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Determination of the Optimal Batch Size for a Manufacturing System with Multiple Deliveries and Random Machine Breakdown

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Abstract: This study is concerned with determination of the optimal batch size for a manufacturing system with multiple deliveries and random machine breakdown. The classic Economic Manufacturing Quantity (EMQ) model assumes a continuous issuing policy for satisfying customer's demands and perfect quality production for all items produced during a production run. However, in a real-life vendor-buyer integrated system, multi-shipment policy is practically used in lieu of continuous issuing policy, while generation of nonconforming items and unexpected breakdown of production equipment are inevitable. The model is developed significantly, taking into account jointly multiple deliveries, machine breakdown and imperfect rework of random defective items.The renewal reward theorem is used to deal with the variable cycle length. An optimal manufacturing batch size that minimizes the long-run average costs for such an imperfect system is derived. An illustrating example is provided to show its practical usages.

Key words: Manufacturing, batch size, machine breakdown, scrap items, multiple deliveries

INTRODUCTION

In the manufacturing sector, when products are produced in-house instead of being acquired from outside suppliers, the Economic manufacturing Quantity (EMQ) model is often utilized to deal with the finite production-inventory replenishment rate in order to minimize the expected overall cost per unit time (Aggarwal, 1974; Tersine, 1994; Silver *et al.*, 1998; Nahmias, 2001).The classic EMQ model implicitly assumes that all items produced are of perfect quality. However, in real-life production systems, due to process deterioration or other controllable and/or uncontrollable factors, generation of defective items is inevitable. Hence, many studies have been carried out to enhance the EMQ model by addressing the imperfect quality issues.

The nonconforming items produced, sometimes, can be reworked and repaired; hence, the overall production-inventory costs can be significantly reduced. For example, production processes in printed circuit board assembly or in plastic injection molding, or in other industries such as chemical, textiles, metal components, etc., sometimes employ rework as an acceptable process in terms of level of product quality. Examples of research that has investigated the effect of rework on EMQ model are surveyed as follows. Yum and McDowell (1987) formulated the allocation of inspection effort problem for a serial system as a 0-1 Mixed Integer Linear Programming (MILP) problem. Their formulation permitted any combination of scrap, rework, or repair at each station and allowed the problem to be solved using standard MILP software packages. Jamal *et al.* (2004) studied the optimal production batch size with rework process at a single-stage production system. Both cases of rework being completed within the same production cycle and rework being done after N cycles are examined. Mathematical models for each case were developed; the optimal batch sizes and total system costs were derived accordingly. Chiu (2007) derived the optimal lot size and back-order level for an EMQ model with backlogging, random defective rate, scrap and imperfect rework process.

In addition to the defective items produced, another unrealistic assumption of classic EMQ model is the continuous inventory issuing policy for satisfying product demand. In real-life vendor-buyer integrated production-inventory system, multiple or periodic deliveries of finished products are commonly used at customer's request. Goyal (1977) studied the integrated inventory model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product

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is procured by a single customer from a single supplier. He gave examples to illustrate his proposed method. Studies have been carried out to address the various aspects of vendor-buyer supply chain optimization issue. Diponegoro and Sarker (2006) determined an ordering policy for raw materials as well as an economic batch size for finished products that are delivered to customers frequently at a fixed interval of time for a finite planning horizon. The problem was then extended to compensate for the lost sales of finished products. A closed-form solution to the problem was obtained for the minimal total cost. A lower bound on the optimal solution was also developed for problem with lost sale. It was shown that the solution and the lower bound were consistently tight. Chiu *et al*. (2011) paid attention to investigation of the joint effect of a discontinuous issuing policy and an imperfect rework process on the optimal replenishment batch size of the EMQ model, developed numerical method for determination of the optimal lot size for a manufacturing system.

