



# Journal of Applied Sciences

ISSN 1812-5654

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## Improved Resampling Particle Filter for Maneuvering Point Target Tracking

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**Abstract:** The sample impoverishment during particles resampling is a challenging problem in using particle filter to track point target. The reason is that the impoverishment will lead to the divergence of particles and failure of tracking. To solve the problem, a new resampling algorithm is presented in this study. The idea is that random particle samples are drawn from the neighborhoods of previous samples with high weights according to Gaussian distribution instead of simple duplication. Therefore, during the resampling, the effect of sample impoverishment is reduced and diversity of particle samples is enriched because of the samples expansion while the low weights samples are discarded. To illustrate proposed method, the resampling algorithm with simple duplication and the resampling algorithm with random drawing samples based on Gaussian distribution are compared. Simulation results show that the tracking failure is reduced in the quantitative criteria-RMSE.

**Key words:** Resampling, particle filter, point target tracking

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### INTRODUCTION

Point target tracking is a tough problem for its indistinguishability in clutter (Zang *et al.*, 2007; Liu and So, 2009; Kural *et al.*, 2010; Wang *et al.*, 2011). It is a non-linear and non-Gaussian estimation problem because of the complex noise constitution and the difficulties to formulate the relationship between the target state and the observation values which are defined as posterior probability density function (Arulampalam *et al.*, 2002). Particle filter which uses sequential Monte Carlo methods for on-line learning within Bayesian framework, is exactly the promising way to tackle the non-linear and non-Gaussian problem. Instead of the integral adopted in Kalman filter, it uses a scatter of particles propagation to represent the required posterior probability distribution of target position without Gaussian noise and linear model limit (Chen *et al.*, 20096; Di *et al.*, 20087). Therefore, particle filter is widely adopted in target tracking (Zang *et al.*, 2007; Liu and So, 2009; Kural *et al.*, 2010; Wang *et al.*, 2011; Arulampalam *et al.*, 2002; Chen *et al.*, 2009; Di *et al.*, 2008).

However, there are two tricky issues in particle filter application (1) It is hard to describe the posterior probability density function  $p(S_k|Z_{1:k})$  in analytical form which is the basement of sampling and (2) Particle's degeneracy occurs in sequence importance sample

(Bar-Shalom and Birniwal, 1982; Yu and Cheng, 2006; Morelande and Challa, 2005). The first issue can be solved by replacing the posterior probability density function  $p(S_k|Z_{k-1})$  by importance density function  $p(S_k|S_{k-1})$ . But the solution lead to big sample variance for its failing to integrate the latest observation value which reduce the filter accuracy. The second issue, particles degeneracy, means after a few iterations, all but one particles have negligible weights. In the extreme condition, when a single particle has a unity and all others have zero weights after iterations, then the whole sample space collapse to one particle which means the total failure of filter (Vadakkepat and Jing, 2006). It is shown by Liu (2008) that the variance of the importance weights can only increase over time and thus it is impossible to avoid the degeneracy phenomenon.

Resampling is a common method to reduce the effects of degeneracy (Zheng *et al.*, 2011; Ristic *et al.*, 2004). It eliminates particles with low weights and focuses on particles with larger weights thus reducing the computation burden. However, resampling introduces the problem of sample impoverishment while reducing the effects of degeneracy. In some situation, when the particles remained after resampling diverges from the true state variable, errors occur in the posterior probability density function  $p(S_k|Z_{k-1})$  estimated by particles propagations for the weights computed by the

observation model are inaccurate which leads to the failure of object tracking. It is seen that the diversity of samples should be kept while eliminating particles with low weights through resampling. This study will discuss how to keep the diversity of samples to guarantee that the target is tracked accurately and quickly.

The rest of the study is arranged as follows: In Section II the problem is formulated. In section III, the basic theory and algorithm of particle filter are presented and then the improved resampling algorithm is introduced in section IV. The proposed algorithm eliminates sample impoverishment and increases tracking accuracy quantified by the Root Mean Square Error (RMSE) criterion. Finally, simulation results and analysis are provided in section V and conclusions are drawn in section VI.

