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A Method Based on the Interval Type Ideal Point for Multiple Attribute Decision Making

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Abstract: A kind of multiple attribute decision making problems is studied, in which the information about the attribute weights is unknown completely and the attribute values are in the forms of interval numbers, a new method to get the priorities of alternatives is given. Based on the interval type ideal point, an optimal model is constructed and a simple formula for obtaining the attribute weights is given. The method has low computational complexity in comparison with the previous researches and is effective for obtaining the priorities of alternatives. An example of practical application is given to show the feasibility and effectiveness of the method.

Key words: Multiple attribute decision making, interval type ideal point, deviation degree, possibility degree

INTRODUCTION

Multiple Attribute Decision Making (MADM) is an important area in modern decision theory, its theories and methods have been widely applied in many fields such as engineering design, economic management and military etc. The methods for the MADM with complete weight information have been widely studied (Cheng, 1987; Chen and Zhao, 1990 and Hwang and Yoon, 1981). However, in fact, the decision maker may have no knowledge about the attribute weight information and the attribute values are usually in the forms of interval numbers. So, the research on the MADM without weight information on alternatives has important theoretical significance and practical value.

Park and Kim (1997) Kim *et al.* (1999), Kim and Ahn, (1999) have researched the MADM problem with incomplete weight information. Bryson, and Mobolurin, 1997, Fan and Zhang, 1999 have proposed methods to discuss the MADM problem with incomplete weight information and the attribute values expressed by interval number. Zhang *et al.* (1999), Xu (2005) have studied the MADM problem with incomplete weight information and the attribute values expressed by linguistic form. Xu (2004), Wei and Wang (2008) have researched the MADM problem without the attribute weight information and the attribute values expressed by linguistic form. However, from the above researches, we can find few researches on the MADM problem, in which weight information is completely unknown, the attribute values are expressed by interval numbers. The aim of this study is to establish an optimal model to solve the above problem.

PRELIMINARIES

In the following, we will introduce important concepts and algorithms about interval numbers (Xu and Da, 2003).

Let $\tilde{a} = [a^-, a^+] = \{x|a^- \le x \le a^+, a \le, a^+ \in R\}$, $\tilde{a}$ is an interval number. Specially, if $a^- = a^+$, $\tilde{a}$ is a real number. The algorithms related to interval numbers are following:

If $\tilde{a} = [a^-, a^+]$ and $b = [b^-, b^+]$, $\beta \ge 0$, then

- $\tilde{a} = \tilde{b}$ if and only if $a^- = b^-$, $a^+ = b^+$
- $\tilde{a} + \tilde{b} = [a^- + b^-, a^+ + b^+]$,
- $\beta\,\tilde{a} = [\beta a^-, \beta a^+]$, specially, if $\beta = 0$, then $\beta\,\tilde{a} = 0$

Let $X = \{X_1, X_2, ..., X_n\} (n \ge 2)$ be the set of alternatives and $U = \{u_1, u_2, ..., u_m\}$ $(m \ge 2)$ be the set of attributes. Let $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ be the decision matrix, $\tilde{a}_{ij} = [a^-_{ij}, a^+_{ij}]$ is the attribute value of alternative X_i with respect to the attribute u_j, i = 1, 2, ..., n, j = 1, 2, ..., m.

In order to avoid the influence of different dimensions on the decision making results, the decision matrix $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ should to be normalized into the dimensionless decision matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$ in which:

$$\tilde{r}_{ij} = [r^-_{ij}, r^+_{ij}] = \{t | 0 \le r^-_{ij} \le t \le r^+_{ij} \le 1\} (i = 1,2,\cdots,n, j = 1,2,\cdots,m)$$

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Then, following proportion transformations are employed (Goh *et al*, 1996):

If $\tilde{a}_{ij}$ is for the benefit, then:

$$r_{ij}^- = \frac{a_{ij}^-}{\sqrt{\sum_{i=1}^{n}(a_{ij}^+)^2}}, \quad r_{ij}^+ = \frac{a_{ij}^+}{\sqrt{\sum_{i=1}^{n}(a_{ij}^-)^2}} \tag{1}$$

If $\tilde{a}_{ij}$ is for the cost, then:

$$r_{ij}^- = \frac{1/a_{ij}^+}{\sqrt{\sum_{i=1}^{n}(1/a_{ij}^-)^2}}, \quad r_{ij}^+ = \frac{1/a_{ij}^-}{\sqrt{\sum_{i=1}^{n}(1/a_{ij}^+)^2}} \tag{2}$$

In order to compare the similarity degree of two interval numbers and realize the order of alternatives, we will introduce the concepts about deviation degree and possibility degree of interval number.

