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Image Compressed Sensing Based on the Double Single-Layer Wavelet Transform

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Abstract: Researches have shown that in the CS algorithm, the sparse representation is increased with the increase of the number of decomposition layers and dividing the image into blocks can greatly shorten the run time. Based on the two points, an improved CS algorithm based on the double single-layer wavelet transform is proposed in this paper. After the first layer wavelet decomposition, the high-pass wavelet coefficients are measured by the single layer wavelet transform CS algorithm and the low-pass wavelet coefficients are preserved. High-pass wavelet coefficients can be recovered by the measurements by using recovery algorithm. Then the image can be reconstructed by the inverse wavelet transform. The simulation experiments were conducted using the Matlab software and three images were chosen as test images. The simulation results demonstrated that compared with the existing algorithms, the proposed algorithm could greatly shorten the run time and improve the quality of the reconstructive images. This will allow the proposed algorithm to be used in applications in image CS field.

Key words: Compressed sensing, sparse representation, wavelet decomposition, Orthogonal matching pursuit

INTRODUCTION

Shannon-Nyquist sampling theorem demonstrates that signals can be exactly recovered from a set of uniformly spaced samples if the sampling rate is at least twice the bandwidth of the signal of interest. This basic principle underlies the majority of signal processing. However, with the development of information technology, the Shannon-Nyquist theorem is facing challenges both on the acquisition hardware and on the subsequent storage. Compressed Sensing (CS) was initiated in 2006 by Donho and Candes, which enables a potentially large reduction in the sampling and computation costs by combining sampling and compression. Since then, CS has become a key concept in various areas and an abundance of theoretical aspects have already been explored.

THE COMPRESSED SENSING THEORY

The key idea of CS is to recover a sparse signal from very few no-adaptive, linear measurements by convex optimization. CS processing consists of the following three parts: (1) Sparse representation: CS builds upon the fundamental fact that many signals can be represented using only a few non-zero coefficients in a suitable basis or dictionary. (2) Sensing: signals are directly sensed in a compressed way at a lower sampling rate. (3) Recovery algorithms: signals are recovered by sparse recovery algorithms after a dimension reduction step.

Let now $x \in R^{N\times 1}$ of length N be our signal of interest. A set $\{\psi_i\}^N_{i=1}$ is an orthogonal basis so that x has a unique representation as a linear combination of these basis vectors. Let $\Psi\{\psi_1, ..., \psi_n\}$ There exist coefficients $\{\theta_i\}^N_{i=1}$ such that:

$$x = \Sigma^N_{i=1} \theta_i \psi_i \quad (1)$$

Signal x is k-sparse if Θ has at most k nonzeros and Ψ is the sparse basis. Depending on the signal, a variety of transformations can be used to provide sparse representations such as Fourier transform, wavelet transform and discrete cosine transform.

Assuming M linear measurements are acquired in measurement systems. The measurement process can be expressed as:

$$y = \Phi\Theta = \Phi\Psi^T x = Ax \quad (2)$$

where, Φ is an M×N matrix which is typically called sensing matrix, $y \in R^M$ and $A = \Phi\Psi^T$. Throughout this paper $M<N$ is always assumed. The original signal x can be recovered from measurements y if the sensing matrix Φ satisfies Restricted Isometry Property (RIP). The best

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choices for sensing matrices are random matrices such as Gaussian matrices, Bernoulli matrices, or uniform random ortho-projectors.

Signal recovery processing attempts to recover x from measurements y by solving an optimization problem of the form:

$$\min\|\Phi\|_{l_0} \text{ s.t.} Ax = \Phi\Psi^T x = y \quad (3)$$

Dual to the unavoidable combinatorial search, this algorithm is NP-hard, thus the closest convex norm is used to substitute the l_0 norm, which is the l_1 norm. This leads to the following minimization:

$$\min\|\Phi\|_{l_1} \text{ s.t.} Ax = \Phi\Psi^T x = y \quad (4)$$

There are various recovery algorithms including interior point method, Gradient Projection (GP), Orthogonal Matching Pursuit (OMP), iterative thresholding as well as kinds of improved algorithm.

