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Predictive Functional Control Based on Second-order plus Time Delay Prediction Model

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Abstract: The traditional predictive functional control usually used a first-order plus time delay prediction model because of the complexity about the control rule's deduction, which cannot descript completely the high order plus large time delay object widespread in the industrial production process. In this study, second-order plus time delay prediction model is used against the problem mentioned above and the control rule is deduced through the solution of generalized Fibonacci sequence. The experiment results have shown that the dynamic response performance of the controller using the second-order plus time delay prediction model is improved significantly and the control effect is better than that of fist-order plus time delay prediction model.

Key words: Predictive functional control, second-order plus time delay prediction model, generalized Fibonacci sequence

INTRODUCTION

Predictive Functional Control (PFC) is a new predictive control algorithm proposed by Richalet and Kuntze, et al. that has the advantages of model predictive control (Kutze *et al*., 1986; Richalet, 1993; Qin and Badgwell, 2003). Since the complexity of control law depends on the structure of predictive functional control, the traditional PFC usually used the first-order, first-order plus time delay or second-order model as the prediction model of controlled object, so that the derivation of the control law is simplified. However, those models mentioned above cannot exactly represent the high order or large time delay process which is commonly encountered in industrial control systems. Thus, in that case, the PFC doesn't give promising results, i.e., the response performance and robustness (Jin *et al*., 1999; Zhang and Wu, 2008).

Dynamic characteristics of most high order or nonlinear industrial process could be usually represented by the second-order plus time delay model(Richalet *et al*., 2009). Hence, in this study, the second-order plus large time-delay model will be used as the prediction model of controlled object to deduce the control law of the PFC. The generalized Fibonacci sequence has been referred to derivate the control law. Some experiments about the high order process have verified the effects of predictive functional control based on second-order plus time delay prediction model.

PREDICTIVE FUNCTIONAL CONTROL PRINCIPLES

Predictive functional control has the higher requirements in terms of the output form of the controller compared with the traditional predictive control algorithm (Clarke *et al*., 1987a, b), so that the tracking of set point can be smoother. In PFC, the control system performance is determined by the structure of control input and the control actions added newly can be expressed as a linear combination of some known basis functions f_n ($n = 1, \ldots, N$):

$$u(k+i) = \sum_{n=1}^{N} \mu_n f_n(i), i = 0, \ldots, P-1 \tag{1}$$

where, f_n ($n = 1, \ldots, N$) is the basis function, μ_n is the linear weighting coefficient; $f_n(\mu)$ is the value of basis function when $t = iT$, where T is the sampling period; P is the time-domain length of prediction and optimization. It depends on the nature of set point to choose the basis functions and the polynomial basis functions are chosen usually.

The system output can be directly predicted by the input utilizing the prediction model. In the SISO system, the model output at time k is consisted of two parts, model free output and model forced output. The former is expressed as $y_m(k) = F_2(x(k))$, $i = 1, \ldots, P$, where x (k) is all known information at time k. F_2 is the mathematical

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representation of the object's prediction model. y_m (k) is computed at time k in the consideration that newly control actions is not added. The model forced output is expressed as:

$$w(i) = \sum_{n=1}^{N} \mu_n g_n(i), i = 1, \cdots, P$$

which is the additional model response when the control actions u (k+i) is considered at time k. The control actions newly added are not temporal independent variables, but the linear combination of basis functions, which is the difference between PFC and traditional predictive control algorithm. Therefore, the corresponding output is the summation of individual output of different basis functions (Richalet *et al.*, 2009).

The error between process output and model predictive output is sent to predict the error of future optimization time-domain and used as the feed forward quantity to compensate the reference trajectory at the same time. The future predictive error is e (k+i) = y (k)-y_m (k), where, y_m (k) is the model output at time k. So, that the predictive output with error compensation could be obtained:

$$yP\,(k+i) = yM(k+i)+w(i)+e(k+i),\ i = 1, ..., P \quad (2)$$

Assuming that the form of reference trajectory is y_r (k+i) , the quadric performance index is defined as:

$$\min\left\{\sum_{i=1}^{P}\left[y_r(k+i) - y_P(k+i)\right]^2\right\} \quad (3)$$

It can be seen as a parameter optimization problem, the goal is to find a set of weighting coefficients $\mu_1, \ldots, \mu_N$, so that the predictive output will be as close as possible to the reference trajectory in the whole optimization time-domain. While the problem is settled, it could be obtained through the μ_n (n = 1, ... , N) that the function input u (k+i)(i = 0, ..., P-1) to be added at time k. The control accuracy mainly depends on the choice of basis functions and the dynamic response is mainly affected by the reference trajectory in PFC, while the length of prediction time-domain plays a major role in system stability and robustness.