**PROBLEM FORMULATION**

The problem of tracking a single, maneuvering, dim point target in clutter using a single sensor is considered in this study. The problem is formulated based on the following assumption:

- The target intension is unknown and can always be detected in every frame of the optical image sequence. It is not concealed and does not disappear
- Target state at time step  $k$ , denoted by  $S_k$ , makes its transition to the next time step, according to a known nonlinear transition model (target state motion model). The model is as follows (Rollason and Salmond, 2001):

$$S_{k+1} = AS_k + B\theta_k \tag{1}$$

$$S = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}, A = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0.5 & 0 \\ 0 & 1 \\ 0 & 0.5 \end{bmatrix}, \theta = \begin{bmatrix} \theta_x \\ \theta_y \end{bmatrix}$$

where  $S_k = [x \ \dot{x} \ y \ \dot{y}]$  is the state vector of target,  $x, y$  and  $\dot{x}, \dot{y}$  represent the position and the velocity of target respectively.

- An initial density for target state is also assumed available:  $S_0 \sim p_0(\cdot)$ . The position state is usually uniform distributed over the observation plane and the velocity state is also uniform distributed over a range estimated by priori knowledge
- The target-originated observation is given by a known nonlinear observation model which depends on the transition density between time steps and measurement likelihood. In particular, for a particle

filter implementation, the particle weight is proportional to its likelihood. Therefore, the observation model that described the weight calculation of the  $p$ th particle is defined as follows (Rollason and Salmond, 2001):

$$q(p) \propto \prod_{i,j \in C(S(p))} l(Z_{i,j} | S(p)) \tag{2}$$

where,  $l(Z_{i,j} | S(p)) = p_{s+N}(Z_{i,j} | S(p)) / p(Z_{i,j})$  is the likelihood ratio of the target  $S = (x, y)$  in the pixel  $(i, j)$ , the distribution  $p_{s+N}(\cdot)$  refers to the probability density function (pdf) of the signal plus the noise, whereas  $p_N(\cdot)$  denotes the pdf of the background noise.  $C(S(p))$  is the set of subscripts of pixels affected by the target  $S(p)$

- The process noise sequences,  $\theta_k$ , the initial states  $S_0$  and the observation noise sequence  $\sigma_k$  are assumed to be mutually independent for all  $k$
- At each time step  $k$ , the sensor produces a set of observation  $Z_{i,j}^k = \{Z_{0,0}^k, Z_{0,1}^k, \dots, Z_{1,0}^k, Z_{1,0}^k, \dots | i \leq M, j \leq N\}$  which consists of the measurements originated from true targets and false alarms due to clutter.  $M, N$  is the size of observation plane

**THEORY AND ALGORITHM OF PARTICLE FILTER**

Particle Filter (PF) is considered in this study to solve the tracking problem because of its ability to tackle the non-linear and non-Gaussian problems. Under the Bayesian framework, Monte Carlo simulation methods are used to approximate the posterior probability function  $p(S_k | Z_{1:k})$  through sampled particles with associated weights. However, during the iteration of weights calculation, some particle samples weights decrease gradually for its irrelative to target state. This phenomenon is called particle degeneracy which can be reduced by the resampling method. Unfortunately, a new problem, sample impoverishment, will be introduced during resampling process.

The particle filter procedure including resampling has four stages including prediction, update, resample and state computation. The procedure is as follows:

- **Initialization:** Sample  $S^i_0$  from the initial prior probability distribution  $p_0(S)$  which is usually set to be uniform distribution over the observation plane and set the weights  $\omega^i_0$  to  $1/N, i = 1: \dots, N$ . For  $i = 1:$  frames
  - **Prediction:** Each sample is passed through the system motion model Eq. 1 to obtain the predicted samples

- **Update:** Once the observation data,  $Z_{k_0}$  is measured, evaluate the importance weight of each predicted sample according to Eq. 2
- **Resample:** Compute the effective sample size:

$$\hat{N}_{eff} = \frac{1}{\sum_{i=1}^N (\omega_k^i)^2}$$

- Compute the target state according the resampled particle samples

End.

If  $\hat{N}_{eff}$  is less than  $N_{threshold}$ , then resample the discrete distribution which means copy the high weight samples and eliminate the low weight samples. This procedure is described as Fig. 1.

