatrix} 0 & 0 \\ 0 & 0.05\cos(k) \end{bmatrix},$$

$$\bar{A}_{13} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ \Delta\bar{A}_{13}(k) = \begin{bmatrix} 0 & 0.04\cos(k) \\ 0 & 0.04\sin(k) \end{bmatrix},$$

$$\bar{A}_2 = \begin{bmatrix} 0 & 1 \\ 0.5 & -1.4 \end{bmatrix},\ \Delta\bar{A}_2(k) = \begin{bmatrix} 0 & 0.09\cos(k) \\ 0.09\sin(k) & 0 \end{bmatrix},$$

$$\bar{B}_2 = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \bar{A}_{21} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix},\ \Delta\bar{A}_{21}(k) = \begin{bmatrix} 0 & 0.04\cos(k) \\ 0 & 0 \end{bmatrix},$$

$$\bar{A}_{23} = \begin{bmatrix} 0 & 0.09 \\ 0.09 & 0 \end{bmatrix}, \Delta\bar{A}_{23}(k) = \begin{bmatrix} 0 & 0.05\cos(k) \\ 0 & 0.05\sin(k) \end{bmatrix},$$

$$\bar{A}_3 = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.3 \end{bmatrix},\ \Delta\bar{A}_3(k) = \begin{bmatrix} 0 & 0.1\cos(k) \\ 0 & 0.2\sin(k) \end{bmatrix}, \bar{B}_3 = \begin{bmatrix} 0 \\ 3 \end{bmatrix},$$

$$\bar{A}_{31} = \begin{bmatrix} 0 & 0.1 \\ 0.02 & 0.1 \end{bmatrix},\ \Delta\bar{A}_{31}(k) = \begin{bmatrix} 0 & 0.04\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix}$$

$$\bar{A}_{32} = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix},\ \Delta\bar{A}_{32}(k) = \begin{bmatrix} 0 & 0.04\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix},$$

$$z_i(k) = \begin{bmatrix} z_{i1}^T(k) & z_{i2}^T(k) \end{bmatrix}^T,\ u_i(k) = \begin{bmatrix} u_{i1}^T(k) & u_{i2}^T(k) \end{bmatrix}^T \quad i = 1, 2, 3$$

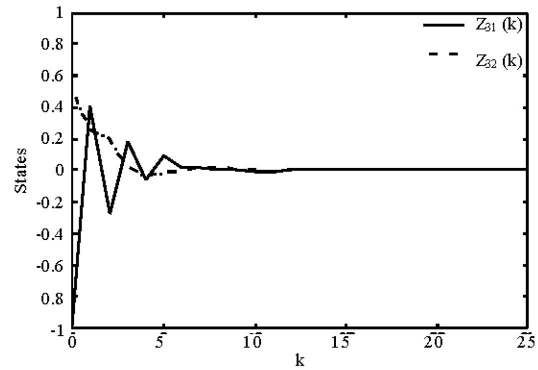

Fig. 3: State responses of subsystem 3

And the time delays and initial condition are:

$$h_{ij} = j, j = 1, 2, 3. \quad \begin{bmatrix} z_1^T(0) & z_2^T(0) & z_3^T(0) & z_4^T(0) \end{bmatrix}^T = \begin{bmatrix} 1 & -0.5 & 2 & 1 \end{bmatrix}^T$$

The above systems is of the form of systems Eq. 8 with:

$$\begin{aligned} & L_{12} = 0.2, L_{22} = 0.09, L_{32} = 0.2, M_{122} = 0.05, \\ & M_{132} = 0.04, M_{212} = 0, M_{232} = 0.05, M_{312} = 0.04, \\ & M_{322} = 0.04, D_1 = D_2 = D_3 = D_{12d} = D_{13d} \\ & = D_{21d} = D_{23d} = D_{31d} = D_{32d}, F(k) = \cos(k). \end{aligned}$$

Solving the LMIs Eq. 14), we obtain:

$$\begin{aligned} & c_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, c_2 = \begin{bmatrix} 0 & \tfrac{1}{2} \end{bmatrix}, c_3 = \begin{bmatrix} 0 & \tfrac{1}{3} \end{bmatrix}, E_{1a} = 0.4, \\ & E_{2a} = 0.09, E_{3a} = 0.1, E_{12ad} = 0, E_{13ad} = 0.12, \\ & E_{21ad} = 0.04, E_{23ad} = 0.15, E_{31ad} = 0.04, E_{32ad} = 0.08 \end{aligned}$$

With the sliding mode controller Eq. 17 in Theorem 2, the simulation results are shown in Fig. 1-3.

In the above figures, one can see that the system is well stabilized with respect to the admissible uncertainties.

CONCLUSION

In this note, we consider the problem of sliding mode control for uncertain discrete-time large-scale systems with delays and unmatched uncertainty. The sliding-mode surface is designed by LMI approach. Then the sliding mode controller satisfied the arriving condition is given.

ACKNOWLEDGMENTS

The authors would like to thank the associate editor and the anonymous reviewers for their constructive comments and suggestions to improve the quality and the presentation of the stduy. This work is supported by National Nature Science Foundation of China under Grant 61073065; Nature Science Foundation of Henan Province under Grant 092300410145; The Education Department of Henan Province Key Foundation under Grant 13A110023, Anyang Normal University Innovation Foundation Project under Grant ASCX/2013-Z29.

REFERENCES

Chen, X., 2006. Adaptive sliding mode control for discrete-time multi-input multi-output systems. Automatica, 4: 427-435.

Gouaisbaut, F., 2002. Robust control of delay systems: A sliding mode control design via LMI. Syst. Control Lett., 46: 219-230.

Janardhanan, S., B. Bandyopadhgay and V.K. Thakar, 2004. Discrete-time output feedback sliding mode control for time-delay systems with uncertainty. Proceedings of the IEEE International Conference on Control Applications, September 2-4, 2004, Taipei, Taiwan, pp: 395-403.

Kau, S.W., Y.S. Liu, L. Hong, C.H. Lee, C.H. Fang and L. Lee, 2005. A new LMI condition for robust stability of discrete-time uncertain systems. Syst. Control Lett., 54: 1195-1203.

Liu, P.L., 2003. Exponential stability for linear time-delay systems with delay dependence. J. Franklin Inst., 340: 481-488.

Mi, Y. and Y.W. Jing, 2006. Sliding mode control for time-delay discrete systems with unmatched uncertainty. Control Eng. China, 13: 560-562.

Mi, Y., J.X. Huang and W.L. Li, 2006. Variable structure control for multi-input discrete systems with control delay. Control Decis., 21: 1425-1428.

Park, J.H., 2004. Robust non-fragile control for uncertain discrete-delay large-scale systems with a class of controller gain variations. Applied Math. Comput., 149: 147-164.

Shyu, K.K., W.J. Liu and K.C. Hsu, 2005. Design of large-scale time-delayed systems with dead-zone input via variable structure control. Automatica, 41: 1239-1246.

Xu, S. and J. Lam, 2005. Improved delay-dependent stability criteria for time-delay systems. IEEE Trans. Autom. Control, 50: 384-387.

Xu, S. and T. Chen, 2004. Robust H_8 control for uncertain discrete-time systems with time-varying delays via exponential output feedback controllers. Syst. Control Lett., 51: 171-183.

Zhang, X.Z., Z.M. Deng and C.C. Gao, 2002. Quasi-sliding mode VSC for discrete linear constant system with time delay. Acta Automatica Sinica, 28: 625-630.