Random breakdown of production equipment is another common and inevitable reliability factors that trouble the production planners and practitioners most. To effectively manage and control the disruption and minimize overall production costs, become the primary task of most manufacturing firms. It is no wonder that determining optimal lot-size (or production uptime) for systems with machine failures has received attention from researchers in recent decades. Example of studies that addressed the machine breakdown issues are surveyed below. Groenevelt *et al*. (1992) studied two production control policies to deal with the machine failures. The first one assumes that the production of the interrupted lot is not resumed (called no resumption (NR) policy) after a breakdown. While the second policy considers that the production of the interrupted lot will be immediately resumed (called Abort/Resume (AR) policy) after the breakdown is fixed and if the current on-hand inventory falls below a certain threshold level. Both of their proposed policies assume that the repair time is negligible and they studied the effects of machine breakdowns and corrective maintenance on the economic lot sizing decisions. Chiu (2007) investigated the optimal run time for EMQ model with scrap, rework and random breakdown. They proposed and proved theorems on conditional convexity of the integrated cost function and on bounds of the production run time. Then, an optimal run time was located by the use of the bisection method based on the intermediate value theorem.

In this article, we generalise the EMQ model for an unreliable manufacturing system, taking into account jointly multiple deliveries, machine breakdown and imperfect rework of random defective items. Since little attention was paid to the aforementioned area, this paper intends to bridge the gap.

FUNDAMENTAL ASSUMPTIONS AND NOTATION

The following assumptions and notation are considered to develop the model.

Assumptions:

- It may randomly produce x portion of defective items at a rate d
- The constant production rate P is larger than the sum of demand rate λ and production rate of defective items d
- During the manufacturing process, all items produced are screened immediately and the unit inspection cost is included in the unit production cost C
- The Abort/Resume (AR) policy is adopted when breakdown occurs. Under such policy, malfunction machine is immediately under repair and the repair time is constant. The interrupted lot will be resumed right after the restoration of machine
- The imperfect quality items fall into two groups, a θ portion of the imperfect quality items is scrap and the other portion $(1-\theta)$ is considered to be rework-able.
- The rework process itself is imperfect, a portion θ_1 $(0\le\theta_1\le1)$ of reworked items fail and become scrap
- The finished items can only be delivered to customers if the whole lot is quality assured at the end of rework

Notation:

K: Setup cost per production run.
Q: Manufacturing batch size to be determined for each cycle
P_1: The production rate of rework process
T: Production time before a random breakdown occurs.
t_1: The production uptime for the proposed EMQ model.
t_r: Time required for repairing and restoring the machine.
t_2: Time required for reworking of defective items.
t_3: Time required for delivering all quality assured finished products
H_1: The level of on-hand inventory when machine breakdown occurs
H_2: The maximum level of on-hand inventory in units when regular production process ends
H_3: The maximum level of on-hand inventory in units when rework process finishes
C_S: Disposal cost per scrap item

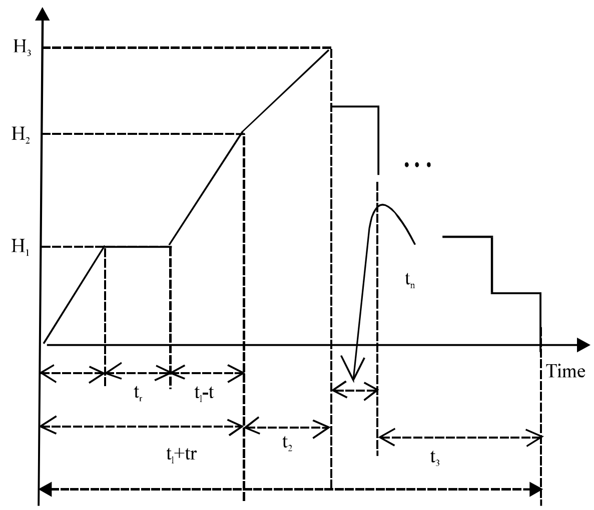

Fig. 1: On-hand inventory of perfect quality items in EMQ model with a multi-delivery policy, machine breakdown and quality assurance issues