It is obvious that after resampling, the sample with high weights are copied to the constituted new sample set with uniform weights. However, it also brings out the sample impoverish problem.

### IMPROVED RESAMPLING ALGORITHM

Resampling with the common method to reduce the effects of particles degeneracy introduces the new problem of sample diversity loss (which is also called impoverish). The expectation of target state  $S_k$  at time  $t = k$  before resampling is expressed in Eq. 3 and the expectation after resampling is Eq. 4:

$$\mu_k = \sum_{i=1}^N S_k^i \omega_k^i \tag{3}$$

$$\hat{\mu}_k = \frac{1}{N} \sum_{i=1}^N \hat{S}_k^i \tag{4}$$

It is obvious that the kinds of particle samples after resampling is less than those before resampling which is called the loss of sample diversity. As a matter of fact, considering the computation complex, the particle samples are chosen as less as possible. Therefore, there are potential probabilities that the sample set is too small to

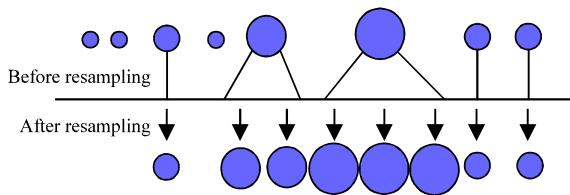


Fig. 1: Resampling procedure

cover the observation plane and to represent initial uniform distribution. Subsequently, during the iteration and propagation the particle samples could diverge instead of converge.

There are two reasons contribute to the divergence: (1) The point target state is only within a very small region comparing the whole observation plane, so the samples in iteration may miss the small region as show in Fig. 2a where the circle represents the target position and the spot is the particle samples and (2) As mentioned in introduction, the weights computed is not accurate enough to correct the tracking bias because the posterior probability density function  $p(S_k|Z_{1:k})$  which is replaced by prior density function  $p(S_k|S_{k-1})$ , leaving the latest observation data to account. Considering the extreme

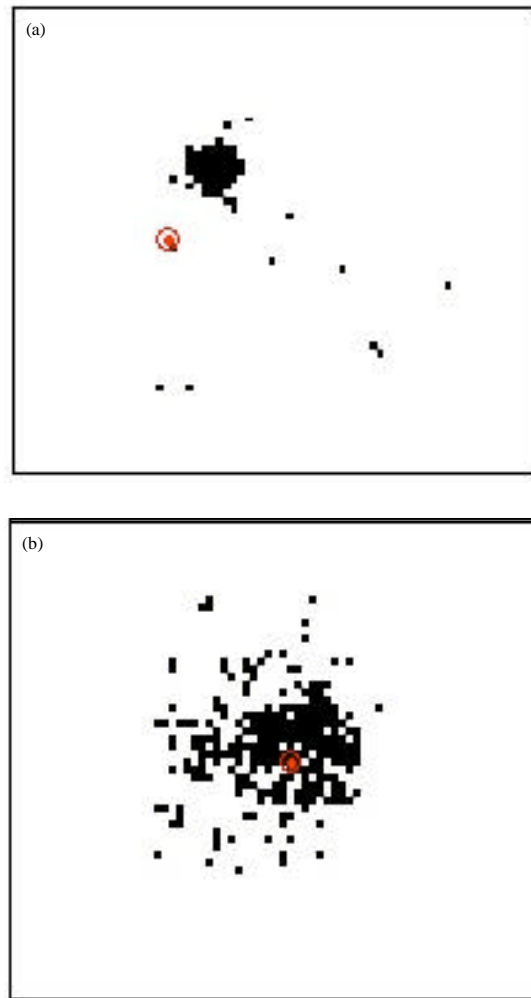


Fig. 2(a-b): The particle distributions for two resampling algorithms (a) Simple duplication and (b) Proposed

condition, when a single particle  $S_{k-1}^i$  has a weight of unity and all others have zero weights, then after resample, the unity weight particle  $S_{k-1}^i$  is duplicated  $N-1$  times and the whole sample space collapses to the  $S_{k-1}^i$ . In the following prediction stage at time  $k$ , the single particle in Eq. 1 is obtained and it may diverge from the true state variable due to the disturbance noise. When the particle  $S_{k-1}^i$  diverges from the true state variable, the estimated posterior probability density function  $p(S_k|Z_{1:k})$  becomes zero resulting in zero weight  $\omega_k^i$  according to Eq. 2 which leads to loss of all samples and subsequent failure in target tracking. It is concluded that the diversity of samples should be kept while discarding particles with low weights through resampling.

