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## Research on Seismic Data Denoising Method by Wavelet Packet Transform

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**Abstract:** Seismic data denoising processing is one of the main tasks to solve in seismic exploration and to improve the signal-to-noise ratio of seismic signals is particularly important. Wavelet packet decomposition techniques are selected to reduce the noise of seismic wave signals and wavelet packet decomposition processing to test signal, synthetic seismic signals and the actual seismic data are done by MATLAB, reducing the high frequency random noise better and retaining the useful signal. The validity and reliability of the wavelet packet noise reduction are verified and the simulation results show that the wavelet packet denoising techniques can effectively reduce noise in seismic signals to improve the resolution.

**Key words:** Wavelet-packet, seismic data, denoising, resolution, simulation

### INTRODUCTION

In seismic exploration, eliminating all interference waves is the key of seismic data processing, the general interference waves include surface waves, high-frequency random noises, side wave, multiples etc., in which high-frequency surface waves and random interference waves have more serious impact on the significant wave and therefore it is need to study the proper filtering method to remove the surface noises and high frequency random interference wave in the seismic record waves to improve the signal to noise ratio (Welford and Zhang, 2004; Li *et al.*, 2003; Danial and Jandyr, 2000). Seismic signal are non-stationary signals, filtering effect is not ideal when the signal and noise bands overlap using time-invariant filtering method (such as traditional Fourier transform) (Liu *et al.*, 2006). The wavelet transform overcome the shortcomings that Fourier transform can not take the time resolution and frequency resolution into account in signal analysis and the time window and frequency domain window can change through mother wavelet compression and shift which can be multi-resolution analysis, so wavelet analysis was used for signal extraction and denoising by many domestic and foreign researchers. Wavelet analysis as a better time-frequency analysis methods, in recent years have been applied to seismic studies and achieved good results (Kong *et al.*, 2005).

Wavelet packet transform is the promotion of wavelet transform, a more elaborate analysis and reconstruction method, in which the frequency band was done multi-level division, the high frequency part can be further decompose can not subdivided in the wavelet analysis. And it can select the appropriate frequency

band adaptively according to the characteristics of the signal being analyzed, to match the frequency spectrum of the signal, thereby enhancing the time-frequency resolution, so wavelet packet has a broader application.

Wavelet packet transform was used for the decomposition of low-frequency seismic data and the decomposition of high-frequency information while lost in the low-frequency decomposition, in order to achieve more precise decomposition than wavelet analysis. Test signals denoising and synthetic seismic records denoising by simulation experiments were achieved and the actual seismic signal denoising were also done, studies showed that wavelet packet transform is very effective in de-noising and can get better denoising results.

### WAVELET PACKET TRANSFORM

**Wavelet packet definition:** In the multi-resolution analysis,  $L^2(\mathbb{R}) = \bigoplus_{j \in \mathbb{Z}} W_j$  show multi-resolution analysis is that the Hilbert space  $L^2(\mathbb{R})$  is decomposed to subspaces  $W_j (j \in \mathbb{Z})$  orthogonal according to a different scale factor  $j$ . In which,  $W_j$  is the closures for the wavelet function  $\varphi(t)$  (wavelet subspace). The wavelet subspace  $W_j$  was done frequency subdivision as binary fractions, in order to achieve the purpose of improving the frequency resolution. The scale subspace  $V_j$  and wavelet subspace  $W_j$  characterize uniformly with a new subspace  $U_j^n$  and order:

$$\begin{cases} U_j^0 = V_j \\ U_j^1 = W_j \end{cases}, (j \in \mathbb{Z}) \quad (1)$$

The Hilbert space orthogonal decomposition  $V_{j+1} = V_j \oplus W_j$  could unify using  $U_j^n$  decomposition:

$$U_{j+1}^0 = U_j^0 \oplus U_j^1, j \in Z \tag{2}$$

Define subspace  $U_j^n$  is the closure space of function  $u_n(t)$ ,  $U_j^{2n}$  is the closure space of  $u_{2n}(t)$  and order  $u_n(t)$  satisfies the following two-scale equation:

$$\begin{cases} u_{2n}(t) = \sqrt{2} \sum_{k \in Z} h(k) u_n(2t - k) \\ u_{2n+1}(t) = \sqrt{2} \sum_{k \in Z} g(k) u_n(2t - k) \end{cases} \tag{3}$$

