Journal of Applied Sciences

ISSN 1812-5654

Journal of Applied Sciences 13 (11): 1984-1987, 2013
ISSN 1812-5654 / DOI: 10.3923/jas.2013.1984.1987

Study of Horizontal Well Fracture Initiation Pressure Based on Nonlinear Constraint Model

[1,2]Wang Liupeng, [1,2]Li Qi and [2]Wang Zhiyue
[1]Key Laboratory of Petroleum Engineering of the Ministry of Education,
Department of Oil-Gas Development Engineering, Collage of Petroleum Engineering,
China University of Petroleum-Beijing, Beijing, P.O. Box 102249, Changping Fuxue Road 18, China
[2]Department of Oil-Field Development, Collage of Petroleum Engineering,
Xi'an Shiyou University, Xi'an, P.P. Box 710065, Dian'zi 2nd Road 18, China

Abstract: Hydraulic fracturing of horizontal well has been a principal stimulation treatment for low permeable reservoirs. Accurately calculate Fracture Initiation Pressure (FIP) is a key factor to the success of hydraulic fracturing operation for horizontal well. Conventional FIP solution model of horizontal well utilize well-bore pressure increment as the iteration step. Therefore, it has disadvantages such as slow convergence speed and lower computational precision. In this paper, a new optimization solution model is proposed for horizontal well FIP calculation. The model transforms the solving process of FIP into the searching problem of minimum objective function under nonlinear constraint condition. Optimization calculation equations for FIP for horizontal well under open hole and perforation completion is established based on the model. The solving flow chart by using outer-point-penalty function method is devised. Verification of the model is completed through calculation of an actual horizontal well data. The results show that proposed model has superiority of faster computational convergence speed and higher calculation precision than the convention models.

Key words: Horizontal well, hydraulic fracturing, fracture initiation pressure, optimization theory, nonlinear constraint model

INTRODUCTION

Accurately calculate Fracture Initiation Pressure (FIP) has been a key factor to the success of horizontal well hydraulic fracturing treatment. In general, the influence factors of FIP includes the in-situ stress magnitude and formation pressure, wellbore pressure and fracturing fluid percolation effect, reservoir temperature, formation rock strength and other rock mechanics properties (Tuman, 1962; Closmann and Phocas, 1978; Zhou 2002; Xu, 2004). At present, the FIP calculation is based on the total stress distribution around the horizontal well. While the tensile stress application to the rock of well bore exceeds the tensile strength, the rock burst and hydraulic fracture generated. Conventional FIP calculation model of horizontal well uses the well-bore pressure increment as iteration step and leads to the disadvantages such as slow convergence speed and lower precision. To improve the inadequateness, a new nonlinear constraint solution model based on optimization theory for horizontal well hydraulic fracturing initiation pressure is proposed in this paper.

STRESS FIELD DISTRIBUTION MODEL AROUND HORIZONTAL WELL

Stress field distribution around open hole horizontal well: Due to the horizontal wellbore axial length is far greater than radial diameter and therefore it is reasonable assumed that wellbore z axis strain is constant under fracturing conditions and that of x, y plane displacement is unrelated with the z axis, the stress distribution around the horizontal wellbore wall is defined as a plane strain problem (define compressive stress and fluid pressure is positive). Assumes that the rock surrounding the well-bore is homogeneous, isotropic and according to the linear elastic stress calculation model, the stress field distribution surrounding horizontal wellbore wall was