Definition 1: Suppose $\tilde{a} = [a^-, a^+]$, $\tilde{b} = (b^-, b^+)$ are two interval numbers, let:

$$d(\tilde{a},\tilde{b}) = \|\tilde{a} - \tilde{b}\| = \sqrt{(b^- - a^-)^2 + (b^+ - a^+)^2} \tag{3}$$

be the deviation degree between $\tilde{a}$ and $\tilde{b}$.

Definition 2: Suppose $\tilde{a} = [a^-, a^+]$, $\tilde{b} = [b^-, b^+]$ are two interval numbers and $l_{\tilde{a}} = a^+ - a^-$ let:

$$p(\tilde{a} = \tilde{b}) = \frac{\min\{l_{\tilde{a}} + l_{\tilde{b}}, \max(a^+ - b^-, 0)\}}{l_{\tilde{a}} + l_{\tilde{b}}} \tag{4}$$

be the possibility degree of $\tilde{a} \geq \tilde{b}$ and $\tilde{a} \geq \tilde{b}$ be the order relationship between $\tilde{a}$ and $\tilde{b}$.

Definition 3: Let $\tilde{r} = (\tilde{r}_1^*, \tilde{r}_1^*, ..., \tilde{r}_m^*)$ be the interval type ideal point, where:

$$\tilde{r}_j^* = [r_j^{-*}, r_j^{+*}] = [\max_i(r_{ij}^-), \max_i(r_{ij}^+)] (j = 1, 2, \cdots, m) \tag{5}$$

MODEL AND METHOD

According to the normalized matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$, the attributes weight vector $w = (w_1, w_2, ..., w_m)^T$ and the algorithms related to interval numbers, the comprehensive attribute value of X_i is:

$$\tilde{z}_i = \sum_{j=1}^{m} \tilde{r}_{ij} w_j \quad (i = 1, 2, \cdots, n) \tag{6}$$

where, w_j is the weight of the attribute u_j and:

$$\sum_{j=1}^{m} w_j^2 = 1 \quad (j = 1, 2, \cdots, m)$$

Obviously, if the alternative X_i is closer to the interval type ideal point, then it is better. The weighted deviation between the alternative X_i and the interval type ideal point $\tilde{r} = (\tilde{r}_1^*, \tilde{r}_1^*, ..., \tilde{r}_m^*)$ is expressed by:

$$\begin{aligned} D_i(w) &= \sum_{j=1}^{m} \|\tilde{r}_{ij} - \tilde{r}_j^*\| w_j \\ &= \sum_{j=1}^{m} d(\tilde{r}_{ij}, \tilde{r}_j^*) w_j \quad (i = 1, 2, \cdots, n) \end{aligned} \tag{7}$$

So, based on the given attribute weight vector $w = (w_1, w_2, ..., w_m$, if $D_i(w)$ is smaller, then the alternative X_i is better. Therefore, we will give the following optimal model:

$$\begin{cases} \min D(w) = (D_1(w), D_2(w), \cdots, D_n(w)) \\ \text{s.t} \quad \sum_{j=1}^{m} w_j^2 = 1 \quad w_j \geq 0 \; j = 1, 2, \cdots, m \end{cases} \tag{8}$$

Generally, these alternatives are fairly competitive and there is no preference relationship among them, so, the above model (8) can be transformed into the following optimal model:

$$\begin{cases} \min D(w) = \sum_{i=1}^{n} D_i(w) = \sum_{i=1}^{n}\sum_{j=1}^{m} d(\tilde{r}_{ij}, \tilde{r}_j^*) w_j \\ \text{s.t} \quad \sum_{j=1}^{m} w_j^2 = 1 \quad w_j \geq 0 \; j = 1, 2, \cdots, m \end{cases} \tag{9}$$