In which  $g(k) = (-1)^k h(1 - k)$ , that is two coefficients have orthogonal relationships. When  $n = 0$ , the above equation becomes:

$$\begin{cases} u_0(t) = \sum_{k \in Z} h_k u_0(2t - k) \\ u_1(t) = \sum_{k \in Z} g_k u_0(2t - k) \end{cases} \tag{4}$$

$u_0(t)$  and  $u_1(t)$  are respectively scale function  $\phi(t)$  and wavelet function  $\varphi(t)$ , Eq. 4 is the equivalent representation of Eq. 2. This equivalent representation were extended to  $n \in Z_+$  (non-negative integer), that is equivalent representation of Eq. 3:

$$U_{j+1}^n = U_j^{2n} \oplus U_j^{2n+1} \quad j \in Z, n \in Z_+ \tag{5}$$

This allow the wavelet subspace further subdivided as binary:

$$\begin{aligned} W_j &= U_j^1 = U_{j-1}^2 \oplus U_{j-1}^3 \\ U_{j-1}^2 &= U_{j-2}^4 \oplus U_{j-2}^5, U_{j-2}^5 = U_{j-2}^6 \oplus U_{j-2}^7 \\ &\dots \\ W_j &= U_{j-k}^{2^k} \oplus U_{j-k}^{2^k+1} \oplus \dots \oplus U_{j-k}^{2^k+m} \oplus \dots \oplus U_{j-k}^{2^k+1-1} \end{aligned} \tag{6}$$

In which orthonormal basis  $\{2^{(j-k)/2} \varphi_{2^k+m}(2^{j-k}t - 1); 1 \in Z_j\}$  are called wavelet packet corresponding to  $k = 1, 2, \dots; j = 1, 2, \dots$  and subspaces  $U_{j-k}^{2^k+m}$ .

Assume  $g_j^n(t) \in U_j^n$ , then  $g_j^n(t)$  can be expressed as:

$$g_j^n(t) = \sum_1 d_1^{j,n} u_n(2^j t - 1)$$

Wavelet packet decomposition are that  $\{d_1^{j,2n}\}$  and  $\{d_1^{j,2n+1}\}$  are obtained by  $\{d_1^{j+1,n}\}$ , that is:

$$\begin{aligned} d_1^{j,2n} &= \sum_k a_{k-2l} d_k^{j+1,n} \\ d_1^{j,2n+1} &= \sum_k b_{k-2l} d_k^{j+1,n} \end{aligned} \tag{7}$$

Wavelet packet reconstruction are that  $\{d_1^{j+1,n}\}$  are obtain by  $\{d_1^{j,2n}\}$  and  $\{d_1^{j,2n+1}\}$ , that is:

$$d_1^{j+1,n} = \sum_k [h_{1-2k} d_k^{j,2n} + g_{1-2k} d_k^{j,2n+1}] \tag{8}$$

Determination of the best wavelet packet basis (optimal tree calculation): A signal with the length  $L = 2^N$  could have  $2^N$  different decomposition, the number of full two fork tree with depth of  $L$  is  $2^N$ , which is so large that it is impossible to enumerate for each case and an optimal signal decomposition method can be obtained by the minimum entropy standard. The traditional entropy standards are shannon, threshold, norm, log energy, sure and so on. Shannon entropy is defined as follows:

$$El(s_i) = -s_i^2 \log(s_i^2) \quad 0 \log 0 = 0 \tag{9}$$

The signals were layer by layer, each decomposed node was calculated entropy. The entropy of a node and its child nodes was compared and the base obtained with minimum entropy is the best wavelet packet basis.

**Noise reduction steps:** Wavelet packet decomposition technique derived from the wavelet transform. The wavelet transform is a local time-frequency analysis which time and frequency are variable. The wavelet transform is with a higher frequency resolution and lower temporal resolution at low frequencies and with low frequency resolution and high temporal resolution at high frequency (Li *et al.*, 2003; Hu *et al.*, 2008).