COMPRESSED SENSING BASED ON THE DOUBLE SINGLE-LAYER WAVELET TRANSFORM

In traditional CS processing based on wavelet transform, firstly, $N \times N$-dimensional image is decomposed by wavelet transform to get the sparse coefficient matrix, then all the wavelet coefficients are measured by using $M\times N$ sensing matrices to obtain the $M\times N$ measurements. Finally, the recovery algorithm and inverse wavelet transform are used to recover the original image.

In the traditional CS algorithm as described above, the number of layers of wavelet decomposition has an important impact on the reconstruction results. Reconstruction effect is enhanced with the increase of decomposition layer, thus 4-5 or more layers are required to meet the requirements. Besides, the wavelet transform divides the image into its low and high-frequency components, the higher frequency components are sparse, while the lowest frequency components which provide a coarse scale approximation of the image cannot be considered to be sparse, so measuring all the coefficients will destroy the correlation and lead to worse quality of the recovered image. To solve these problems, an improved CS algorithm based on the single layer wavelet transform was proposed in paper, which only measures the high-pass wavelet coefficients of the image but preserves the low-pass wavelet coefficients. For the reconstruction, by using the recovery algorithm, high-pass wavelet coefficients can be recovered by the measurements. Then the image can be reconstructed by the inverse wavelet transform. The algorithm improves the quality of the recovered image significantly.

As already mentioned, the sparse representation is better if the number of decomposition layers increases. And dividing the image into blocks can greatly shorten the run time. Based on the above two points, an improved CS algorithm based on the double single layer wavelet transform will be proposed in this study. After the first layer wavelet decomposition, the high-pass wavelet coefficients are measured by the single layer wavelet transform CS algorithm and the low-pass wavelet coefficients are preserved. Fig. 1 shows the sparse representation via the double single-layer wavelet transform. On one hand double decomposition layers can improve the quality of reconstructed image; on the other hand, the second wavelet transform which is

Fig. 1: Sparse representation of cameraman via the double single-layer wavelet transform

equivalent to the block processing can greatly reduce the time cost. The algorithm is as follows:

- After a wavelet decomposition of N×N image, four wavelet sub-band coefficients $\{LL_1, LH_1, HL_1, HH_1\}$ are obtained
- A second wavelet decomposition is used for LH_1, HL_1, HH_1 and three sets of wavelet coefficients $\{LH\_LL_1, ..., LH\_HH_1\}$, $\{HL\_LL_1, ..., HL\_HH_1\}$, $\{HH\_LL_1, ..., HH\_HH_1\}$ are gained
- Selecting the appropriate value of M and let M×N/4 Gaussian random matrix which obeys (0,1/N) be the sensing matrix. The coefficient matrices can be obtained after measuring X_LH_1, X_HL_1, X_HH_1 (X = LH, HL, HH), while the low-pass coefficients X_LL_1 of each set are preserved
- By using OMP algorithm, $\{LH\_\widetilde{LH}_1, ..., LH\_\widetilde{HH}_1\}$, $\{HL\_\widetilde{LH}_1, ..., HL\_\widetilde{HH}_1\}$, $\{HH\_\widetilde{LH}_1, ..., HH\_\widetilde{HH}_1\}$ could be recovered by the measurements, together with LH_LL_1, HL_LL_1, HH_LL_1, three high-pass sub-band coefficients $\widetilde{LH}_1, \widetilde{HL}_1, \widetilde{HH}_1$ could be reconstructed by the inverse wavelet transform
- Then the image could be reconstructed by the inverse wavelet transform of $\widetilde{LH}_1, \widetilde{HL}_1, \widetilde{HH}_1$ and LL_1.

SIMULATION RESULTS

According to the previous steps, this simulation chose Daubechies wavelet (db) as wavelet basis, Gaussian random matrix which obeys (0, 1/N) as sensing matrix and OMP algorithm as recovery algorithm.

