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Assessment of Different Matrix-fracture Shape Factor in Double Porosity Medium

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Abstract: Shape factor is required in modeling naturally fractured reservoirs represented by double porosity theory. The function of shape factor is to define the leakage term between the matrix block and fracture conduits. Different geometric assumption of the double porosity medium resulted in different shape factors and consequently the leakage term. The influence of various shape factors based on different geometric assumptions on their matrix-fracture transfer rate has been studied in this study. Comparison of different shape factor performance is accomplished via the dimensionless pressure and time. This study proposed a new relation that measures the impact of different shape factor in the consequence flow rate in fractured reservoir. It was discovered that higher value of shape factor contributes to higher rate of change of flow rate. This correlation can aid in deciding the appropriate shape factor for modeling double porosity medium.

Key words: Matrix-fracture, shape factor, double porosity, naturally fractured reservoirs, transfer rate

INTRODUCTION

According to Sarma and Aziz (2006), Naturally Fractured Reservoir (NFR) is a complex system with irregular fractures network, vugs and matrix blocks. They added that NFR can be defined as a reservoir having a connected fractures network which has significant higher permeability than matrix. This implies that production of hydrocarbons is highly dependent on the matrix-fractures interaction. This paper assessed the influence of different shape factor in modeling the matrix-fracture transfer rate.

NFR can be modeled by using double porosity concept. The double porosity concept was introduced by Barenblatt *et al*. (1960), while Warren and Root (1963) were the first to use double porosity concept in reservoir simulation. Double porosity concept is having two separate partial differential equations to define matrix and fractures flow. Matrix usually have low permeability and high storativity, while fractures have high permeability but low storativity. This suggest that matrix function as a main source of hydrocarbons while, fractures become the flow path of hydrocarbon production. For this reason, interaction between matrix and fractures should be considered. This interaction can be described by using a transfer function. The matrix-fracture transfer function was given by Warren and Root (1963) as:

$$q = \sigma \frac{kV}{\mu}(p_m - p_f) \quad (1)$$

Equation 1 showed that the matrix-fracture transfer function which requires shape factor to governs the flow.

The initial double porosity model of Warren and Root (1963) used assumption of pseudosteady flow and the NFR system is simplified into blocks of matrix and fractures set which looks like sugar cubes (Fig. 1). Each cube is known as matrix that contained in within a

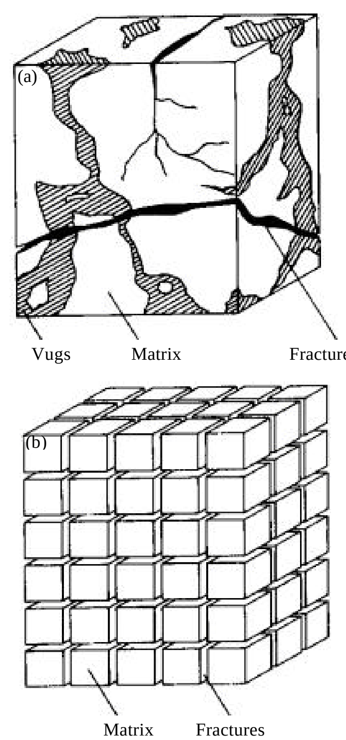

Fig. 1(a-b): Actual reservoir block and sugar cubes model

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systematic array of identical and rectangular parallelepipeds. Matrix is assumed to be homogenous and isotropic. All the fractures are continuous and may have different spacing and width to simulate certain degree of anisotropy.