By solving the above model (9), we can obtain:

$$w_j = \frac{\sum_{i=1}^{n} d(\tilde{r}_{ij}, \tilde{r}_j^*)}{\sqrt{\sum_{j=1}^{m}(\sum_{i=1}^{n} d(\tilde{r}_{ij}, \tilde{r}_j^*))^2}} (j = 1, 2, \cdots, m) \tag{10}$$

After normalized, the weight vector $w = (w_1, w_2, ..., w_m)^T$ can be transformed into the optimal weight vector

$$w_j^* = \frac{\sum_{i=1}^{n} d(\tilde{r}_{ij}, \tilde{r}_j^*)}{\sum_{j=1}^{m}\sum_{i=1}^{n} d(\tilde{r}_{ij}, \tilde{r}_j^*)} (j = 1, 2, \cdots, m) \tag{11}$$

If we calculate the attribute weights through the research from literature (Xu and Sun, 2002), then we

need to calculate the deviation degree mn(n-1)/2 times. However, if we calculate the attribute weights through (11), then we only need to calculate the deviation degree mn times. Obviously, if $n \geq 3$, the computational complexity of (11) is lower than it of the research from literature (Xu and Sun, 2002).

Based on the optimal weight vector $w^* = (w_1^*, w_2^*, \ldots, w_m^*)^T$, the comprehensive attribute values $\tilde{z}_i$ (i = 1, 2,..., n) of all the alternatives can be calculated. Because $\tilde{z}_i$ = (i = 1, 2,..., n) are still interval numbers, it is inconvenient to rank the alternatives, so we will calculate the possibility degree of $\tilde{z}_i$ (i=1, 2,..., n) by using Eq. 4 and establish the possibility degree matrix $P = (p_{ik})_{n\times m}$ where $p_{ik} = p(\tilde{z}_i \geq \tilde{z}_k)$ (i, k = 1, 2,..., n). The matrix P is a complementary judgement matrix, the priority vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ of P can be given by using the following Eq.:

$$\omega_i = \frac{1}{n(n-1)}\left(\sum_{k=1}^{n} p_{ik} + \frac{n}{2} - 1\right) \quad (i = 1, 2, \cdots, n) \tag{12}$$

And further, the best alternative will be given if we rank all the alternatives based on the components of the priority vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$.

Based on the above discussion, we develop a new method to solve the multiple attribute decision making problems, in which the information about the attribute weights is unknown completely and the attribute values are in the forms of interval numbers. The method involves the following steps:

- **Step 1:** Let $X = \{X_1, X_2, \ldots, X_n\}$ ($n \geq 2$) be the set of alternatives, $U = \{u_1, u_2, \ldots, u_n\}$ ($m \geq 2$) be the set of attributes and $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ be the decision matrix where $\tilde{a}_{ij} = [a_{ij}^-, a_{ij}^+]$ is the attribute value of alternative X_i with respect to the attribute uj (i = 1, 2,..., n, j = 1, 2,..., m)
- **Step 2:** According to (1) and (2), the decision matrix $\tilde{A} = (\tilde{a}_{ij})_{n\times m}$ normalized into the dimensionless decision matrix $\tilde{R} = (\tilde{r}_{ij})_{n\times m}$
- **Step 3:** According to (11), we obtain the optimal weight vector $w^* = (w_1^*, w_2^*, \ldots, w_m^*)^T$
- **Step 4:** According to (6), we obtain the comprehensive attribute values $\tilde{z}_i$ (i = 1, 2,..., n)
- **Step 5:** Utilize (4) to get the possibility degree of $\tilde{z}_i$ (i = 1, 2,..., n) and establish the possibility degree matrix $P = (p_{ik})_{n\times m}$
- **Step 6:** Utilize (12) to calculate the priority vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of the possibility degree matrix $P = (p_{ik})_{n\times m}$ and rank all the alternatives based on the components the priority vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ and then get the best alternative.

ILLUSTRATIVE EXAMPLE

The following example of practical application is given to show the feasibility and effectiveness of the method. (The example is from the literature (Xu and Sun, 2002).

