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Evaluating Volatility of Price Risk in Electricity Market Based on GARCH and Value-at-risk Theory

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Abstract: The restructuring and deregulation in electric power industry has created the extreme short-time price volatility and heightened the importance of risk management in competitive electricity markets. Accurately characterizing the electricity price volatilities is the foundation to effectively evaluate the price risk in electricity market. With the system demand for electricity as an exogenous explanatory variable, a model to estimate value-at-risk via., a GARCH specification (GARCH-VaR) is proposed, in which the seasonalities, heteroscedasticities, lepkurtosises, heavy-tails and volatility-clustering can be jointly addressed. The impacts of probability distribution assumption and the time-varying features of parameters of the proposed model on the accuracy of VaR estimation are analyzed for three innovation's distributions: normal, student-t and skewed student-t. The numerical example based on the historical data of the PJM electricity market shows that the GARCH-VaR model with conditional skewed student-t innovations performs better in predicting one-period-ahead VaR but the one with Gaussian distribution underestimates the higher quantiles and the one with student-t distribution overestimates the lower quantiles. These results present several potential implications for electricity markets risk quantifications and hedging strategies.

Key words: Value-at-risk, GARCH model, Probability distribution assumption, time-varying parameters

INTRODUCTION

The distinctive characteristics of electric energy which cannot be effectively stored through time and space and need instantaneous balance for supply and demand make electricity price present highly unusual volatility and occasional extreme movements of magnitudes rarely seen in traditional financial markets, thus the power market participants are facing enormous risk of loss. If volatility of price risk in electricity market cannot be effectively identified, assessed and managed, it is possible to cause disastrous consequences for electricity market participants. Once financial risk occurs in electricity market, there have more serious negative effects on society and economy than in traditional financial market. For example, during 2000-2001, what defined the period as California's electricity crisis, California has suffered tremendous economic disruption from the upheaval of its electricity markets. Two investor-owned utilities, PG and E and SCE, had racked up $12 billion of under collections and were on the verge of bankruptcy, bringing the attention of the financial markets to the electric power industry (Liu *et al*., 2007). Therefore, how to effectively make an accurate assessment on the price of volatility risk in electricity market has become an urgent and important problem.

Value-at-Risk (VaR) is a risk management tool to quantify the level of risk exposure in advance. With VaR as the risk measure, the purchasing risk of electric utility is calculated using a normal distribution based Delta model (Zhang and Zhou, 2004.). A copula based method to estimate the VaR of electric utility has been proposed, in which the fluctuating properties and correlation of load and electricity price, respectively described by normal distribution and copula function (Zhong and Li, 2007). The trading risk in energy markets has been dealt with by GARCH based variance-covariance approaches, showing that GARCH with normal distribution errors (N-GARCH) underestimates the actual risk during the sample period (Sadeghi and Shavvalpour, 2006). Considering that N-GARCH cannot effectively address the skewness and kurtosis in the data of profit and loss, a resampling method based on a bias-correction step and the bootstrap has been developed, further improving the VaR forecasting accuracy of the N-GARCH model (Hartz *et al*., 2006). By introducing fatter-tail fractal distribution to describe the distribution of spot and future prices, the optimal strategies of electric purchase portfolio for power distribution companies have been constructed on the basis of traditional invest portfolio model (Wang *et al*., 2009). By utilizing system surplus capacity percent, a hybrid method combining Monte-Carlo simulation and random sampling with replacement is used to evaluate the trading risk faced by an electric utility based on the historical data of Zhejiang electricity market, avoiding the distribution assumption on electricity prices but its estimating accuracy critically depends on the typicality of selected sample (Zhou *et al*., 2004). With electricity applications in mind, the VaR calculating model that accommodates autoregression and weekly seasonalities in both the conditional mean and volatility of returns, as well as leverage effects via an EGARCH specification is proposed, in which extreme value theory (EVT) is adopted to explicitly model the tails of the return distribution. Compared to the parametric models and simple historical simulation methods, the proposed EVT-based evaluating model performs well in forecasting out-of-sample VaR (Chan and Gray, 2006, Gong *et al*., 2009). With EGARCH-based model, the impacts of distribution assumption on VaR estimation accuracy are analyzed for three innovation's distributions: normal, student-t and General Error Distribution (GED). The numerical example based on the historical data of the PJM market shows that the model with GED performs better in predicting VaR (Wang *et al*., 2012).