Compared with the wavelet analysis, wavelet packet decomposition technique is a more sophisticated analysis and reconstruction. In the wavelet analysis, the signal is broken down into the low-frequency rough part and high frequency details section. Then only the low-frequency parts are for the second layer of decomposition, the high frequency part were not dealt with and process in this way continuously. Due to the frequency resolution became low with the increase of the scale in wavelet analysis, so the resolution was poor at high frequencies.

Wavelet packet decomposition technique were not only for the decomposition of low-frequency part of the signal and also for the decomposition of the high-frequency part. Therefore, the wavelet packet

decomposition technique could do a more detailed description of the high frequency part of the signal, with a stronger signal analysis capabilities.

Wavelet denoising made using of noise and signal having different properties at each scale in the wavelet spectral (conversion modulus maxima) to remove the noise spectrum and then apply the wavelet transform reconstruction rules to reconstruct the original signal. The idea of the wavelet packet noise reduction were the same as the basic wavelet, the only difference is that the wavelet packet analysis provide a more complex and more flexible analytical tools, because that wavelet packet analysis did subdivision on low-frequency part and high-frequency part of last layer the same time, with more accurate local analysis capabilities. The wavelet packet noise reduction steps as follows (Hu *et al.*, 2008; Wu and Liu, 2008; Zhang and Ulrych, 2003).

**First was wavelet packet decomposition of the signal:**

Choose a wavelet and determine a wavelet decomposition level N, then do N-layer wavelet packet decomposition on the seismic waves signal S. Described with a three-tier decomposition, the wavelet packet decomposition tree were shown in Fig. 1. In Fig. 1a represented a low frequency, D represented high frequency, the end serial number represented wavelet packet decomposition layers, namely the scale number. The original signal S is equivalent to:

$$S = AAA_3 + DAA_3 + ADA_3 + DDA_3 + AAD_3 + DAD_3 + ADD_3 + DDD_3 \quad (10)$$

**Second was to calculate the optimal tree:** Compute the best tree for a given entropy criteria, commonly used entropy criteria were Shannon, threshold, norm, log energy, sure and user and so on.

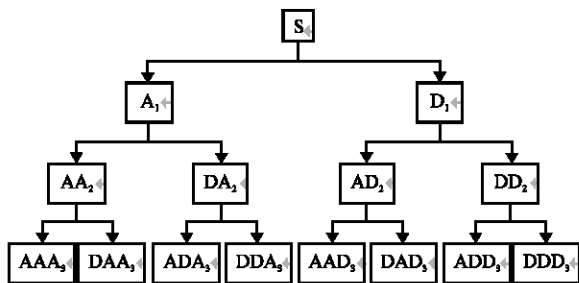


Fig.1: Wavelet packet decomposition technique

**Third was thresholding quantization of wavelet packet decomposition coefficients:**

Select a threshold and do coefficients thresholding quantization for each wavelet packet decomposition coefficients. Wavelet transform coefficient values are compared with a threshold, it is believed that the values smaller than the threshold value were generated by the noise and set to zero, the values greater than the threshold values were corresponding to the signal mutation point and retained in order to achieve the purpose of denoising. In the process of de-noising there were usually three treatments (Wei *et al.*, 2008; Wen *et al.*, 2002; Hu *et al.*, 2008): Force denoising, the default threshold denoising, given soft (or hard) threshold denoising.

Soft threshold was used and threshold size was  $t = \sigma\sqrt{2\log(N)}$ , N is the signal length,  $\sigma$  is the noise standard deviation:

**Fourth was signal reconstruction:** Do wavelet packet reconstruction on signals according to the L layer wavelet packet decomposition low-frequency coefficients of the original signal and high frequency coefficients after threshold quantization processing.

**SIMULATION**

**Simulation signal denoising:** In order to verify the effectiveness of the wavelet packet denoising, MATLAB noismima signal and noisbump signals were used to illustrate this (Hu, 2003; Li *et al.*, 2006).