PREDICTIVE FUNCTIONAL CONTROL ALGORITHM

Assuming that the second-order plus time delay model is:

$$G_m(s) = \frac{Ke^{-\tau s}}{(T_1 s+1)(T_2 s+1)} \quad (4)$$

When the sampling period is T, Eq. 4 is discretized and Eq. 5 can be obtained:

$$y_{k+2} = \alpha_1 y_{k+1} + \alpha_2 y_k + \alpha_3 u_{k+2-L} \quad (5)$$

Where:

$$L = \tau/T;\ \alpha_1 = \frac{2T_1T_1 + T(T_1+T_2)}{T_1T_2 + T(T_1+T_2)+T^2};$$

$$\alpha_2 = -\frac{T_1T_2}{T_1T_2 + T(T_1+T_2)+T^2};$$

$$\alpha_3 = \frac{T^2K}{T_1T_2 + T(T_1+T_2)+T^2}$$

$$e\,(k) = y_p\,(k)-y_m\,(k) \quad (6)$$

$$y_r\,(k+H) = \beta.y_p\,(k)+(1-\beta).c \quad (7)$$

where, $\beta = \exp(-T_s/T_r)$, c is the step value.

According to Eq. 5, the actual process output is:

$$y_p\,(k+2) = \alpha_1 y_p\,(k+1)-\alpha_2 y_p(k)+\alpha_3 u\,(k+2-L) \quad (8)$$

The model output is:

$$y_m\,(k+2) = \alpha_1 y_m\,(k+1)-\alpha_2 y_m\,(k)+\alpha_3 u\,(k+2-L) \quad (9)$$

While using the above equation as prediction model, the system response speed will be decreased because of the time delay of model. Reference to the Smith predictor control, the process model without time delay is adopted to correct the measurements.

Then, the model output without time delay is:

$$y_{mav}(k) = \alpha_1 y_{mav}(k-1) + \alpha_2 y_{mav}(k-2) + \alpha_3 u(k), k \ge 3 \quad (10)$$

To meet the requirements of multi-step predictive control, the problem can be simplified to solve the generalized Fibonacci sequence equations when the prediction step is H and with the assumption of u (k+1) = u (k+2) = ...u (k+H).

$$y_m\,(k)-\xi = \alpha_1\,(y_m(k-1)-\xi)+\alpha_2\,(y_m\,(k-2)-\xi),\ k\ge 3$$

Defining $Y_k = y_k-\xi$, so that:

$$(1-\alpha_1-\alpha_2)\xi = \alpha_3 u \quad (11)$$

Then, $Y_k = \alpha_1 Y_{k-1} + \alpha_2 Y_{k-2}$, which is the generalized Fibonacci sequence (Zhang, 2005).

Two roots A and B will be obtained by solving equation; it can be divided into the following three conditions.

If $\Delta = \alpha^1_2 + 4\alpha_2 > 0$, the equation has two different roots:

$$A, B = \frac{\alpha_1 \pm \sqrt{\alpha_1^2 + 4\alpha_2}}{2}$$

and $Y_k = c_1 A^k + c_2 B^k$, k = 1, 2. It will has:

$$c_1 = \frac{Y_2 - BY_1}{A^2 - AB}, c_2 = \frac{Y_2 - AY_1}{B^2 - AB}$$

So, that:

$$y_k = \frac{(y_2 - By_1)}{A^2 - AB} A^k + \frac{y_2 - Ay_1}{B^2 - AB} B^k + A^k \frac{B-1}{A^2 - AB} \xi + B^k \frac{A-1}{B^2 - AB} \xi + \xi \quad (12)$$