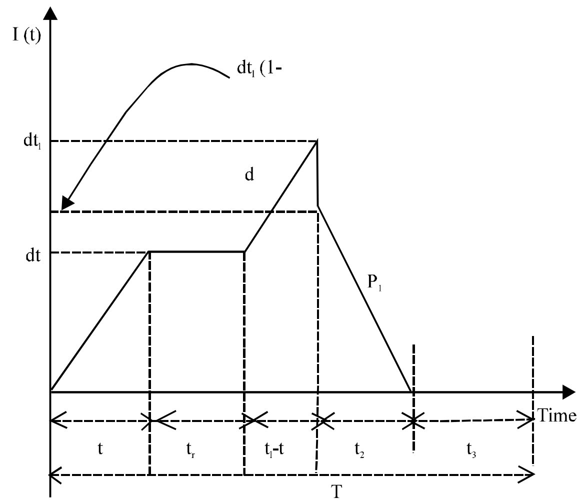

Fig. 2: On-hand inventory of defective items in EMQ model with a multi delivery policy, machine breakdown and quality assurance issues

C_R: Unit rework cost
h: Unit holding cost
h_1: Holding cost for each reworked item
h_2: Holding cost for each item kept by customer
φ: Over all scrap rate per cycle (sum of scrap rates in t_1 and t_2)
T: The production cycle length
K_1: Fixed delivery cost per shipment
C_T: Unit delivery cost per item shipped to customers
N: The number of fixed quantity installments of the finished batch to be delivered by request to customers
t_n: A fixed interval of time between each installment of finished products delivered during production downtime t_3

MATHEMATICAL MODELLING

From Fig. 1, one obtains the level of on-hand inventory H_1 when machine breakdown occurs; the level of inventory H_2 when regular production process ends; the maximum level of on-hand inventory H_3 when rework process finishes; the cycle length T; the production uptime t_1; time required for reworking of defective items t_2; time required for delivering all finished products t_3, as follows:

$$H_1 = (P-d)\,t \tag{1}$$

$$H_2 = H_1+(P-d)\,(t_1-t) \tag{2}$$

$$H_3 = H_2+(P_1-d_1)\,t_2 \tag{3}$$

$$t_1 = \frac{Q}{P} \tag{4}$$

$$t_2 = \frac{xQ(1-\theta)}{P_1} \tag{5}$$

$$t_3 = nt_n \tag{6}$$

$$t_3 = nt_n \tag{7}$$

The on-hand inventory of defective items produced during the production uptime t are as follows (Fig. 2). Among them a θ portion is scrap and the other (1-θ) portion of defective items is considered to be rework-able.

$$d\,(t+t_1-t) = Pxt_1 = xQ \tag{8}$$

During the rework process, a portion θ of reworked items fail and become scrap. The maximum level of scrap items φxQ is:

$$\varphi xQ = [\theta+(1+\theta)\,\theta_1]\,xQ \tag{9}$$

During delivery time t_3, n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time. Cost for each delivery is:

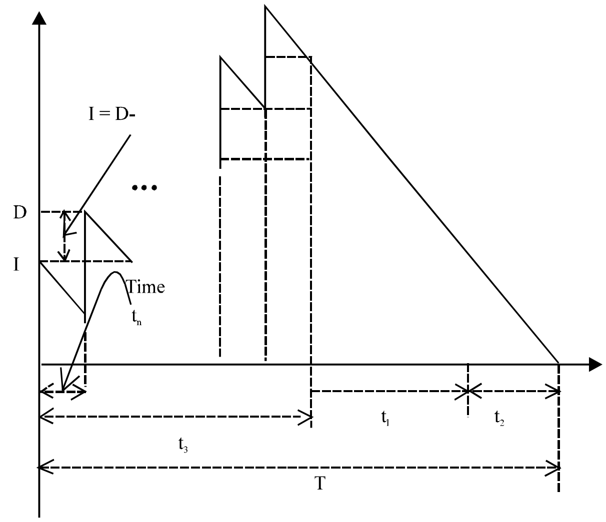

Fig. 3: On-hand inventory at the customer's end when installments of the finished batch are delivered

$$K_1 + C_T\left(\frac{H}{n}\right) \tag{10}$$

And total delivery costs for n shipments in a cycle are:

$$n\left[K_1 + C_T\left(\frac{H}{n}\right)\right] = nK_1 + C_T Q(1-\varphi x) \tag{11}$$