Corresponding Author: Wang Liupeng, Key Laboratory of Petroleum Engineering of the Ministry of Education, Department of Oil-Gas Development Engineering, Collage of Petroleum Engineering, China University of Petroleum-Beijing, Beijing, P.O. Box 102249, Changping Fuxue Road 18, China

derived (Risnes *et al*., 1982; Hossain *et al*., 1999; Russel *et al*., 2009; Fallahzadeh *et al*., 2010):

$$\begin{cases} \sigma_r = P_w \\ \sigma_\theta = -P_w + (\sigma_v + \sigma_h + \Delta\sigma\cos^2\beta) - 2(\sigma_v - \sigma_h - \Delta\sigma\cos^2\beta)\cos 2\theta \\ \sigma_z = \sigma_h + \Delta\sigma\sin^2\beta - 2\nu\,(\sigma_v - \sigma_h - \Delta\sigma\cos^2\beta)\,\cos 2\theta \\ \tau_{r\theta} = \tau_{rz} = 0 \\ \tau_{\theta z} = \Delta\sigma\sin 2\beta\cos\theta \end{cases} \tag{1}$$

In Eq. 1, σ_v, σ_H, σ_h = vertical principal stress, maximum horizontal principal stress, minimum horizontal principal stress, MPa,

σ_r, σ_θ, σ_z = horizontal wellbore wall radial stress, tangential stress, axial stress at angle θ on the horizontal wellbore wall under cylindrical coordinate system, MPa; $\tau_{r\theta}$, τ_{rz}, $\tau_{\theta z}$ = horizontal wellbore wall shear stress on the horizontal wellbore wall under cylindrical coordinate system, Mpa; $\Delta\sigma$ = σ_H-σ_h horizontal stress anisotropy, MPa; β = azimuth angle, horizontal wellbore anticlockwise turned angle w.r.t σ_h, degree; θ = Initiation angle, horizontal wellbore anticlockwise turned angle w.r.t σ_v, degree; P_w = horizontal wellbore pressure, Mpa; v = Poisson's ratio of horizontal well-bore surrounding rock, dimensionless.

Stress field distribution around perforation completion horizontal well: Stress field distribution surrounding perforation completion horizontal wellbore wall is distinguish from the open hole completion, perforation tunnel and horizontal wellbore are two different size of the hole in the perpendicular intersection, by superposition of stress concentration in wellbore and perforation hole intersection, the stress distribution in junction of perforation and sidewall boundary was given:

$$\begin{cases} \sigma_r = P_w \\ \sigma_{\theta'} = -2P_w(1+\cos 2\theta') + 2[\sigma_v(1+2v\cos 2\theta) + 2\sigma_h v\cos 2\theta + \Delta\sigma(\cos^2\beta(1+2v\cos 2\theta) \\ \quad -\sin^2\beta)]\cos 2\theta' - 2[\sigma_v - \sigma_h - \Delta\sigma\cos^2\beta]\cos 2\theta(1+\cos 2\theta') \\ \quad +[\sigma_v(1-2v\cos 2\theta) + \sigma_h(2+2v\cos 2\theta) + \Delta\sigma(1+2v\cos^2\beta\cos 2\theta)] \\ \sigma_z = \sigma_h + \Delta\sigma\sin^2\beta - 2\nu(\sigma_v - \sigma_h - \Delta\sigma\cos^2\beta)\cos 2\theta \\ \tau_{r\theta} = \tau_{rz} = 0 \\ \tau_{\theta z} = \Delta\sigma\sin 2\beta\cos\theta \end{cases} \tag{2}$$

In Eq. 2,θ = perforation azimuth angle, anticlockwise turned angle w.r.t σ_v, degree,

θ'=horizontal wellbore anticlockwise turned angle w.r.t σ_z, degree,

$\sigma_{\theta'}$ = tangential stress in wellbore and perforation hole intersection, MPa.

Fracture initiation criterion: According to tensile strength criterion, when any of the principal stress on wellbore wall exceeds the tensile strength of the rock, then the fracture initiates. Due to the radial stress σ_r is one of the principal stresses, so other two principal stresses can be calculated by using the theory of combined stresses, thus the expression is:

$$\begin{cases} \sigma_1 = \sigma_r \\ \sigma_2 = \frac{1}{2}[(\sigma_\theta + \sigma_z) + \sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_{\theta z}^2}] \\ \sigma_3 = \frac{1}{2}[(\sigma_\theta + \sigma_z) - \sqrt{(\sigma_\theta - \sigma_z)^2 + 4\tau_{\theta z}^2}] \end{cases} \tag{3}$$

In Eq. 3, Initiation pressure calculation of perforation completion, substitute as $\sigma_{\theta'}$ as σ_θ.