Li and Sun (2010) have explored the risk in assumption of distribution for innovations by comparing the estimated accuracy of Delta-Gamma-Cornish-Fisher and Delta-normal model, showing that the selection of innovation probability distribution plays very important role for the validity and stability of VaR estimates. Up to now electric power energy cannot be stored economically and therefore demand for electric power energy has an untempered effect on electricity prices, exhibiting special features: mean reversion, seasonality, heteroscedasticity, skewness, lepkurtosis and extreme fast-reverting spikes. Incorporating the pricing dynamics of electricity is of vital importance in price of volatility risk assessment for all market participants for their survival under deregulated environment. In this study, with load as an exogenous explanatory variable, a model to estimate VaR via a GARCH specification (GARCH-VaR) is proposed, in which the features of electricity prices-seasonalities, heteroscedasticities, skewnesses and lepkurtosises can be jointly addressed. A comparative analysis have been made on the estimated results for normal, student-t and skewed student-t GARCH-VaR models to evaluate the impacts of probability distribution assumption and the time-varying features of the model parameters on the accuracy of VaR estimates. The numerical example based on the historical data of the PJM electricity market shows that the skewed student-t GARCH-VaR model with time-varying parameters performs better in predicting one-period-ahead VaR but the one with Gaussian distribution underestimates the GARCH-VaR quantiles below the 5% significance level and the one with student-t distribution overestimates the quantiles above the 2.5% significance level. These results present several potential implications for electricity markets risk quantifications and hedging strategies.

VaR ESTIMATION MODEL

Value-at-Risk (VaR) is one of the most intuitive and comprehensible risk measures. VaR puts a monetary value on the risk that arises from holding an asset. Assuming normal markets and no trading in a given portfolio, VaR is defined as a threshold value such that the probability that the worst loss on the portfolio over a target horizon exceeds this value is the given probability level. The VaR of the portfolio with a confidence interval c is:

$$\mathrm{VaR}_c = \inf\{x \in R \mid \Pr\mathrm{ob}(\Delta P \geq x) \leq 1 - c\} \tag{1}$$

where, Prob(.) denotes the portfolio probability distribution and ΔP the portfolio losses over the given holding period.

For a given time horizon t, suppose that the system demand for electricity is Q_t, the retail price to ultimate customers is P_0, the spot price is $p_t = \overline{p_t} + \varepsilon_t$, where $\overline{p_t}$ is the expected price and ε_t is the random shock. The trading losses of an electric utility over the target horizon t is:

$$\Delta P_t = Q_t(p_t - P_0) = Q_t(\overline{p_t} - P_0 + \varepsilon_t) \tag{2}$$

Let Q_t and P_0 be constant, $F_\varepsilon(x|I_{t-1})$ be the cumulative distribution function of ε_t conditional on the information set I_{t-1} available at time t-1. The VaR of an electric utility in the specified period t with the pre-assigned probability level c, denoted by $\mathrm{VaR}_{c,t}$, is:

$$\begin{aligned} 1 - c &= \Pr\mathrm{ob}\ (\Delta P_t \geq \mathrm{VaR}_{c,t}) \\ &= \Pr\mathrm{ob}\ (Q_t(\overline{p}_t - P_0 + \varepsilon_t) \geq \mathrm{VaR}_{c,t}) \\ &= \Pr\mathrm{ob}\left(\varepsilon_t \frac{\mathrm{VaR}_{c,t} - Q_t(\overline{p}_t - P_0)}{Q_t}\right) \\ &= \int_{\frac{\mathrm{VaR}_{c,t}}{Q_t} - (\overline{p}_t - P_0)}^{\infty} dF_\varepsilon(x \mid I_{t-1}) \\ &= 1 - F_\varepsilon\left(\frac{\mathrm{VaR}_{c,t}}{Q_t} - (\overline{p}_t - P_0) \middle| I_{t-1}\right) \end{aligned} \tag{3}$$

Now we obtain:

$$\mathrm{VaR}_{c,t} = Q_t(\overline{p_t} - P_0 + F_\varepsilon^{-1}(c \mid I_{t-1})). \tag{4}$$

where, F_c^{-1} is the quantile function defined as the inverse of the distribution function F_c. Therefore, calculating VaR does require some knowledge of the underlying asset distribution.