Total holding costs of finished products during t_3 at manufacturer's end can be obtained as follows (refer to Appendix 1):

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)H_3 t_3 = h\left(\frac{n-1}{2n}\right)H_3 t_3 \tag{12}$$

Total holding costs for items kept at customer's end are as follows (Fig. 3 and also refer to Appendix 2).

$$\frac{h_2}{2}\left[\frac{H_3 t_3}{n} + (T - tr)(H_3 - \lambda t_3)\right] \tag{13}$$

Total production-inventory-delivery cost per cycle TC (Q) consists of variable production cost, setup cost, variable rework cost, disposal cost, fixed and variable delivery cost, holding cost at the manufacturer's end during production uptime t_1, reworking time t_2 and delivery time t_3, the holding cost at the customer's end during t_3 and variable holding cost for items reworked. Therefore, the overall production-inventory- delivery cost per cycle TC (Q) is:

$$\begin{aligned} TC(Q) = {} & CQ + K + C_R[x(1-\theta)Q] + C_S[x\varphi Q] + nK_1 + \\ & C_T[Q(1-\varphi x)] + h\left[\begin{array}{l} \frac{H_1 + dt}{2}(t) + (H_1 + dt)t_r + \\ \frac{H_1 + dt + H_2 + dt_1}{2}(t_1 - t) + \frac{H_2 + H_3}{2}(t_2) \end{array}\right] \\ & + h\left(\frac{n-1}{2n}\right)H_3 t_3 + h_1\frac{P_1 t_2}{2}(t_2) + \frac{h_2}{2}\left[\frac{H_3}{n}t_3 + (T - tr)(H_3 - \lambda t_3)\right] \end{aligned} \tag{14}$$

The production cycle length is not constant due to the assumption of random scrap rate and a uniformly distributed random breakdown is assumed to occur in the production period. Thus, one can use the renewal reward theorem in inventory cost analysis to deal with the variable cycle length and the integration of TC (Q) to cope with the random breakdown happening in period t_1. The expected total production-inventory costs per breakdown happening in period t_1 unit time can be calculated as follows:

$$E[TCU(Q)] = \frac{E\left[\int_0^{t_1} TC(Q)\cdot(1/t_1)\,dt\right]}{E\left[\int_0^{t_1} Q(1-\theta x)/\lambda\cdot(1/t_1)\,dt\right]} \tag{15}$$

$$\begin{aligned} E[TCU(Q)] = {} & \frac{C\lambda}{1-\varphi E[x]} + \frac{(K + nK_1)\lambda}{Q(1-\varphi E[x])} + \frac{C_R E[x](1-\theta)\lambda}{(1-\varphi E[x])} + \frac{C_S E[x]\varphi\lambda}{1-\varphi E[x]} + C_T\lambda \\ & + \frac{hQ\lambda}{2P(1-\varphi E[x])} + \frac{hQ\lambda}{2P_1(1-\varphi E[x])}\left[(2E[x] - (E[x])^2 - \varphi(E[x])^2)(1-\theta)\right] \\ & + \left(1-\frac{1}{n}\right)\left[\frac{hQ(1-\varphi E[x])}{2} - \frac{hQ\lambda}{2P} - \frac{hQE[x](1-\theta)\lambda}{2P_1}\right] + \frac{h_1(E[x])^2(1-\theta)^2 Q\lambda}{2P_1(1-\varphi E[x])} \\ & + \left(\frac{1}{n}\right)\frac{h_2 Q}{2}(1-\varphi E[x]) + \left(1-\frac{1}{n}\right)\frac{h_2 Q\lambda}{2P} + \frac{h_2 Q}{2}\left[\left(1-\frac{1}{n}\right)\frac{E[x](1-\theta)\lambda}{P_1}\right] + \frac{\lambda t_r}{2(1-\varphi E[x])} \\ & - \frac{h_2 t_r}{2}\left[\frac{\lambda(P - d + 2\lambda)}{P(1-\varphi E[x])} + \frac{\lambda(P_1 - d_1 + 2\lambda)E[x](1-\theta)}{P_1(1-\varphi E[x])} - \lambda + \frac{\lambda^2 t_r}{Q(1-\varphi E[x])}\right] \end{aligned} \tag{16}$$