To keep the diversity of samples, an expansion resampling based on Gaussian distribution is adopted instead of the simple-duplication resampling. The new particles are sampled from the neighborhoods of the high weight particles  $S_k^{ij}$  based on the Gaussian distribution,  $S_k^{ij} \sim \mathcal{N}(S_k^i, \alpha^2)$ .  $S_k^{ij}$  is the particle needed to be replaced in resampling.  $S_k^i$  is the particle sample with high weights which is remained as basement of resampling.  $\alpha^2$  determines the variance of sampling distribution which is adapted to the effective number of particles  $\hat{N}_{eff}$  and the variance  $\epsilon^2$  of the state variable:

$$\alpha^2 \propto \frac{\epsilon^2}{\hat{N}_{eff}}$$

When  $\hat{N}_{eff}$  decreases, the sampling region is expanded to obtain more “different” particles. Variance  $\epsilon^2$  determines the tracking precision of the state variable. Larger  $\epsilon^2$  means a wider varying area of state variables and sampling region is expanded accordingly and vice versa. Figure 2b shows that after resampling from the neighborhood of the focused particles, the samples cover the target true state by enriching the diversity.

### SIMULATION AND ANALYSIS

Here, the process of tracking a single, maneuvering, dim point target in clutter using a single sensor is simulated. The performance of the resampling with simple duplication and the proposed resampling algorithms are quantified by the criteria RMSE of tracking accuracy between the estimated and the true target trajectory.

All the simulations shown in this section are implemented in MATLAB version 2010a, using an Intel CPU 3.30 GHz, 16 GB RAM PC. The tracking simulation parameters are set as follows: discrete sample time = 1 sec, the size of optical image sensor is  $100 \times 100$  pixels, the target and its extension pixels is  $3 \times 3$  pixels which is less

than 0.15% of a  $100 \times 100$  pixels image. Therefore, the target is called dim point target according to SPIE (International Society for Optical Engineering) definition. The target intensity  $I = 20$  and the extent of sensor’s blurring represented by  $\sigma_{psf} = 0.7$ . The background noise in the simulation was assumed to be a zero-mean Gaussian with variance  $\delta_N^2 = 81$  for every pixel. The driving noise in the motion model (Eq. 1) which represents acceleration, is a zero-mean Gaussian process of variance  $\theta_k^2 = 1$ . The optical image sequence in the experiment is simulated with low SNR = 7 dB (SNR: Signal Noise Ratio). The SNR is defined as follows:

$$SNR = 10 \times \log_{10} \frac{I^2}{\delta_N^2}$$

According to the parameters set above, a sequence of 30 frames of data has been simulated. The particle filter proposed by Salmond to track point target in optical image sequence is adopted. The parameters used in particle filter are listed in Table 1.

The resampling algorithm with simple duplication is compared with the proposed resampling algorithm in the same tracking model and parameters. The simulation result is shown in Fig. 3-5. Figure 3 and 4 show the actual and estimated target position in X and Y coordinates of two

Table 1: Tracking parameter

Parameter	Value
Resampling method	Residual resampling
Target intensity	Unknown
Particle number	2000
Initial velocity state distribution	Un (-3:3)
Initial state position	Unknown
Initial position state distribution	Un (0:100)