GARCH-VaR model: The seasonality in the electricity spot market is particularly obvious over the day and over the week. We therefore include a general formulation for sinusoidal function in the model to capture the possibility of having many cycles per year. Assuming that p_t, d_t, ε_t and z_t denote the electricity spot price, the system load, the random shock and the normalized innovation at time t, h_t denotes the conditional variance of ε_t, then the GARCH model depicting the changing rule of electricity spot price at time t can be formulated as follows:

$$\begin{aligned}
&p_t = p_t + \varepsilon_t \\
&\overline{p}_t = f(t) + \gamma(B)d_t^2 + \varphi(B)p_t + k(B)\varepsilon_t \\
&f(t) = \alpha_0 + \alpha_1 d_{wkd} + \sum_{i=1}^{m}\alpha_{1i}\sin\left(\frac{2\pi i}{365}t + \alpha_{2i}\right) \\
&\gamma(B) = \gamma_0 + \gamma_1 B + \gamma_2 B^2 + \ldots + \gamma_u B^u \\
&\varphi(B) = \varphi_1 B + \varphi_2 B^2 + \varphi_3 B^3 + \ldots + \varphi_v B^v \\
&k(B) = 1 + k_1 B + k_2 B^2 + \ldots + k_q B^q \\
&\varepsilon_t = \sqrt{h_t}z_t,\ z_t \mid I_{t-1} \sim D(0, 1) \\
&h_t = \beta_0 + \sum_{i=1}^{r_h}\beta_{1i}h_t + \sum_{i=1}^{S_h}\beta_{2i}\varepsilon_{t-i}^2
\end{aligned} \tag{5}$$

where, B is the backshift operator, d_{wkd} is a dummy variable that takes a value of 1 if the observation is in weekday and zero otherwise; u, v and q, respectively denote the lagged orders of d_t^2, p_t and ε_t in the mean equation; r_h and s_h denote the lagged order of h_t and ε_t^2 in the conditional variance equation; m is the number of changing cycles of electricity price series per year, the amplitude and location of the peak can be, respectively captured by $\alpha^*_1 = (\alpha_{11},..., \alpha_{1m})$ and $\alpha^*_1 = (\alpha_{21},..., \alpha_{2m})$; $\alpha = (\alpha_0, \alpha_1, \alpha^*_0, \alpha^*_2)$, $\beta = (\beta_0, \beta_{11},..., \beta_{1r_h},... \beta_{2S_h})$, $\gamma = (\gamma_0, \ldots, \gamma_u)$, $\varphi = (\varphi_1,\ldots, \varphi_p)$, $\kappa = (\kappa_1,\ldots, \kappa_q)$ and $\theta = (\theta_0,\ldots, \theta_v)$ are the parameters to be estimated.

Given $F_z(x|I_{t-1})$, the conditional distribution function of standardized innovation z_t and using Eq. 4, the VaR of the electric utility over the target horizon t with the confidential level c, $VaR_{c,t}$, is defined as:

$$VaR_{c,t} = Q_t(\overline{p_t} - P_0 + \sqrt{h_t}F_z^{-1}(c \mid I_{t-1})). \tag{6}$$

Residuals distribution assumption: Before parameters calibration, assumption on the distribution of random errors needs to be made. In order to effectively depict the kurtosis and fat-tail of electricity price, we assume that the Probability Density Function (PDF) for the standardized innovation z_t, a white nose process with zero mean and constant variance equal to 1, is consistent with skewed student-t probability distribution. The PDF of skewed student-t distribution can be expressed as:

$$\begin{aligned}
&f(x \mid I_{t-1}) = b_t c_t\left(1 + \frac{1}{n_t - 2}\left(\frac{b_t x + a_t}{1 \pm \lambda_t}\right)^2\right)^{-\frac{n_t+1}{2}} \\
&1 \pm \lambda_t = \begin{cases} 1 + \lambda_t,\ \forall x \geq -a_t / b_t \\ 1 + \lambda_t,\ \forall x < -a_t / b_t \end{cases} \\
&a_t = 4\lambda_t c_t\left(\frac{n_t - 2}{n_t - 1}\right) \\
&b_t = 1 + 3\lambda_t^2 - a_t^2 \\
&c_t = \frac{1}{\sqrt{\pi(\eta_t - 2)}\Gamma(\eta_t / 2)}\Gamma\left(\frac{\eta_t + 1}{2}\right)
\end{aligned} \tag{7}$$