CONVEXITY OF E [TCU (Q)] AND THE OPTIMAL SOLUTION

The optimal replenishment lot size can be obtained by minimizing the expected cost function E [TCU (Q)]. Differentiating E [TCU (Q)] with respect to *Q*, the first and the second derivatives of E [TCU (Q)] are shown in Eqs. 17 and 18:

$$\begin{aligned} \frac{\partial E[TCU(Q)]}{\partial Q} = {} & \frac{1}{1-\varphi E[x]}\left[\frac{h\lambda}{2P} - \frac{nK_1\lambda}{Q^2} - \frac{K\lambda}{Q^2}\right] \\ & + \frac{h_1(E[x])^2\lambda(1-\theta)^2}{2P_1(1-\varphi E[x])} + \frac{h\lambda(1-\theta)E[x][2 - E[x](1+\varphi)]}{2P_1(1-\varphi E[x])} \\ & + \left(1-\frac{1}{n}\right)\left[\frac{h(1-\varphi E[x])}{2} - \frac{h\lambda}{2P} - \frac{hE[x](1-\theta)\lambda}{2P_1}\right] \\ & + \frac{1}{n}\frac{h_2}{2}(1-\varphi E[x]) + \left(1-\frac{1}{n}\right)\frac{h_2\lambda}{2P} \end{aligned} \tag{17}$$

$$\frac{\partial^2 E[TCU(Q)]}{\partial Q^2} = \frac{2(K+nK_1)\lambda - h_2 t_r^2 \lambda^2}{Q^3(1-\varphi E[x])} \quad (18)$$

If :

$$t_r < \sqrt{\frac{2(K+nK_1)}{h_2\lambda}}$$

also Eq. 18 is positive. Hence E [TCU (Q)] is a strictly convex function for all Q. The optimal replenishment lot size Q* can be obtained by setting the first derivative of E [TCU (Q)] equal to zero. (refer to Eq. 17):

$$\frac{dE[TCU(Q)]}{dQ} = 0 \quad (19)$$

One obtains the following after rearrangement:

$$\frac{nK_1\lambda + K\lambda - 1/2h_2 t_r^2\lambda^2}{(1-\varphi E[x])Q^2} = \frac{1}{1-\varphi E[x]}\left[\frac{h\lambda}{2P}\right] + \frac{h_1(E[x])^2\lambda(1-\theta)^2}{2P_1(1-\varphi E[x])} + (1-\frac{1}{n})\left[\begin{array}{l}\frac{h(1-\varphi E[x])}{2} - \frac{h\lambda}{2P} \\ -\frac{hE[x](1-\theta)\lambda}{2P_1}\end{array}\right] + \frac{h\lambda(1-\theta)E[x]}{2P_1(1-\varphi E[x])}\cdot[2-E[x]-\varphi E[x]] \quad (20)$$

$$Q^* = \sqrt{\frac{2(K+nK_1)\lambda - h_2 t_r^2\lambda^2}{\left\{\begin{array}{l}\frac{h\lambda}{P} + \frac{h\lambda}{P_1}[2E[x]-(E[x])^2-\varphi(E[x])^2](1-\theta) \\ +\left[(\frac{n-1}{n})h + \frac{1}{n}h_2\right](1-\varphi E[x])^2 + (\frac{n-1}{n})(h_2-h) \\ \cdot\left(\frac{\lambda}{P} + \frac{E[x](1-\theta)\lambda}{P1}\right)(1-\varphi E[x]) + \frac{h_1(E[x])^2(1-\theta)\lambda}{P1}\end{array}\right\}}} \quad (21)$$