SHAPE FACTOR

Most NFR modeling require shape factor in the matrix-fracture transfer function, although, some approaches do not require shape factor (Firoozabadi and Thomas, 1990). Shape factor is commonly used and it is a crucial parameter in matrix-fracture transfer function. Warren and Root (1963) have defined rectangular shape factor as:

$$\sigma = \frac{4n(n+2)}{L_e^2} \tag{2}$$

where, n refers to the number of fractures sets and:

$$n = 1; \quad L_e = L_x \tag{3}$$

$$n = 2; \quad L_e = 2L_xL_y/(L_x + L_y) \tag{4}$$

$$n = 3; \quad L_e = 3L_xL_yL_z/(L_xL_y + L_xL_z + L_yL_z) \tag{5}$$

In subsequent years, Kazemi *et al*. (1976) developed new shape factor for their simulator using finite difference method. Their shape factor for rectangular geometry is:

$$\sigma = 4\left(1/L_x^2 + 1/L_y^2 + 1/L_z^2\right) \tag{6}$$

Ueda *et al*. (1989) research concluded that Kazemi's shape factor needed to be multiplied by a factor of 2 or 3 in order to get more realistic pressure distributions. Their works were later supported by Lim and Aziz (1995), which showed that Kazemi's shape factor needed to be adjusted with factor of ~2.5.

Coats (1989) derived shape factor that are doubled of Kazemi's shape factor. Coat's shape factor for rectangular geometry is:

$$\sigma = 8\left(1/L_x^2 + 1/L_y^2 + 1/L_z^2\right) \tag{7}$$

The method used by Coats (1989) is Fourier finite sine transform and integration. Fourier transformation was also used by Chang (1993) and Lim and Aziz (1995) to arrive at another shape factor different from Coats (1989). Coat's work became the main references for both of them. They continued Coat's work but with different boundary conditions. By using pressure boundary conditions, they arrived at similar shape factor for rectangular geometry Eq. 8:

$$\sigma = \pi^2\left(1/L_x^2 + 1/L_y^2 + 1/L_z^2\right) \tag{8}$$

Lim and Aziz (1995) added that the total amount of mass entered a system at time t, M_t with corresponding mass after infinite time, M_8 can be expressed as in Eq. 9. In additions, the matrix-fracture transfer rate can be expressed as in Eq. 10:

$$\frac{M_{time}}{M_\infty} = \frac{\overline{\rho}_m - \rho_i}{\rho_f - \rho_i} = \frac{\overline{p}_m - p_i}{p_f - p_i} \tag{9}$$

$$Q = -\rho\phi c_t \frac{\partial \overline{p}_m}{\partial t} \tag{10}$$

Equation 11-13 are the analytical solutions given by Lim and Aziz (1995) for single phase flow in fracture. The solutions can be differentiated with respect to time and related with Eq. 10 to obtain respective shape factor:

$$n = 1; \quad \frac{\overline{p}_m - p_i}{p_f - p_i} = 1 - 0.81\exp\left[\frac{-\pi^2 kt}{\phi\mu c_t L^2}\right] \tag{11}$$

$$n = 2; \quad \frac{\overline{p}_m - p_i}{p_f - p_i} = 1 - 0.69\exp\left[\frac{-5.78kt}{\phi\mu c_t R^2}\right] \tag{12}$$

$$n = 3; \quad \frac{\overline{p}_m - p_i}{p_f - p_i} = 1 - \left(\frac{8}{\pi^2}\right)^3 \exp\left[\frac{-\pi^2 t}{\phi\mu c_t}\left(\frac{k_x}{L_x^2} + \frac{k_y}{L_y^2} + \frac{k_z}{L_z^2}\right)\right] \tag{13}$$

Meanwhile, Chang (1993) has derived another shape factor using constant flow rate boundary conditions which are:

$$\sigma = 12^2\left(1/L_x^2 + 1/L_y^2 + 1/L_z^2\right) \tag{14}$$

On the other hand, Quintard and Whitaker (1995) used assumption of infinite permeability in the fracture to set the boundary value problem for double porosity flow. By solving using Fourier series for rectangular geometry, they reached conclusion of shape factors which are:

$$\sigma = 12^2\left(1/L_x^2\right) \tag{15}$$

$$\sigma = 14.22^2\left(1/L_x^2 + 1/L_y^2 + 1/L_z^2\right) \tag{16}$$

$$\sigma = 16.54^2\left(1/L_x^2 + 1/L_y^2 + 1/L_z^2\right) \tag{17}$$

Table 1 summarized all the shape factor for rectangular geometries.