where, $\Gamma(.)$ is a Gamma function, λ_t and η_t are the conditional skewness and degree of freedom of skewed student-t distribution. If we denote the upper and lower limits of conditional degree of freedom by U_η and L_η, the upper and lower limits of conditional skewness by U_λ and L_λ, we can rewrite λ_t and η_t as:

$$\begin{aligned}
&\eta_t = L_n + \frac{U_\eta - L_\eta}{1 + \exp(-\omega t)} \\
&\omega_t = \delta_0 + \sum_{i=1}^{r_\eta}\delta_{1i}\varepsilon_{t-i} + \sum_{i=1}^{s_\eta}\delta_{2i}\varepsilon_{t-i}^2 + \sum_{i=1}^{v_\eta}\delta_{3i}\omega_{t-i} \\
&\lambda_t = L_\lambda + \frac{U_\lambda - L_\lambda}{1 + \exp(-\mu_t)} \\
&\mu_t = \tau_0 + \sum_{i=1}^{r_\lambda}\tau_{1i}\varepsilon_{t-i} + \sum_{i=1}^{S_\lambda}\tau_{2i}\varepsilon^3{}_{t-i} + \sum_{i=1}^{v_\lambda}\tau_{3i}\mu_{t-i}
\end{aligned} \tag{8}$$

where, $\delta = (\delta_0, \delta_{11}, \cdots, \delta_{1r_\eta}, \delta_{21}, \cdots, \delta_{2s_\eta},, \delta_{31}, \cdots, \delta_{3v_\eta})$ and $\tau = (\tau_0, \tau_{11}, \cdots, \tau_{1r_\lambda}, \tau_{21}, \cdots, \tau_{2s_\lambda}, \tau_{31}, \cdots, \tau_{3v_\lambda})$ are the estimated parameters; r_η, s_η and v_η are the lagged orders of ε_t, ε^2_t and ω_t in the equation of conditional degree of freedom; r_λ, s_λ and v_λ are the lagged orders of ε_t, ε^3_t and μ_t in the equation of conditional skewness. When λ_t=0, the skewed student-t distribution degenerates to student-t distribution.

Model estimation method: Now there have been various estimation methods for the parameters of the GARCH-VaR model, Gebizlioglu *et al*. (2011) have shown that the Maximum Likelihood Estimator (MLE) performs better for the large samples. Along this line, we estimate the parameters of the proposed forecasting models by maximizing conditional log-likelihood function under different assumption for residuals' probability distribution.

Let $\xi = (\alpha, \gamma, \varphi, \kappa, \beta, \gamma, \tau)$, the log-likelihood function for all observations corresponding to z_t, a random variable with skewed-t distribution, is:

$$L(\xi) = \sum_{t=1}^{T} l_t(\xi) = \sum_{t=1}^{T}\left(\ln\left(\frac{b_t c_t}{\sqrt{h_t}}\left(1+\frac{1}{\eta_t-2}\left(\frac{b_t z_t + a_t}{1\pm\lambda_t}\right)^2\right)^{-\frac{\eta_t+1}{2}}\right)\right) \quad (9)$$

where, $l_t(\xi) = \ln f_{z_t}(z_t \mid I_{t-1})$ is the log-likelihood function for one observation at period t, T is the sample volume. By maximizing $L(\xi)$, $\hat{\xi}$, the estimated values of parameters ξ can be obtained. It is important to note that the log-likelihood function $L(\xi)$ is highly nonlinear. The starting values of parameters ξ must be selected with care. In order to improve the accuracy of estimation, a successive approximation method, namely using the parameters estimated from simpler models as starting values for more complex one, is used in this study.