ξ is obtained from Eq. 12 and substituted in Eq. 11:

$$u_k = \frac{1 - \alpha_1 - \alpha_2}{\alpha_3} \frac{y_k - \frac{(y_2 - By_1)}{A^2 - AB} A^k - \frac{(y_2 - Ay_1)}{B^2 - AB} B^k}{A^k \frac{(B-1)}{A^2 - AB} + B^k \frac{(A-1)}{B^2 - AB} + 1} \quad (13)$$

If $\Delta = \alpha_1^2 + 4\alpha_2 = 0$, the equation has two equal roots $A = B = \alpha_1/2$, then $Y_k = (c_1 + kc_2)A^k$, k = 1, 2. It will has:

$$c_1 = \frac{2AY_1 - Y_2}{A^2}, c_2 = \frac{(Y_2 - AY_1)}{A^2}$$

So, that:

$$y_k = A^{k-2}(2Ay_1 - y_2) + kA^{k-2}(y_2 - Ay_1) + (1 - 2A)A^{k-2}\xi + k(A-1)A^{k-2}\xi + \xi \quad (14)$$

$$u_k = \frac{1 - \alpha_1 - \alpha_2}{\alpha_3} \frac{y_n - A^{k-2}(2Ay_1 - y_2) - kA^{k-2}(y_2 - Ay_1)}{1 + (1 - 2A)A^{k-2} + k(A-1)A^{k-2}} \quad (15)$$

If $\Delta = \alpha_1^2 + 4\alpha_2 < 0$, the equation has two conjugate imaginary roots A (cosB±i sin B), where:

$$\begin{cases} A = \sqrt{\left(\frac{\alpha_1}{2}\right)^2 + \left(\frac{-\alpha_1 - 4\alpha_2}{2}\right)^2} = \sqrt{-\alpha_2} \\ B = \arctan \frac{\sqrt{-\alpha_1^2 - 4\alpha_2}}{\alpha_1} \end{cases}$$

Then, $Y_k = A^k(c_1 \cos kB + c_2 \sin kB)$, k = 1, 2. It will has:

$$c_1 = \frac{2A\cos BY_1 - Y_2}{A^2}, c_2 = \frac{Y_2 \cos B - A\cos 2BY_1}{A^2}$$

So, that:

$$y_k = A^{k-2}(2A\cos By_1 - y_2)\cos B + A^{k-2}(y_2 - A\cos 2By_1)\sin kB + A^{k-2}\xi(1 - 2A\cos B)\cos kB + A^{k-2}\xi(A\cos 2B - \cos B) + \xi \quad (16)$$

$$u_k = \frac{1 - \alpha_1 - \alpha_2}{\alpha_3} \frac{\begin{pmatrix} y_k - A^{k-2}(2Ay_1 \cos B - y_2)\cos kB \\ -A^{k-2}(y_2 - A\cos 2By_1)\sin kB \end{pmatrix}}{\begin{pmatrix} 1 + A^{k-2}(1 - 2A\cos B)\cos kB \\ +A^{k-2}(A\cos 2B - \cos B) \end{pmatrix}} \quad (17)$$

The relation between y_m (k+H) and y_m (k) is:

$$y_m(k+H) = \alpha_1 y_m(k-1+H) + \alpha_2 y_m(k-2+H) + \alpha_3 u(k+H), k \geq 3 \quad (18)$$

According to the three conditions mentioned above: When $\Delta = \alpha_1 2 + 4\alpha_2 > 0$, substituted in Eq. 13:

$$u(k+H) = \cdots = u(k+1) = \frac{1 - \alpha_1 - \alpha_2}{\alpha_3} \frac{\begin{pmatrix} y_m(k+H) - \frac{(y_m(k) - By_m(k-1))}{A^2 - AB} A^{H+2} \\ -\frac{y_m(k) - Ay_m(k-1)}{B^2 - AB} B^{H+2} \end{pmatrix}}{A^{H+2}\frac{B-1}{A^2 - AB} + B^{H+2}\frac{A-1}{B^2 - AB} + 1} \quad (19)$$