Let Cameraman, Lena, Rice be the test images. Different values of M were taken to get the different compression ratios. Since the sensing matrix is random, every time the value of M changed, the program would run three times to obtain the average value as result. Fig. 2 shows the simulation results in different compression ratios. The average run time of each image is showed in Table 1.

Simulation results demonstrated that the run time of the program and the quality of the reconstructive image had great improvements. The peak signal-to-noise ratio (PSNR) of the proposed algorithm was improved about 1~2dB and the time cost reduced by 2/3.

Table 1: Average run time of each image

Test image	Average run time (t sec^{-1}) Single layer	Double single-layer
Cameraman	46.9385	16.81367
Lena	38.74971	12.03314
Rice	44.95357	15.47929

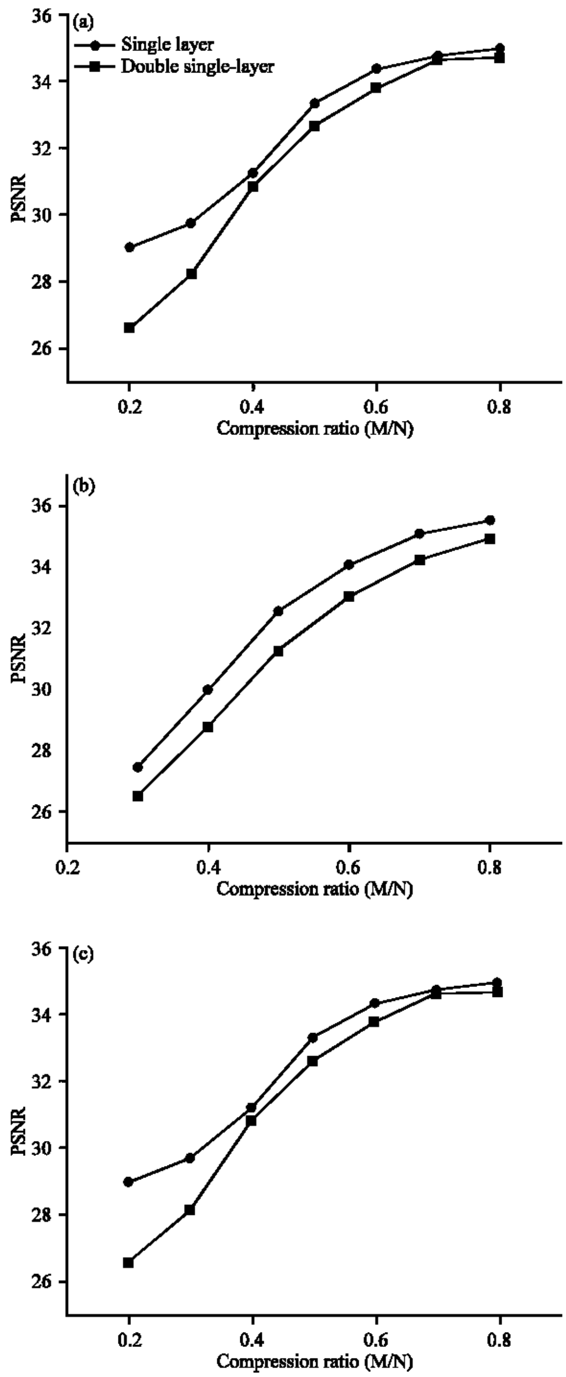

Fig. 2(a-c): Simulation results in different compression ratios, (a) Lena image (b) Cameraman image and (c) Rice image

CONCLUSION

CS is an exciting, rapidly growing, field that has attracted considerable attention. This paper briefly introduces the theory of compressed sensing and CS algorithms. Since the existing compressed sensing algorithms have its own disadvantages, an improved image CS algorithm based on the double single-layer wavelet transform is proposed. According to the simulation results of test images, the proposed algorithm can greatly shorten the run time and improve the quality of the reconstructive images. This will allow the proposed algorithm to be used in applications in image CS field. Further work can be carried out to research the algorithm performance of the image with image and promote the use of CS in practical applications.

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