Backtesting for VaR estimates: It is of crucial importance to assess the accuracy of VaR estimates, as they are only useful insofar as they accurately characterize risk. Backtesting or verification testing is the way that we verify whether projected losses are in line with actual losses. The most widely known backtesting method based on failure rates has been suggested by Kupiec (1995). Kupiec's test measures whether the number of violation exceptions (losses larger than estimated VaR) is in line with the expected number for the chosen confidence interval. Denoting the number of times that the actual portfolio returns fall outside the VaR estimate as N and the total number of observations as T, we may define the number of violation exceptions as:

$$N = \sum_{t=1}^{T} I_t(\Delta P_t > VaR_{c,t}), \quad (10)$$

where, $I_t(.)$ is an indicator function. Under the null hypothesis that the VaR estimated model is correct at a pre-assigned confidence interval, the number of violation exceptions N should follow a binomial probability distribution:

$$P(N|T,\alpha) = \binom{T}{N}\alpha^N(1-\alpha)^{T-N}, \quad (11)$$

where, T is the sample size and α corresponding to the significance level chosen for the VaR approach. If the sample size T is input and α is set to one minus the level of confidence, the binomial function produces the likelihood that a specific number of VaR breaks is to occur.

The observed failure rate N/T should act as an unbiased measure of α = 1-c as sample size is increased. Assuming that the proposed model is accurate, the following Likelihood-Ratio (LR):

$$LR = -2\log\left(\frac{(1-c)^N c^{T-N}}{\left(\frac{N}{T}\right)^N\left(1-\frac{N}{T}\right)^{T-N}}\right) \quad (12)$$

is asymptotically χ^2 (chi-squared) distributed with one degree of freedom. If the value of LR exceeds the critical value of the χ^2 distribution, the null hypothesis will be rejected and the model is deemed as inaccurate. On the contrary, the null hypothesis will be accepted and the model should be considered correct.

EMPIEICAL RESULT

The PJM is organized as a day-ahead market. Participants submit their buying and selling bid curves for each of the next 24 h. Then the market operator aggregates bids for each hour and determines market clearing prices and volumes for each hour of the following day. In this study, a total of 1197 observations of average daily electricity spot prices in dollars per megawatt hour ($/MWh) and average daily loads in gigawatt (Gw) are employed to validate the performance of the VaR calculating model. The sample period begins on 1st June 2007 and ends on 9th September 2010. As shown in Fig. 1.

Table 1 presents some descriptive statistics for the average daily electricity spot price and load series. It can be seen from Table 1 that electricity prices and loads are quite volatile, highly non-normal, clearly skewed rightward and with a median well below the mean. In fact the nulls of normality of electricity price and load series are rejected with the Jarque-Bera test. This is typical of electricity spot prices in a competitive market.

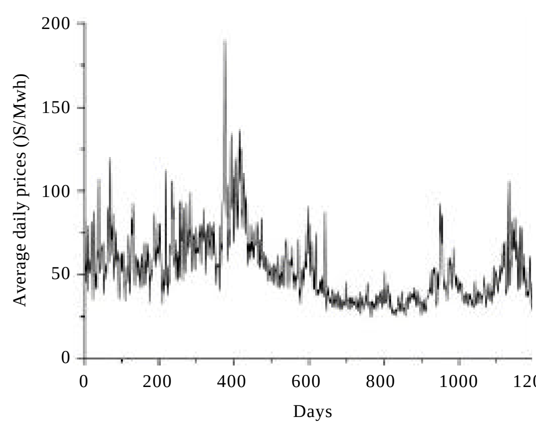

Fig. 1: Average daily electricity spot prices in the PJM electricity market

Table 1: Descriptive statistics of the sample

Statistics	Price ($/MWh)	Load (GW)
Mean	53.520410	81.192210
Median	49.970680	79.892210
Maximum	189.655700	115.783900
Minimum	24.874940	58.345860
Std. Dev.	20.201580	10.505600
Skewness	1.420081	0.375318
lepkurtosis	6.594566	2.582759
Jarque-Bera	1046.748000	36.785060
p-value	0.000000	0.000000