Figure 2 are the comparison chart before and after noismima signal de-noising and the db2 wavelet was selected to three-layer decomposition and adjust the threshold. Figure 3 showed the comparison chart before

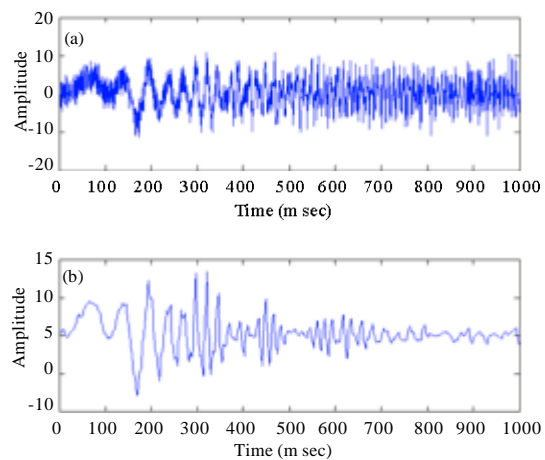


Fig. 2(a-b): Noismima signal denoising (a) Noismima original signal and (b) Noismima de-nosing signal

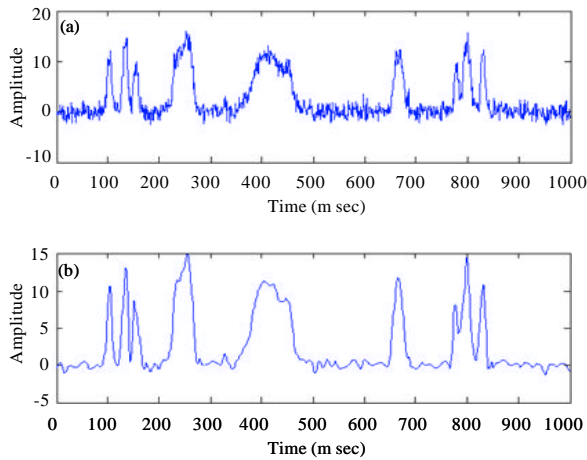


Fig. 3(a-b): Noisbump signal denoising synthetic seismogram denoising (a) Noismima original signal and (b) Noismima de-nosing signal

and after the noisbump signal de-noising and the db4 wavelet was selected to three-layer decomposition noisbump signal. It can be seen from Fig. 2 and 3, wavelet packet decomposition could effectively filter out the high frequency noise components in the the original signal, restore the essential characteristics and of a good noise reduction performance (Liu *et al.*, 2012).

Ricker wavelet model in time-domain is as follows:

$$r(t) = [1 - 2(\pi f_p t)^2] e^{-(\pi f_p t)^2} \quad (11)$$

Seismic trace convolution model in seismic exploration is:

$$x(t) = s(t) * r(t) + n(t) \quad (12)$$

In Eq. 11  $s(t)$  is seismic wavelet,  $r(t)$  are random reflection sequences,  $x(t)$  are synthetic seismic records,  $n(t)$  are the noise signals. Seismic data simulation was done by using MATLAB, common shot gather profiles are synthesized with a low-frequency ricker wavelet and a high-frequency ticker wavelet superimposed, specific parameters: A depth of one hundred meters, upper reflective layer velocity of two thousand meters every second, lower speed of three thousand meters every second, the minimum offset of ten meters, the number of channels of fifty, a record length of six hundred millisecond. Figure 4a is the synthetic common shot gathers, Fig. 4b is common shot gathers with 5 db random noise, Fig. 4c is the common shot gathers after the wavelet packet denoising.