When $\Delta = \alpha_1^2 + 4\alpha_2 = 0$, substituted in Eq. 15:

$$u(k+H) = \cdots = u(k+1) = \frac{1 - \alpha_1 - \alpha_2}{\alpha_3} \frac{\begin{pmatrix} y_m(k+H) - A^H(2Ay_m(k-1) - y_m(k)) \\ -(H+2)A^H(y_m(k) - Ay_m(k-1)) \end{pmatrix}}{1 + (1 - 2A)A^H + (H+2)(A-1)A^H} \quad (20)$$

When $\Delta = \alpha_1^2 + 4\alpha_2 < 0$, substituted in Eq. 17:

$$u(k+H-1) = \cdots = u(k+1) = \frac{1 - \alpha_1 - \alpha_2}{\alpha_3} \frac{\begin{pmatrix} y_m(k+H-1) \\ -A^{H-1}(2Ay_m(k-1)\cos B - y_m(k))\cos(H+1)B \\ -A^{H-1}(y_m(k) - Ay_m(k-1)\cos 2B)\sin(H+1)B \end{pmatrix}}{\begin{pmatrix} 1 + A^{H-1}(1 - 2A\cos B)\cos(H+1)B \\ +A^{H-1}(A\cos 2B - \cos B) \end{pmatrix}} \quad (21)$$

At time k+H:

$$y_m(k+H) = y_r(k+H) - e(k) = (c - y_p(k))(1 - \beta) + y_m(k)$$

For the process output y_P (k+L) is unknown at time k, the measurements y_P (k) at current moment need to be corrected to construct the new output y_{pav} (k) involved the future predictive information:

$$y_{pav}(k) = y_p(k)+y_{mav}(k)-y_{mav}(k-(L-1)) \tag{22}$$

Substitute Eq. 11 into one of Eq. 19, 20 or 21, the control law of next time will be obtained.

EXPERIMENT STUDY

The PFC algorithm based on second-order plus time delay prediction model is implemented on the SIMATIC process control system PCS 7 to control a high order heat-exchanger that has large time delay. The flow of cooling water is used as the manipulated variable to control the outlet temperature of the material to be cooled. Through the least square method (Wang *et al*., 2001), the first-order plus time delay model and second-order plus time delay model are obtained respectively, which will be used as the prediction model of PFC, respectively.

First-order plus time delay model is:

$$G_m(s) = \frac{-0.55e^{-20s}}{(35.1s+1)}$$

Second-order plus time delay model is:

$$G_m(s) = \frac{-0.55e^{-20s}}{(437.4975s^2+34.4871s+1)}$$

When the system is stable, the set point of heat-exchanger outlet temperature is decreased from 53-40°C firstly and then, the flow of materials being cooled is decreased to 30% of the original flow. The output curve is shown in Fig. 1 and 2, where the first-order plus time delay model is adopted in Fig. 1 and the second-order plus time delay model is adopted in Fig. 2. In the two figures, curve 1 is the outlet temperature response curve of materials being cooled; curve 2 shows the variable curve of the cooling water valve. The disturbance of the materials being cooled is added from the plotted point of dotted line.

The experimental results have shown that no matter the first-order plus time delay model or the second-order plus time delay model is used as the prediction model, the predictive control have good performance to resist the disturbance. However, it also can be seen that when the second-order plus time delay model is used as prediction model, it gives a promising results, such as the system obtains the quicker response speed, smaller overshoot and shorter regulate time, the control output is more stable and is impacted little of disturbance.