Table 1: Shape factors for rectangular geometry

Sets of fractures n	Warren and Root (1963)	Kazemi *et al.* (1976)	Coats (1989)	Chang (1993)	Chang (1993), Lim and Aziz (1995)	Quintard and Whitaker (1995)
1	$12/L_e^2$	4 L	8 L	12 L	π^2 L*	12.00 L
2	$32/L_e^2$	4 L	8 L	12 L	π^2 L*	14.22 L
3	$60/L_e^2$	4 L	8 L	12 L	π^2 L*	16.54 L

Table 2: General data used in comparison study

Parameter	Value
Blocks dimension ($f_i \times f_t \times f_t$)	100×100×100
Matrix porosity (fraction)	0.29
Matrix permeability (mD)	1
Fracture porosity (mD)	0.01
Fracture permeability (mD)	90
Initial reservoir pressure (psi)	6000
Bottom hole pressure (psi)	5500
Productivity index (rb-cp/day-psi)	1

COMPARISON OF SHAPE FACTORS

The reservoir problem selected for this comparison study was a multiphase depletion run with 5×3×2 blocks and only one production well at (1,1,1). The 5×3×2 blocks were selected for this purpose to allow 3-ways flow between the blocks during simulation. There is no injection well and only one production well. This is to avoid complication of the problems which later would complicate the results analysis. The production runs with no flow constraints. Table 2 describes the details of the reservoir problem. The additional reservoir properties used for this comparison are from the Sixth SPE Comparative Solution Project (Firoozabadi and Thomas, 1990).

COMPARISON APPROACH

Lim and Aziz (1995) has provided analytical derivation of shape factors for single phase flow.

In the effort of deriving shape factors, they have shown that the total amount of mass entered a system at time t, M_t and the corresponding mass after an infinite time, M_8 can be expressed as in Eq. 9. The Equation 9 is known as a dimensionless pressure, P_D. This dimensionless pressure is a function of dimensionless time as shown in Eq. 18.

The equivalent fractures length, L_e is given in the Eq. 3-4:

$$n = 1,\ L^* = 1/L_x^2$$
$$n = 2,\ L^* = 1/L_x^2 + 1/L_y^2$$
$$n = 3,\ L^* = 1/L_x^2 + 1/L_y^2 + 1/L_z^2$$

and the analytical expressions for single phase flow is shown in the Eq. 11-13:

$$t_d = \frac{kt}{\phi \mu c_t L^2} \quad (18)$$

It is desired to know the effect of different shape factors in multiphase flow NFR simulation. The comparison is done by representing the simulation results in P_D and t_D. A basic double porosity simulator is used to solve the reservoir problem. The simulator solves the pressure and saturation by using Implicit Pressures Explicit Saturation (IMPES) method. When P_D is plotted against t_D, the gradient (P_D/t_D) gives indication of the matrix-fracture transfer rate. From Eq. 19, it is shown that the matrix-fracture transfer rate is proportional to the gradient (P_D/t_D). Eq. 19 can be found by extending the analytical solution given by Lim and Aziz (1995). The derivation new correlation is shown in the appendix:

$$Q = \frac{\partial p_d}{\partial t_d}\left(\frac{\rho k}{\mu L^2}(p_i - p_f)\right)$$

This relation is for analyzing the results. In additions, the results are compared against the Lim and Aziz (1995) analytical solutions.

Single set of fractures: Shape factor for single set of fractures is used when fractures are assumed to be present only in one direction (Fig. 2). The flow between matrix and fracture is assumed to be in a single direction. The direction of fractures is not necessary has to be in x-axis, but it can be at any axis such as straight y-axis or slanted x-y axis. This also applies to double and triple set of fractures.