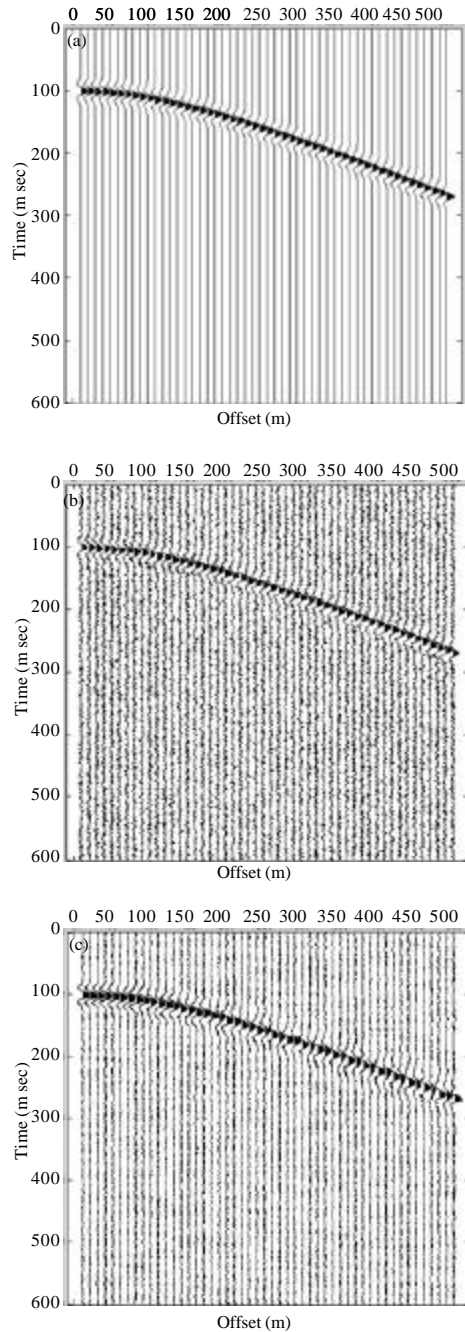


Fig. 4(a-c): (a) Synthetic common shot gathers (b) Common shot gathers with noise and (c) Common shot gathers after the wavelet packet denoising

### TEST AND ANALYSIS

The interference noise is inevitable in real seismic data acquisition and the collected data must be denoised.



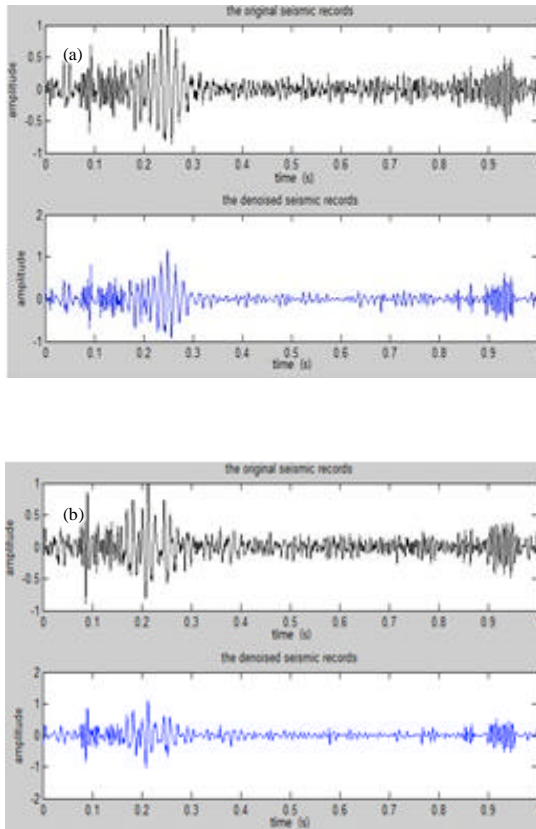


Fig. 5(a-b): (a) Data processing of channel one (b) Data processing of channel two

In the test Germany SUMMIT distributed seismograph were used for observation seismic data of a monitoring point, channels were twenty-six, channel spacings were one kilometers, offset distances (two shot distances) were 10 km, gun dot distances were 2 km, the detector arrays and source arrays were with one line, recording time was 6 sec. Actual seismic data are generally stored in the format of segy, it need to convert and to remove the channel header and volume header in real data processing. Fig. 5a, b were two channels data extracted from the real seismic data and the denoising data after the noise elimination part of the data. It can be seen from Figure 5a and b, the wavelet packet analysis technology could better remove a lot of noise mixed in seismograph sampled signals and restore the original signal.