Special cases: Suppose all items produced are of perfect quality (i.e., x = 0) and the breakdown factor is not considered (t_r = 0), the model becomes the same as the classic EMQ model with a multi-delivery policy:

$$TC_1 = CQ + K + h\left[\frac{H}{2}(t_1) + \left(\frac{n-1}{2n}\right)Ht_3\right] + nK_1 + C_TQ + \frac{h_2}{2}\left[\frac{H}{n}t_3 + T(H-\lambda t_3)\right] \quad (22)$$

The expected production-inventory-delivery cost E [TCU_1 (Q)] for this special model can be derived as follows:

$$E\left[TCU_1(Q)\right] = C\lambda + \frac{(K+nK_1)\lambda}{Q} + C_T\lambda + \frac{hQ\lambda}{2P} + \left(\frac{n-1}{n}\right)\left(\frac{hQ}{2} - \frac{hQ\lambda}{2P}\right) + \left(\frac{1}{n}\right)\frac{h_2Q}{2} + \left(1-\frac{1}{n}\right)\frac{h_2Q\lambda}{2P} \quad (23)$$

Convexity of E [TCU_1(Q)] can be proved as shown in Eq. 21 and optimal lot size Q* can also be derived accordingly, as shown in Eq. 22:

$$\frac{\partial^2 E[TCU(Q)]}{\partial Q^2} = \frac{2(K+nK_1)\lambda}{Q^3} > 0 \quad (24)$$

and:

$$Q^* = \sqrt{\frac{2(K+nK_1)\lambda}{\left\{\frac{h\lambda}{P} + \frac{(n-1)h}{n}[1-(\frac{\lambda}{P})]\right\}}} \quad (25)$$

NUMERICAL EXAMPLE

Assume a product can be manufactured at a rate of 60,000 units per year and this item has experienced a at demand rate of 3400 units per year. During production process a random defective rate is assumed to be uniformly distributed over the interval [0, 0.3] and among defective items a portion θ = 0.1 is considered to be scrap and other portion can be reworked and repaired at a rate P_1 = 2200 units per year. During the rework process, a portion θ_1 = 0.1 of reworked items fails and becomes scrap. Additional values of parameters used in this example are given below: C = \$100 per item, C_R = \$60 per item reworked, C_S = \$20 per scrap item, tr = 0.018 years. h = \$20 per item per year, h_1 = \$40 per item reworked per unit time (year), h_2 = \$80 per item kept at the customer's end per unit time, n = 3 installments of the finished batch are delivered per cycle, K = \$20,000 per production run, K_1 = \$4,350 per shipment, a fixed cost, C_T = \$0.1 per item delivered. The optimal replenishment lot size Q* = 1693 can be calculated from Eq. 21.The optimal expected production-inventory -delivery cost E [TCU (Q*)] = \$7477 can also be computed from the Eq. 16.

CONCLUSION

We extend a vendor-buyer integrated model of Chiu *et al.* (2011), considering the effect of an imperfect production process subject to random breakdown. When breakdown occurs, the Abort/Resume (AR) policy is adopted. Under such policy, malfunction machine is immediately under repair and the repair time is constant. The interrupted lot will be resumed right after the restoration of machine. The model is developed, taking into account jointly multiple deliveries, machine breakdown and imperfect rework of random defective items.

The mathematical modelling is used in this study. The renewal reward theorem is utilized to deal with the variable cycle length of the proposed system. The long-run average production-inventory-delivery cost function is derived and proved to be convex. The closed-form solutions in terms of optimal replenishment lot size are obtained. A numerical example is provided to demonstrate its practical usage. A possible extension of this work may be set in the direction of considering other probability distributions of machine breakdown.