The comparison result for single set of fractures is presented in Fig. 5. By applying Eq. 19, it is deduced that steeper gradient shows higher matrix-fracture transfer rate. At t_D<0.5, Kazemi's shape factor has showed much lower transfer rate as compared with other researcher's shape factor. It also require twice the t_D value to reach the same P_D value. Note that all the curve gradient is very high at early t_D and becoming lower as t_D increases. This indicate that flow is high at early time and decreasing with time. This curve behaviour is associated with rate of change of flow Eq. 19:

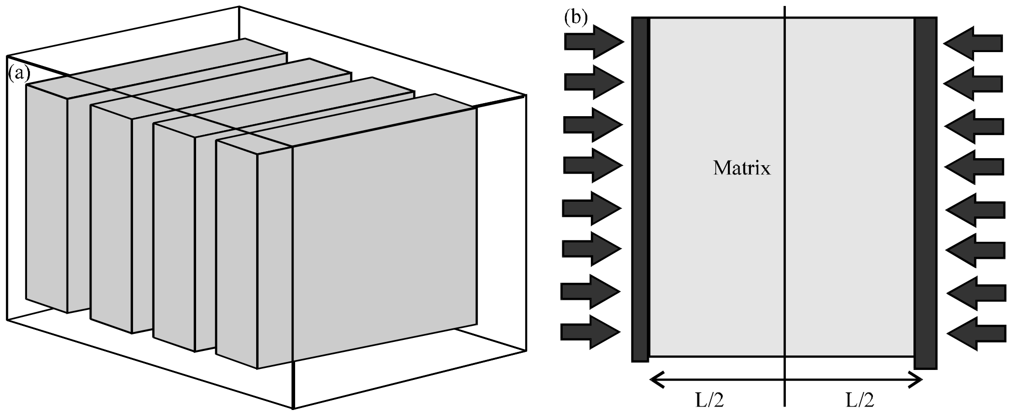

Fig. 2(a-b): (a) Matrix block with single set fractures and (b) Flow boundary between matrix and fractures

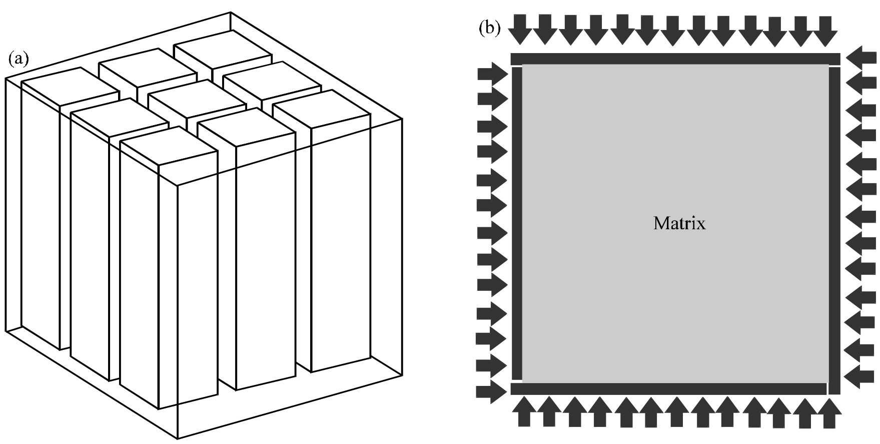

Fig. 3(a-b): (a) Matrix block with double set fractures and (b) Flow boundary between matrix and fractures

$$\Delta \text{ gradient} = \frac{\partial^2 P_d}{\partial t_d^2} \quad (19)$$

Conversely, Warren and Root (1963) shape factor yield the highest Δgradient. The Warren and Root (1963) shape factor flow is the highest at early t_D and then, it dropped quickly. In general, all the results converges into P_D and the curve gradient becoming linear. This indicate that the flow is becoming steady. When the gradient is 0, it literally means that no flow between matrix-fracture and it occurs at $P_D = 1$. At $P_D = 1$, the p_m must be equal to p_f since the initial pressure, p_i always constant. This supported by Eq. 1 whereby there must be a pressure difference to initiate the flow.

As stated earlier, analytical solution from Lim and Aziz (1995) is based on single phase flow and direct comparison cannot be made. However, it can be observed that the analytical solutions has much higher Δg as compared with others. This denotes that the analytical solutions flow is very high at initial t_d and then decreases rapidly. The reservoir problem is a multiphase flow problem whereby the transfer of fluids are much more complex. The components that present in a multiphase flow is water, oil and gas. Total multiphase matrix-fracture transfer rate is a summation of all the components, while single phase matrix-fracture transfer rate is having only the oil components. Analytical solution cannot be compared directly with results of other shape factors but it can serves as a reference line. This can be used to detect shape factor results that yield faster transfer rate by comparing the gradient and P_D.