Figure 6 is a noisy seismic record and the signal-to-noise ratio deteriorates due to the presence of noise in the seismic profile. Figure 7 is denoising seismic record with a wavelet packet threshold method denoising. It can be seen that parts noises of the fifteenth to twenty-sixth

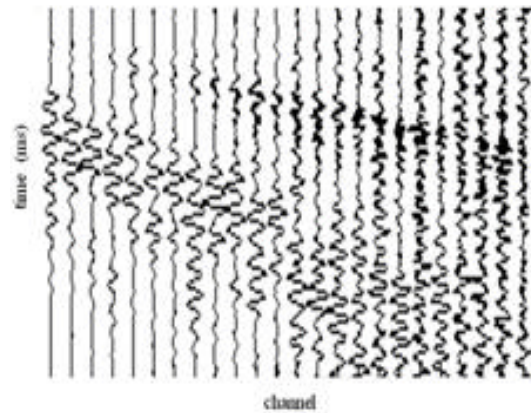


Fig. 6: Noisy seismic records

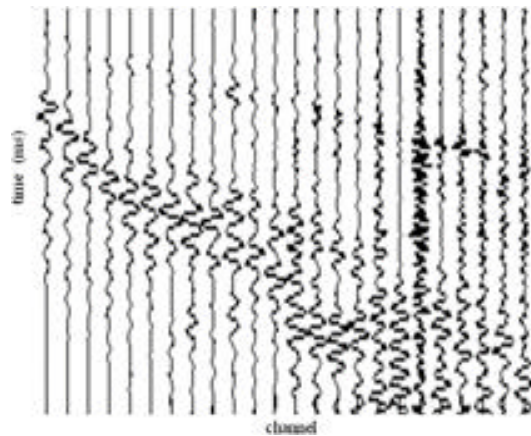


Fig. 7: Denoising seismic records

have been eliminated through noise elimination processing, the waveform significantly improved, the denoising effect is better, seismic profile phase axis is clearer and signal to noise ratio increased by about 18.6 dB than the original sampling signals.

**CONCLUSION**

Signals with noises would affect the data analysis effect and need effectively be filtered out. Wavelet packet decomposition technique decompose to the low-frequency part and high-frequency part of the signal, could effectively remove the noise in the sampled signals and improve the signal to noise ratio of seismic data. It not only suppress the noises but also retain useful signal and the visual effects are good which prove the effectiveness of the method.

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#### **REFERENCES**

- Hu, C., G. Li and T. Zhou, 2008. MATLAB 7.x Systems Analysis and Design-Wavelet Analysis. 3rd Edn., Xidian University Press, China.
- Hu, G., 2003. Digital Signal Processing: Theory, Algorithm and Implementation. 3rd Edn., Tsinghua University Press, China.
- Kong, X., J. Wu and J. Yue, 2005. Research on seismic signal denoising using wavelet transform. *Comput. Aided Eng.*, 14: 52-56.
- Li, J., Z. Wang, Y. Chen and X. Tang, 2003. A research on the noise-elimination method of pre-stack seismic data by wavelet packet transform. *J. ChengDu Univ. Technol. (Sci. Technol. Edn.)*, 30: 541-546.
- Li, Y., S. Zhang and K. Xu, 2006. Application of wavelet transfer in seismic signal denoise. *Northwestern Seismol. J.*, 28: 159-162.
- Liu, J., S. Han and J. Ma, 2006. Application of wavelet analysis in seismic data denoising. *Prog. Geophys.*, 21: 541-545.
- Liu, S., E. Gao and C. Guo, 2012. Seismic signal wavelet-packet denoising and fast spectrum analysis. *Applied Mech. Mater.*, 229-231: 1772-1776.
- Trad, D.O. and J.M. Travassos, 2000. Wavelet filtering of magnetotelluric data. *Geophysics*, 65: 482-491.
- Wei, H., S. Huang and F. Xu, 2008. Application of the wavelet packet denoising in pretreatment of seism signal. *Seismol. Res. Northeast China*, 24: 45-49.
- Welford, J.K. and R.F. Zhang, 2004. Ground-roll suppression from deep crustal seismic reflection data using a wavelet-based approach: A case study from Western Canada. *Geophysics*, 69: 877-884.
- Wen, L., Z. Liu and Y. Ge, 2002. Several methods of wavelet denoising. *J. Hefei Univ. Technol.*, 25: 167-172.
- Wu, Z. and T. Liu, 2008. Wavelet transform methods in seismic data noise attenuation. *Prog. Geophys.*, 23: 493-499.
- Zhang, R.F. and T.J. Ulrych, 2003. Physical wavelet frame denoising. *Geophysics*, 68: 225-231.