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Appendix 1:

Computations of the holding cost of finished products during t_3 (i.e., the very last term in Eq. 12) are as follows:

- When n = 1, the total holding cost in delivery time $t_3 = 0$ are:

$$h\left(\frac{H}{2}\times\frac{t_3}{2}\right)=h\left(\frac{1}{2^2}\right)Ht_3 \tag{26}$$

- When n = 2, the total holding costs in delivery time t_3 become:

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_3=h\left(\frac{(n-1)}{2n}\right)Ht_3 \tag{27}$$

- When n = 3, the total holding costs in delivery time t_3 are:

$$h\left(\frac{2H}{3}\times\frac{t_3}{3}+\frac{H}{3}\times\frac{t_3}{3}\right)=h\left(\frac{2+1}{3^2}\right)Ht_3 \tag{28}$$

- When n = 4, the total holding costs in delivery time t_3 are:

$$h\left(\frac{3H}{4}\times\frac{t_3}{4}+\frac{2H}{4}\times\frac{t_3}{4}+\frac{H}{4}\times\frac{t_3}{4}\right)=h\left(\frac{3+2+1}{4^2}\right)Ht_3 \tag{29}$$

Therefore, the following general term for total holding costs during delivery time t_3 can be obtained:

$$h\left(\frac{1}{n^2}\right)\left(\sum_{i=1}^{n-1} i\right)Ht_3=h\left(\frac{(n-1)}{2n}\right)Ht_3 \tag{30}$$

Appendix 2:

Computations of the holding cost at the customer’s end during t_3 are as follows. Because n instalments (fixed quantity D) of the finished lot are delivered to the customer at a fixed interval of time t_n, one has the following:

$$D=\frac{H}{n}\quad t_n=\frac{t_3}{n} \tag{31}$$

At the customer’s end, the demand between shipments is (λt_n). If we let I denote number of items that will be left over after satisfying the demand during each fixed interval of time t_n (refer to Fig. 3), then:

$$I=D\text{-}\lambda t_n \tag{32}$$

From Fig. 3, one can calculate the average inventory as follows:

- Average inventory:

$$\begin{aligned}&=\left[\left(\frac{D+I}{2}\right)t_n\right]+\left(\frac{nI}{2}\right)(t_1+t_2+t_r)\\&+\left[\frac{(D+I)+[(D+I)-\lambda t_n]}{2}\right]+\cdots\\&+\left[\frac{(D+(n-1)I)+[(D+(n-1)I)-\lambda t_n]}{2}t_n\right]\end{aligned} \tag{33}$$

Substituting Eq. 31 in Eq. 32, the average inventory becomes:

- Average inventory:

$$\begin{aligned}&=\left(D-\frac{\lambda}{2}t_n\right)t_n+\left(D+I-\frac{\lambda}{2}t_n\right)t_n+\left(D+2I-\frac{\lambda}{2}t_n\right)t_n\\&+\cdots+\left(D+(n-1)I-\frac{\lambda}{2}t_n\right)t_n+\left(\frac{nI}{2}\right)(t_1+t_2+t_r)\\&=n\left(D-\frac{\lambda}{2}t_n\right)t_n+\frac{n(n-1)}{2}It_n+\left(\frac{nI}{2}\right)(t_1+t_2+t_r)\end{aligned} \tag{34}$$

Substituting Eq. 29 through 31 in Eq. 33, the following general term for average inventory at the customer’s end can be obtained:

$$\begin{aligned}&=\left(D-\frac{\lambda}{2}t_n\right)t_n+\left(D+I-\frac{\lambda}{2}t_n\right)t_n+\\&\cdots+\left(D+(n-1)I-\frac{\lambda}{2}t_n\right)t_n+\left(\frac{nI}{2}\right)(t_1+t_2+t_r)\\&=\frac{1}{2}[\frac{H_3t_3}{n}+(T-tr)(H_3-\lambda t_3)]\end{aligned} \tag{35}$$

Therefore, total holding cost for items kept at the customer’s end is:

$$\frac{h_2}{2}[\frac{H_3t_3}{n}+(T-tr)(H_3-\lambda t_3)] \tag{36}$$

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