Two and three set of fractures: Shape factor for two sets of fractures is used when fractures assumed a matchstick model (Fig. 3) and three sets of fractures is used when fractures assumed sugarcube model (Fig. 4).

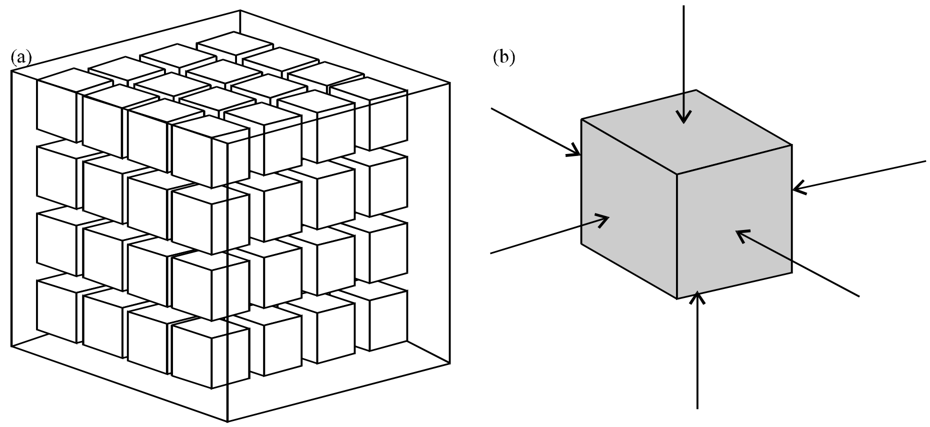

Fig. 4(a-b): (a) Matrix block with triple set fractures (b) Flow boundary between matrix and fractures

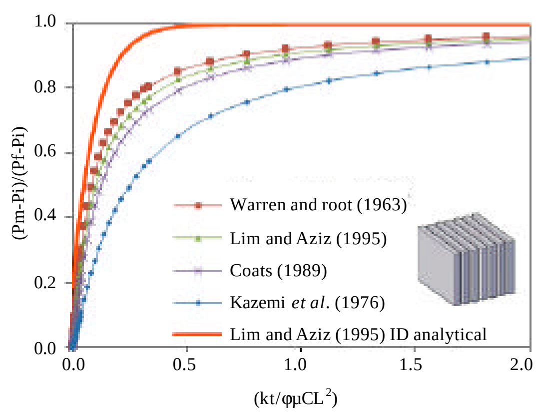

Fig. 5: Single set of fractures

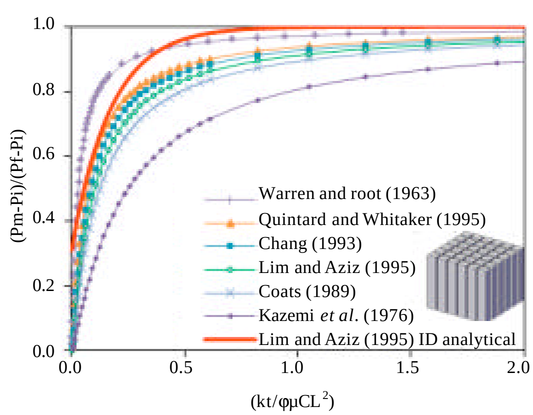

Fig. 6: Two sets of fractures

The comparison for two and three set of fractures are presented in Fig. 6 and 7, respectively. Both the results can be analyzed using the same approach