From Eq. 3 it is clear that σ_2 is maximum tensile stress of horizontal wellbore wall, considering the effect of pore pressure, the fracture may crack when the stress of the wellbore wall satisfies the following condition, the corresponding P_w is initiation pressure, θ (or θ'perforation well) for fracture azimuth,

$$\sigma_2 - P_p = \sigma_t \tag{4}$$

where, P_p is pore pressure, MPa; σ_t is rock tensile strength, MPa.

NOLINEAR CONSTRAINTS NUMERICAL MODEL FOR FIP

Because of the maximum stress σ_2 is associated with wellbore pressure P_w, the conventional calculation method assumeP_w increase from the minimum value, until satisfies the formula above, the final wellbore pressure solving by Eq. 4 named P_{wf} is the FIP and corresponding θ_f (or θ_f' for perforation well) is the fracture initiation azimuth. Conventional solving process requires iteration calculation, solving speed is slow and the solving precision depends on increment of wellbore pressure P_w. Based on optimization theory, nonlinear constraints numerical model of FIP for open hole and perforating horizontal well is established, respectively.

Numerical model for open hole horizontal well: In solving process for FIP by using Eq. 4, P_w and θ are unknown parameters, for the purposes of generating nonlinear constrained numerical model, following definition is made at first:

$$x_1 = \cos\theta, x_2 = \sin\theta, x_3 = \sigma_\theta, A = \sigma_v + \sigma_h + \Delta\sigma\cos^2\beta, B = \sigma_v - \sigma_h - \Delta\sigma\cos^2\beta,$$
$$C = \sigma_h + \Delta\sigma\sin^2\beta, D = P_p + \sigma_t, E = \Delta\sigma\sin 2\beta \tag{5}$$

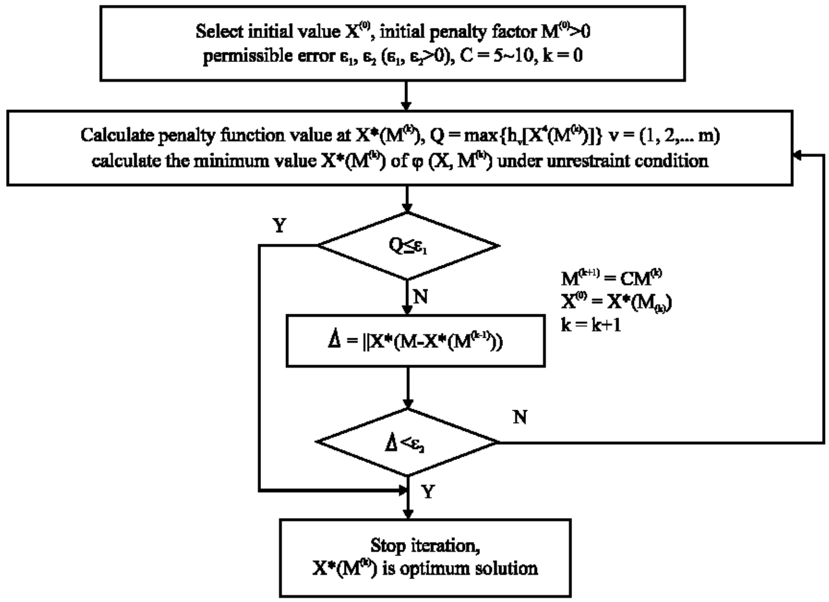

Fig. 1: FIP solving flowchart by using outer point penalty function method

Then the optimization equation can be written as follows:

$$\begin{cases} \min P_w = f(x_1, x_2, x_3) = A - 2B(x_1^2 - x_2^2) - x_3 \\ \text{s.t. } \sigma_z = C - 2\upsilon B(x_1^2 - x_2^2) \\ \quad E^2 x_1^2 - D^2 + (D - \sigma_z)x_3 + D\sigma_z = 0 \\ \quad x_1^2 + x_2^2 = 1 \end{cases} \tag{6}$$