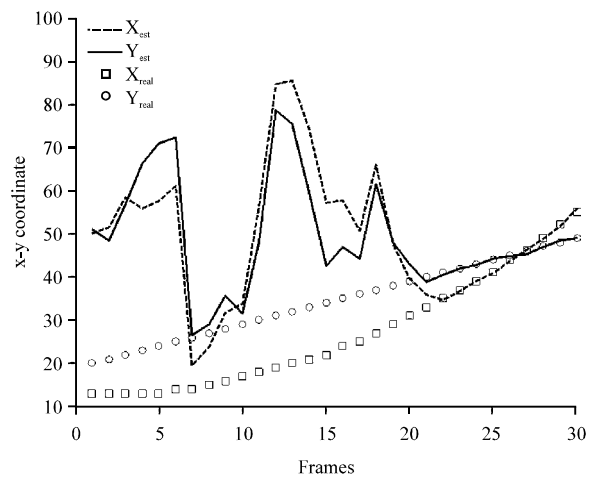


Fig. 3: Tracking result using resampling algorithm with simple duplication

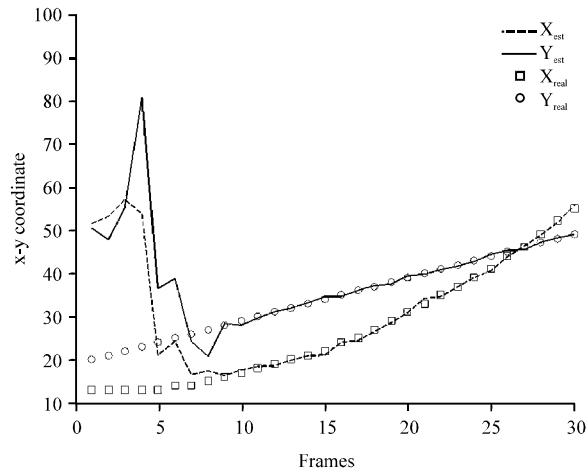


Fig. 4: Tracking result using the proposed resampling algorithm

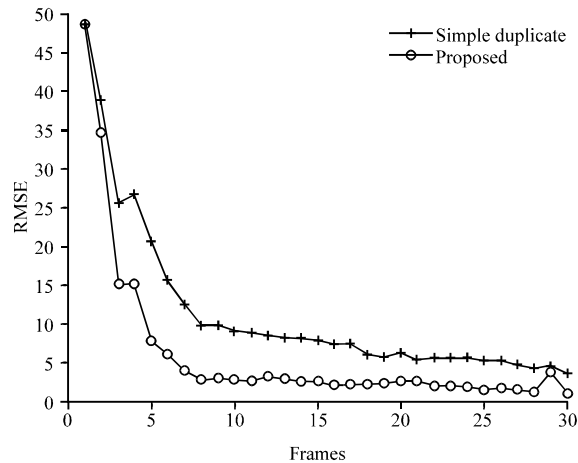


Fig. 5 comparing results of RMSE after 100 times runs of two algorithms

resampling algorithms. It is clear that the proposed resampling algorithm takes fewer frames to follow the true trajectory compared with the resampling with simple duplication which is the result of increasing the particles diversity by expanding the resampling particles distribution.

Monte Carlo simulations of 100 runs with 2000 particle number were performed in order to evaluate the performance of the proposed algorithm in terms of tracking accuracy criteria RMSE. Figure 5 shows the tracking error of the two methods. It is concluded that the tracking errors of the proposed method from frame 1-30 stays lower than the traditional algorithm.

Table 2: Tracking RMSE of two algorithms

Algorithms	Max RMSE	Min RMSE	Mean RMSE
Simple duplicated	48.45	4.12	11.26
Proposed	48.43	1.41	6.06

Table 2 shows that the tracking accuracy represented by RMSE is reduced in all respects including max, min and mean RMSE of 30 frames.

### CONCLUSION

In this study, a new resampling algorithm is proposed to reduce the sample impoverishment which is the of derivative of resampling. Instead of simple duplication in resampling, it enriches the sample diversity by expanding the sample set based on the Gaussian distribution of particle sample. The expected qualitative performance of the proposed algorithm is verified by simulation. This improvement increases the kinds of particles samples and guarantees that the computed states follow the real target trajectory.

However, the variance of Gaussian distribution of high weight particles in resampling is still a problem to discuss. If the variance is too big, it will result in the failure of particle convergence. If the variance is too small, the expansion is negligible to enrich the particles diversity. Therefore, the variance adopted in resampling remains an open issue.

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