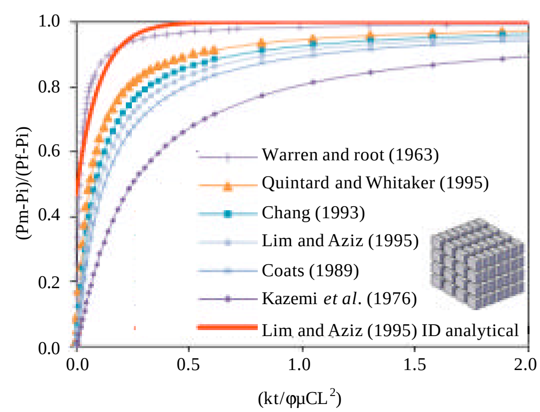

Fig. 7: Three sets of fractures

discussed for the parallel plate model. It is interesting to note that at early t_D, Warren and Root (1963) shape factor yields the highest flow as compared with the analytical solution.

In general, Warren and Root (1963) has the highest rate of change of flow while Kazemi *et al.* (1976) shape factor produces the lowest rate of change of flow. These two models can be taken as the two extreme bounds where (Quintard and Whitaker, 1995; Chang, 1993; Lim and Aziz, 1995 and Coats, 1989) shape factors produce intermediate rate of change of flow.

It is apparent that higher shape factor will results in high rate of change of flow. The flow behavior is such that there is an initial high flow between matrix-fracture and then followed by quick drop of flow rate. Figure 8 is an example of 15×1×1 line injection model investigated using different shape factors. It showed that the oil production using shape factor from Warren and

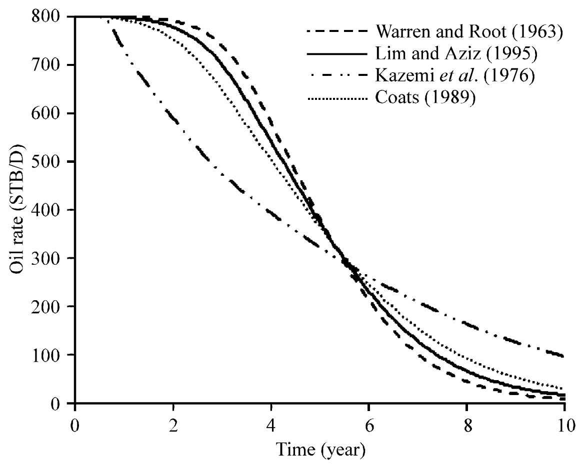

Fig. 8: Comparison of different shape factors for a line injection problem taken from Almengor *et al.* (2002)

Root (1963) has the highest productivity before the fifth year, followed by a quick drop in the productivity index. Kazemi *et al.* (1976) shape factor resulted in the lowest productivity among the models in the initial time but the production picked up after the fifth-year. Not surprisingly, oil production using shape factors from Coats (1989) and Lim and Aziz (1995) are bounded in between the results by Warren and Root (1963) and Kazemi *et al.* (1976).

CONCLUSION

Comparison of shape factors is presented in dimensionless parameters, P_D and t_D. Gradient P_D/t_D are proportional to the matrix-fracture transfer rate. Higher gradient P_D/t_D indicate higher transfer rate. The dimensionless comparison displays the different flow behavior of different shape factors.

NOMENCLATURE

t = time, T
L = length, L
V = volume, L^3
q = matrix-fracture transfer rate, L^3/T
Q = matrix-fracture transfer rate, $M/(L^3T)$
p = pressure, $M/(LT^2)$
k = absolute permeability, L^2
σ = shape factor, $1/L^2$
μ = viscosity, M/(LT)
n = set of fractures
ϕ = porosity, fraction
c = compressibility, fraction
ρ = density, M/L^3

SUBSCRIPTS

i = Initial
t = Total
m = Matrix
∞ = Infinity
f = Fracture
d = Dimensionless

APPENDIX

Derivation of dimensionless correlation: Equation 13 can be differentiated with time to obtain Eq. A1:

$$\frac{\partial}{\partial t}\left(\frac{\bar{p}_m - p_i}{p_f - p_i}\right) = \frac{\pi^2}{\phi\mu c_t}\left(\frac{k_x}{L_x^2} + \frac{k_y}{L_y^2} + \frac{k_z}{L_z^2}\right)\left(\frac{p_f - \bar{p}_m}{p_f - p_i}\right) \quad \text{(A1)}$$