Numerical model for perforation horizontal well: For fracture initiation calculation of perforation completion horizontal well, the unknown parameters are P_w and θ', in order to establish the optimization equation, following definition is made at first:

$$\begin{aligned} & x_1 = \cos 2\theta', x_2 = \sigma_{\theta'}, \\ & A = [\sigma_v(1 + 2v\cos 2\theta) + 2\sigma_h v\cos 2\theta + \Delta\sigma(\cos^2\beta(1 + 2v\cos 2\theta) - \sin^2\beta)], \\ & B = [\sigma_v - \sigma_h - \Delta\sigma\cos^2\beta]\cos 2\theta, \\ & C = [\sigma_v(1 - 2v\cos 2\theta) + \sigma_h(2 + 2v\cos 2\theta) + \Delta\sigma(1 + 2v\cos^2\beta\cos 2\theta)], \\ & D = P_p + \sigma_t, E = \Delta\sigma\sin 2\beta\cos\theta, F = \sigma_h + \Delta\sigma\sin^2\beta, \end{aligned} \tag{7}$$

Then the optimization equation can be written as follows:

$$\begin{cases} \min P_w = f(x_1, x_2) = (2 + 2x_1)^{-1}(2Ax_1 - x_2 - 2B(1 + x_1) + C] \\ \text{s.t. } \sigma_z = F - 2\upsilon B \\ \quad \tau_{\theta z} = E \\ \quad E^2 - D^2 + D(\sigma_z + x_2) - \sigma_z x_2 = 0 \\ \quad -1 \le x_1 \le 1 \end{cases} \tag{8}$$

Computational flowchart of nonlinear constraint model: The optimization calculation model (6), (8) is minimum value solving problem under the equality constraint condition, it is available to use the external penalty function method solving the solution(Chen and Zhao, 2002, 2011; Huang, 2009; Liu and Shan, 2011), the basic idea is, by constructing penalty function, convert the constraint problem into a series of unconstrained optimization problems, then using unconstrained optimization method to calculate, the fracture initiation calculation model of outer penalty function form to see Eq. 9, the calculation process is shown in Fig. 1:

$$\begin{cases} \min f(x_1, x_2, x_3) \\ \text{s.t. } h_v(x^*) = 0 \end{cases} \\ \varphi(X, M^{(k)}) = f(X) + M^{(k)} \sum [h_v(X)]^2 \tag{9}$$

where, $M^{(k)}$ is penalty factor, a set of ascending value greater than zero, must satisfies the condition of Eq. 10:

$$0 < M^{(0)} < M^{(1)} < \cdots < M^{(k)} < M^{(k+1)} < \cdots \to \infty \tag{10}$$

CALCULATION RESULT COMPARISION

Given an open hole completion horizontal well DP1 in Changqing oilfield to verify the new model, the basic parameters of DP1 as shown in Table 1:

According to the parameters in Table 1, the initial parameters to calculation are setting as follows, for new

Table 1: Basic parameters of DP1 well for fracture initiation calculation

Mechanical parameter			
Vertical stress (MPa)	40	Horizontal max stress (MPa)	45
Pore pressure (MPa)	20	Horizontal min stress (MPa)	30
Rock mechanics parameter			
Young's modulus (MPa)	10000	Poisson ratio	0.3
Horizontal wellbore parameter			
Inclination angle of horizontal well(degree)	90		
Azimuth angle of horizontal well(degree)	30		

Table 2: Comparison of computation results of DP1

	Result comparison			
	New model	Conventional model	Absolute error	Relative error
FIP (MPa)	61.35	62.00	0.65	1.05%
Initiation azimuth (degree)	0.00	0.00	0.00	0.00
Loop	32.00	1500.00	1200.00	80.00%

model set $[x_1, x_2, x_3] = [1, 0, 0]^T$, for convention model set $[P_w, \theta] = [0, 0]^T$ and $[\Delta P_w, \Delta\theta] = [1, 15]^T$. The calculated results by using new model and conventional model are shown in Table 2, respectively.