Table 2: Parameters estimates of GARCH-VaR model

	Normal	t	Var. FD.	Var. FD.SK.
α_0	1.2830***	0.9344***	0.7896**	0.8639***
α_1	-2.6623***	-2.0047***	-1.8039***	-1.7840***
γ_0	0.0070***	0.0065***	0.0063***	0.0062***
γ_1	-0.0068***	-0.0063***	-0.0061***	-0.0061***
φ_1	0.9868***	0.9841***	0.9820***	0.9844***
α_{12}	-0.2447***	0.3053***	0.3155***	0.2926***
α_{22}	3.5105	277.3000***	277.5900***	281.9700***
α_{14}	-0.1933***	-0.1519***	-0.1373***	-0.1315***
α_{24}	-51.7760***	-323.7900***	-323.3500***	-322.8200***
α_{16}	-0.1583***	0.1025**	0.0886**	-0.0783**
α_{26}	10.7870***	-18.9950***	-18.1980***	-109.2600***
α_{112}	0.1640***	-0.1056**	-0.0996*	-0.1154**
α_{212}	3.5509**	17.0870***	17.3340***	18.4260***
α_{124}	-0.2357***	-0.1817**	-0.1744***	-0.2070***
α_{224}	-21.3460***	-81.7230***	-81.4450***	-81.0760***
α_{152}	-1.0666***	0.9821***	0.9759***	0.9734***
α_{252}	14.047***	-94.7380***	-94.7570***	-94.8860***
κ_1	-0.2813***	-0.2724***	-0.2657***	-0.3050**
κ_2	-0.2624***	-0.2529***	-0.2383***	-0.2355***
κ_3	-0.1675***	-0.1527***	-0.1781***	-0.1750***
β_0	0.2388**	0.2750***	0.2532**	0.2270**
β_{11}	0.8011***	0.8330***	0.8228***	0.8421***
β_{21}	0.2191***	0.1722***	0.2140***	0.1786***
δ_0		5.5971***	-1.2305***	-0.8825***
δ_{11}			0.1850***	0.2070***
δ_{21}			-0.0046***	-0.0037**
δ_{31}			0.4441***	0.5282***
τ_0				0.1877**
τ_{11}				-0.0377
τ_{21}				0.0003
τ_{31}				0.6119***

"*", "**" and "***", respectively indicate statistical significance of estimated parameters at 90, 95 and 99% confidence interval. "Normal" denotes normal distribution; "t" student-t; "Var. FD." t with time-varying degree of freedom; "Var. FD. SK." skewed-t with time-varying skewness and degree of freedom

Estimates of GARCH-VaR model: Analyzing the correlation coefficient, the partial correlation coefficient and the time changing trend chart of the sample data, the values of m, u, v, q, r_h, s_h, r_η, s_η, v_η, r_λ, s_λ, v_λ in the GARCH model can be identified. In our situation, they are equal to 52, 1, 1, 3, 1, 1, 1, 1, 1, 1, 1, 1. Table 2 shows the results of the maximum likelihood estimation for the proposed GARCH model.

Analyzing the data in Table 2, we can draw the following conclusions: Firstly, concerning the mean equation, the t-statistics of α_{1i}, α_{2i}, $\forall i \in (2,4,6,12,24,52)$, associated to the seasonal effects, are significant at the 95% confidence interval, showing that there exist weekly, monthly, quarterly and semi-annual cycles in the sample periods. Secondly, the t-statistic for α_1 is significant at the 99% confidence level, suggesting that the impacts of load on the average daily electricity prices for weekday and weekend are more different. Thirdly, volatility is found to be persistent since the coefficient of lagged volatility, β_{11}, is positive and significant at the 99% confidence level, indicating high conditional variance is followed by high conditional variance. Fourthly, the t-statistics of β_{21}, δ_{21}, are significant at 95% confidence interval, indicating that the volatility of conditional variance, degree of freedom will be strengthened by external shocks. Specifically, the degree of freedom manifests obviously time-varying features, since the coefficients δ_{11}, δ_{21} and δ_{31} are significant.