By using finite approximations for first degree derivative, Eq. A1 can be rewritten as Eq. A2:

$$\frac{(P_d)_2 - (P_d)_1}{\Delta t} = \frac{\pi^2}{\phi\mu c_t}\left(\frac{k}{L^2}\right)\left(\frac{p_f - \bar{p}_m}{p_f - p_i}\right) \quad \text{(A2)}$$

For Sufficient small points and if p_f = Constant, then:

$$\frac{(\bar{p}_m)_2 - (\bar{p}_m)_1}{\Delta t} = \frac{\pi^2}{\phi\mu c_t}\left(\frac{k}{L^2}\right)(p_f - \bar{p}_m) \quad \text{(A3)}$$

Substitute Eq. A3 into the given Eq. 10, we get:

$$Q = \frac{-\rho\pi^2}{\mu}\left(\frac{k}{L^2}\right)(p_f - \bar{p}_m) \quad \text{(A4)}$$

If:

$$\left(\frac{k}{\phi\mu c_t L^2}\right)$$

is constant/equivalent, while t varies, then Eq. A2 can be rewrite into Eq. A5:

$$(p_f - \bar{p}_m) = \frac{(P_d)_2 - (P_d)_1}{(t_d)_2 - (t_d)_1} \times \frac{1}{\pi^2} \times (p_f - p_i) \quad \text{(A5)}$$

Substitute Eq. A6 into Eq. A5, we get Eq. A6:

$$Q = \frac{\partial p_d}{\partial t_d}\left(\frac{\rho k}{\mu L^2}(p_i - p_f)\right) \quad \text{(A6)}$$

REFERENCES

Almengor, J.G., G. Penuela, M.L. Wiggins, R.L. Brown, F. Civan and R.G. Hughes, 2002. User's guide and documentation manual for boast-nfr for excel. The University of Oklahoma, Office of Research Administration, Norman, Oklahoma, USA., DE-AC26-99BC15212.

Barenblatt, G.I., I.P. Zeltov and I.N. Kochina, 1960. Basic concepts in the theory of seepage of homogeneous liquids in fissured rocks. J. Applied Math. Mech., 24: 1286-1303.

Chang, M.M., 1993. Deriving the shape factor of a fractured rock matrix. Technical Report NIPER-696, (DE93000170), National Institute for Petroleum and Energy Research, Bartlesville, Oklahoma, USA., pp: 40.

Coats, K.H., 1989. Implicit compositional simulation of single-porosity and dual-porosity reservoirs. Proceedings of the SPE Symposium on Reservoir Simulation, February 6-8, 1989, Houston, Texas, USA.

Firoozabadi, A. and L.K. Thomas, 1990. Sixth SPE comparative solution project: Dual-porosity simulators. J. Petroleum Technol., 42: 762-763.

Kazemi, H., L.S Merrill, K.L. Porterfield and P.R. Zeman, 1976. Numerical simulation of water-oil flow in naturally fractured reservoirs. SPE J., 16: 317-326.

Lim, K.T. and K. Aziz, 1995. Matrix-fracture transfer shape factors for dual-porosity simulators. J. Petroleum Sci. Eng., 13: 169-178.

Quintard, M. and S. Whitaker, 1995. Transport in chemically and mechanically heterogeneous porous media. II: Comparison with numerical experiments for slightly compressible single-phase flow. Adv. Water Resour., 19: 49-60.

Sarma, P. and K. Aziz, 2006. New Transfer functions for simulation of naturally fractured reservoirs with dual-porosity models. SPE J., 11: 328-340.

Ueda, Y., S. Murata, Y Watanabe and K. Funatsu, 1989. Investigation of the shape factor used in the dual-porosity reservoir simulator. Proceedings of the Society of Petroleum Engineers Asia-Pacific Conference, September 13-15, 1989, Sydney, Australia.

Warren, J.E. and P.J. Root, 1963. The behaviour of naturally fractured reservoirs. SPE J., 3: 245-255.