In order to develop new products, there are five investment projects x_i (i = 1, 2,...,5) to be chosen. The index system includes four attributes: u_1-investment amount, u_2-expected NPV, u_3-profit, u_4-loss of risk, the attribute values are expressed by interval numbers, as listed in the following Table 1:

The attribute u_2-expected NPV and u_3- profit are for the benefit; the attribute u_1-investment amount and u_4-loss of risk are for the cost, the information about the attribute weights is unknown completely. To select the best investment project, the following steps are included:

- **Step 1:** Utilize Table 1 to build the decision matrix $\tilde{A}$:

$$\tilde{A} = \begin{pmatrix} [5,7] & [4,5] & [4,6] & [0.4,0.6] \\ [10,11] & [6,7] & [5,6] & [1.5,2] \\ [5,6] & [4,5] & [3,4] & [0.4,0.7] \\ [9,11] & [5,6] & [5,7] & [1.3,1.5] \\ [6,8] & [3,5] & [3,4] & [0.8,1] \end{pmatrix}$$

- **Step 2:** Utilize (1), (2) to transform the decision matrix $\tilde{A}$ into the dimensionless decision matrix $\tilde{R}$:

$$\tilde{R} = \begin{pmatrix} [0.40,0.71] & [0.32,0.50] & [0.32,0.65] & [0.43,0.98] \\ [0.25,0.35] & [0.47,0.69] & [0.40,0.65] & [0.13,0.26] \\ [0.46,0.71] & [0.32,0.50] & [0.24,0.44] & [0.37,0.98] \\ [0.25,0.39] & [0.40,0.59] & [0.40,0.76] & [0.17,0.30] \\ [0.35,0.59] & [0.24,0.50] & [0.24,0.44] & [0.26,0.49] \end{pmatrix}$$

- **Step 3:** Utilize (11) to get the optimal weight vector:

$$w^* = (0.206, 0.182, 0.193, 0.419)^T$$

Utilize (6) to get the comprehensive attribute values of the five investment projects as follows:

$$\tilde{z}_1 = [0.383, 0.773], \tilde{z}_2 = [0.269, 0.432],$$
$$\tilde{z}_3 = [0.354, 0.733], \tilde{z}_4 = [0.273, 0.460],$$
$$\tilde{z}_5 = [0.271, 0.503]$$

- **Step 4:** Utilize (4) to calculate the possibility degree of $\tilde{z}_i$ (i = 1, 2,..., 5) and build the possibility degree matrix:

Table 1: Decision matrix Ã

	u_1	u_2	u_3	u_4
x_1	[5, 7]	[4, 5]	[4, 6]	[0.4, 0.6]
x_2	[10, 11]	[6, 7]	[5, 6]	[1.5, 2]
x_3	[5, 6]	[4, 5]	[3, 4]	[0.4, 0.7]
x_4	[9, 11]	[5, 6]	[5, 7]	[1.3, 1.5]
x_5	[6, 8]	[3, 5]	[3, 4]	[0.8, 1]

$$P = \begin{pmatrix} 0.5 & 0.911 & 0.545 & 0.867 & 0.808 \\ 0.089 & 0.5 & 0.144 & 0.454 & 0.408 \\ 0.455 & 0.856 & 0.5 & 0.813 & 0.756 \\ 0.133 & 0.546 & 0.187 & 0.5 & 0.451 \\ 0.192 & 0.592 & 0.244 & 0.549 & 0.5 \end{pmatrix}$$

- **Step 5:** Utilize (12) get the priority vector of the possibility degree matrix P:

$$\omega = (0.2566, 0.1547, 0.2441, 0.1659, 0.1789)^T$$

- **Step 6:** Rank all the investment projects and select the best one according to the ω (i = 1,2,...,5):

$$x_1 \succ x_3 \succ x_5 \succ x_4 \succ x_2$$

Thus, the most desirable investment project is x_1. (The result is identical with the literature (Xu and Sun, 2002).

CONCLUSION

In this study, a new method is proposed to discuss the multiple attribute decision making problems, in which the information about the attribute weights is unknown completely and the attribute values are in the forms of interval numbers. Based on the interval type ideal point and the idea that the alternative is better if it is closer to the interval type ideal point, we have established an optimal model and a simple formula to obtain the attribute weights. The priorities of alternatives can be given by the optimal model. We have proposed that the new method has low computational complexity in comparison with the previous researches and is effective for obtaining the priorities of alternatives. Specifically, if we calculate the attribute weights through the research from literature (Xu and Sun, 2002), then we need to calculate the deviation degree mn (n-1)/2 times, if we calculate the attribute weights through (9), then we only need to calculate the deviation degree mn times.

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