According to the contrast of computational results in Table 2, the conventional FIP calculation precision depends on the wellbore pressure increment, the convergence speed is slow and precision is low, while using the new optimization model to calculate, the results of FIP precision increase 1.05% and saving 80% convergence speed. It is shown that the new model is more applicable than the conventional one.

CONCLUSION

This study has presented a nonlinear constraint model for calculation of hydraulic FIP for open hole or perforation completion horizontal well. The numerical model is based on the optimization theory and the solving flowchart is put forward in the paper by using outer point penalty function method. The comparison of FIP shows that the proposed optimization model has faster calculation convergence speed and higher calculation accuracy than the conventional one. Through application of computer technology to solve the proposed optimization model in the paper can provide a fast and high precision calculation method, it is better able to satisfy the site hydraulic fracture treatment requirements.

ACKNOWLEDGMENT

We are grateful to the National Natural Science Foundation China, National Science and Technology Major Project and Oil and Natural Gas Engineering Key Discipline of Shaanxi Province for their financial supported under Grant No. 51074125, No. 2011ZX05013-003 for this study.

REFERENCES

Chen, J.B. and L.J. Zhao, 2002. Optimization Theory and Application. 1st Edn . Shaanxi People's Publishing House, China

Closmann, P.J. and D.M. Phocas, 1978. Thermal stress near a heated fracture in transversely isotropic oil shale. Soc. Petroleum Eng. J., 18:59-74.

Fallahzadeh, S.H., S.R. Shadizadeh, P. Pourafshary and M.R. Zare, 2010. Modeling the perforation stress profile for analyzing hydraulic fracture initiation in a cased hole. Proceedings of the Nigeria Annual International Conference and Exhibition, July 31-August 7, 2010, Society of Petroleum Engineers, Tinapa-Calabar, Nigeria, pp: 9-18.

Hossain, M.M., M.K. Rahman and S.S. Rahman, 1999. A comprehensive monograph for hydraulic fracture initiation from deviated well-bores under arbitrary stress regimes. Proceedings of the SPE Asia Pacific Oil and Gas Conference and Exhibition, April 20-22, 1999, Society of Petroleum Engineers, Jakarta, Indonesia, pp:1-11.

Huang, P., 2009. Optimization Theory and Methods. 1st Edn. Tsinghua University Press, China, ISBN: 9787302191537

Liu, F. and R. Shan, 2011. A kind of revised penalty function self-correcting algorithm. J. Chongqing Technol. Bus. Univ., 28: 8-13.

Risnes, R., R.K. Bratli and P. Horsrud, 1982. Sand stress around a wellbore. Soc. Petroleum Eng. J., 22:883-898. DOI: 10.2118/9650-PA.

Russel, R., E. Mike and B.A. Robert, 2009. Horizontal, near-wellbore stress effects on fracture initiation. Proceedings of the SPE Rocky Mountain Petroleum Technology Conference, April 14-16, 2009, Society of Petroleum Engineers, Denver, Colorado, pp: 5-22.

Tuman, V.S., 1962. Thermal stresses around a wellbore and their small effect on velocity logging. SPE J., 2:303-308.

Xu, Y.B., 2004. Research of basic theory for horizontal well hydraulic fracturing. Ph.D. Thesis, Department of Oil and Gas Field Development, Faculty of Petroleum Engineering, Southwest Petroleum University, China.

Zhao, J.J., 2011. Optimization Techniques with the MATLAB Optimization Toolbox. 2nd Edn. China Machine Press, China, ISBN: 9787111332053.

Zhou, D.Y., 2002. Study of hydraulic fracture fundamental theory and design method for ERD wells. Ph.D. Thesis, Department of Oil and Gas Field Development, Faculty of Petroleum Engineering, Southwest Petroleum University, China.