Table 3: Backtesting results of VaR estimates

Percentage	Exception	Normal	t	Var. FD.	Var. FD.SK.
90	Expected	120	120	120	120
	Real	115	94	82	116
	LR	0.2075	6.570**	14.67***	0.1283
95	Expecteds	60	60	60	60
	Real	63	43	37	54
	LR	0.1717	5.513**	10.56***	0.6214
97.5	Expected	30	30	30	30
	Real	42	20	18	29
	LR	4.449**	3.816*	5.672**	0.0296
99	Expected	12	12	12	12
	Real	22	12	10	9
	LR	6.805***	0.001	0.3469	0.8142
99.5	Expected	6	6	6	6
	Real	18	6	7	6
	LR	15.73***	0.000	0.1640	0.000

"*", "**" and "***", respectively indicate statistical significance of estimated parameters at 90, 95 and 99% confidence interval. "Normal" denotes normal distribution; "t" student-t; "Var. FD." t with time-varying degree of freedom; "Var. FD. SK." skewed-t with time-varying skewness and degree of freedom

VaR backtesting: Without loss of generality, in this study we assume that an electric utility has the obligation to serve 1MW of load 24 hours a day and the retail price has been frozen at a level equivalent to 0$/MWh. The Kupiec's test results are shown in Table 3. It can be seen from Table 3 that the model with normal distribution underestimates the quantiles below the 5% significance level and that the one with student-t distribution overestimates the quantiles above the 2.5% significance level whereas the null hypothesis cannot be rejected for the skewed student-t GARCH-VaR model with time-varying skewness and degree of freedom in each significance level. Summarizing the results for the Kupiec's test, our method is able to improve the VaR forecasts so much that VaR predictions are obtained which are insignificantly different from the proposed downfall probability.

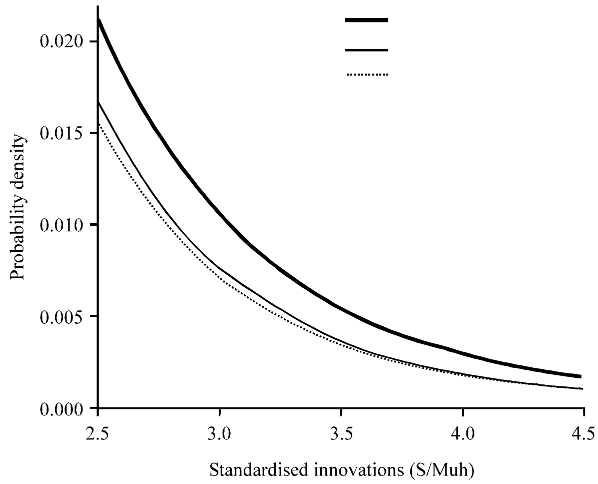

Fig. 2: Right tails of probability density functions for t, Var FD. and Var FD.SK.

Figure 2 shows the right tails of probability density function for t distribution, t distribution with time-varying degree of freedom and skewed-t distribution with time-varying skewness and degree of freedom. As can be seen from Fig. 2 and Table 3, the t distribution with time-varying degree of freedom has the thinnest right-tail and the VaR estimate effectiveness is the worst; the t distribution has fatter right tail than the t distribution with time-varying degree of freedom, the estimate accuracy of the former is better than the latter. In contrast, the skewed-t distribution with time-varying skewness and degree of freedom has the heaviest right tail which can more effectively portray the kurtosises and fat-tails of electricity prices, the VaR estimate accuracy is also best.

CONCLUSION

The distinctive characteristics of electric energy which cannot be effectively stored through time and space and need instantaneous equilibrium of supply and demand make electricity price present highly volatility and occasional extreme movements of magnitudes rarely seen in markets for traditional financial assets, thus price risk identification, evaluation and management in electricity market are more important than in traditional markets. Considering various influencing factors on electricity prices and their pertinence, a model to estimate VaR via a GARCH specification with load as an exogenous explanatory variable is proposed, in which the seasonalities, heteroscedasticities, volatility-clustering and heavy-tails can be jointly addressed. The impacts of probability distribution assumption and the time-varying features of parameters on the accuracy of VaR estimation are analyzed for three innovation's distributions: normal, student-t and skewed student-t. The numerical example based on the historical data of the PJM electricity market shows:

- VaR provides participants in the electricity market with a method for easily quantifying their risk exposure to movements in electric power prices
- The skewed student-t GARCH-VaR model with time-varying parameters performs better in estimating one-period-ahead VaR but the one with normal distribution underestimates the quantiles below the 5% significance level and the one with student-t distribution overestimates the quantiles above the 2.5% significance level

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