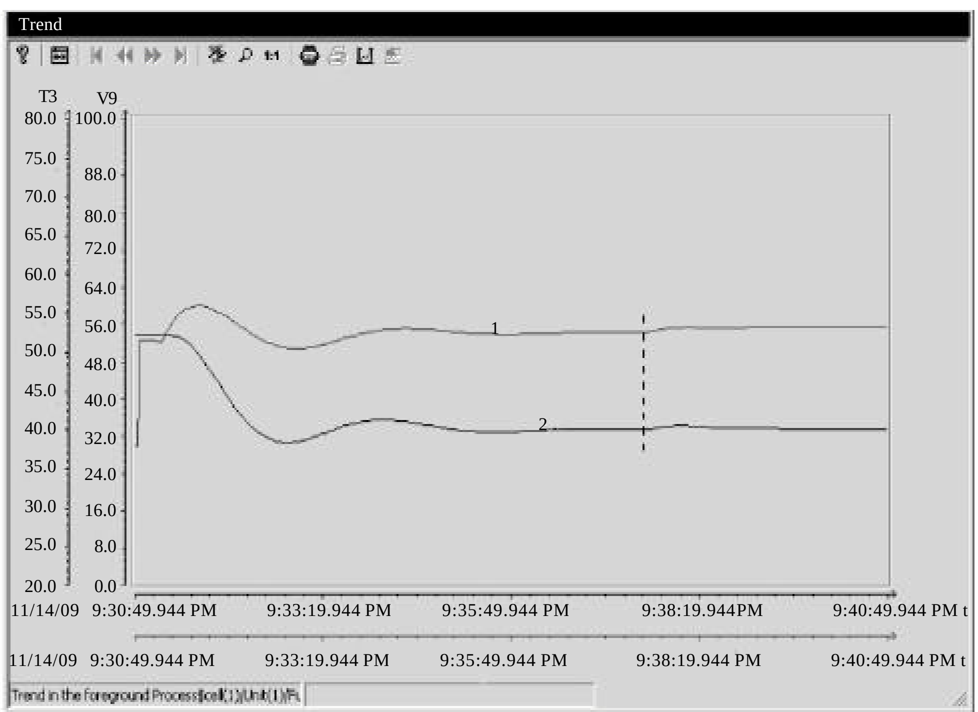

Fig. 1: Response curve of first-order plus time delay prediction model

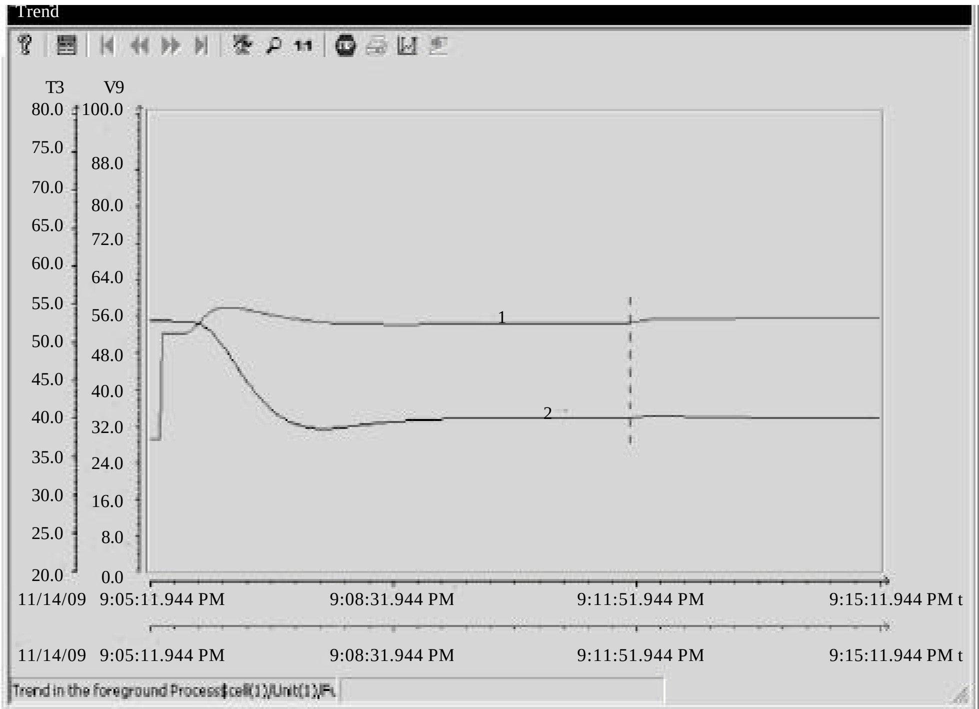

Fig. 2: Response curve of second-order plus time delay prediction model

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CONCLUSION

The second-order plus time delay model is used as the prediction model in PFC and the control law is derived with generalized Fibonacci sequence. This method is more suitable to apply in the industrial process with high order and large time delay objects, which will improve the response speed and control performance and have good robustness. The approach has been implemented on the SIMATIC PCS 7; the results have shown the well reliability and feasibility for the industrial process control.

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