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Multi-facet Community Detection from Bipartite Networks

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Abstract: Detecting communities from networks is one of the important and challenging research topics in social network analysis, especially from bipartite network. In unipartite network, communities are usually represented as sets of nodes within which connections are dense but between which connections are sparse. However, communities in unipartite networks are not suitable to bipartite network, because there is only one-to-one correspondence between communities of different types. In this study we propose an algorithm for detecting communities from bipartite network based on ant colony optimization. Present algorithm allows many-to-many correspondence between communities in different parts. Experimental results demonstrate that tour algorithm can extract multi-facet communities from bipartite networks and obtain high quality of community partitioning.

Key words: Detecting communities, unipartite network, bipartite network, ant colony optimization

INTRODUCTION

Various kinds of real-world complex systems could be modeled as complex networks (Barabasi and Oltvai, 2004), where nodes (or vertices) represent the objects and edges represent the interactions among these objects. An important feature of most of the real world networks typically is community structure (Barber, 2007; Broder *et al*., 2000; Davis *et al*., 1941). The nodes of networks can often be divided into distinct groups, such that nodes within the same groups are similar to each other in some sense, whereas nodes from different groups are dissimilar.

Communities in unipartite networks are relatively independent in the structure and it is believed that each of them may correspond to some fundamental functional unit or may be similar in some sense. For example, a community in genetic networks (Dorigo *et al*., 1996) often contains genes with similar functions and a community on the World Wide Web Girvan and Newman (2002) may correspond to web pages related to similar topics. So what is the definition of community in bipartite networks? If we accept the definition in unipartite networks that communities consist of densely linked nodes, a community in an authors-papers cooperation network should contain both authors and papers, because there is no edge between authors or study. That is to say, there is only one-to-one correspondence between author-communities and study-communities, as shown in Fig. 1a.

Why should a community contain the different types of objects? A community in unipartite networks is just the same type of objects which are similar to each other in some sense but in real-world bipartite networks are often more complex than that. The communities in each part may gave many-to-many correspondence as shown in Fig. 1b. For instance, in a researcher-topic network, a group of researchers in mathematics may have interest in topic groups of optimization, combination theory and statistics and another group of researchers in computer science

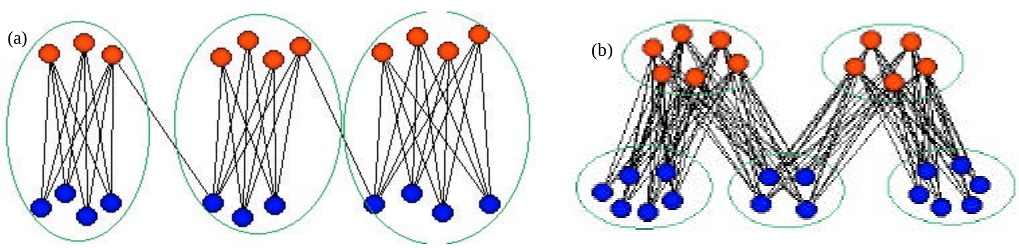

Fig. 1(a-b): Communities in bipartite networks, (a) One-to-one correspondence between different types of communities and (b) Many-to-many correspondence between different types of communities

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also have interest in those topic groups. Two different communities in researcher part correspond to the same set of communities in the topic part. Similarly, two different communities in the topic part may correspond to the same set of communities in researcher part. Therefore, if we adopt a clustering method that ignores the diverse nature of the human character, it would be difficult to find the common ground among people.

n this study, we propose an algorithm for detecting community of many-to-many correspondence from bipartite networks based on ant colony optimization. The algorithm transforms the problem of detecting communities into a combinatorial optimization problem and allows many-to-many correspondence between communities. Experimental results on some real world social bipartite networks demonstrate that our algorithm can not only accurately identify the number of communities of a bipartite network but also obtain higher quality of community detection.

RELATED WORKS

In unipartite networks, communities are often modeled as sets of nodes within which connections are dense but between which connections are sparse. To evaluate the quality of a particular division of a network into communities, Newman introduced a qualitative measure called modularity (Guimera *et al*., 2007). A widely used and quite successful method for the identification of communities in unipartite networks is maximization of a modularity function.

one type at a time. As we know, two node types are not treated symmetrically. Barber (2007) extends the definition of Newman's modularity in unipartite network to be appropriate for bipartite networks and presents a bipartite modularity based on the assumption that there is one-to-one correspondence between communities of different node type. He also proposed an algorithm called adaptive BRIM for detecting community structure by maximizing this bipartite modularity.

Liu and Murata (2009b) first introduced the label propagation algorithm for community detection in unipartite networks. The algorithm first assigns every node a unique label, then at every step in the iterative process, each node adopts the label that most of its neighbors currently have. Finally, densely connected group of nodes with identical label forms a community. Long *et al*. (2007) improved the Label Propagation Algorithm (LPA) and propose a new algorithm to make it more suitable for bipartite networks. Their algorithm is ready to be parallelized for real time community analysis on large-scale bipartite networks. Newman (2006) also proposed an algorithm called LP and BRIM for community detection in large-scale bipartite networks. The algorithm is a joint strategy of LP and BRIM, which employs LP to search an initial division and then use BRIM to find the final community division.

Porter *et al*. (2009) extended the k-clique community detection algorithm for bipartite networks. The algorithm retains all of the advantages of k-clique algorithm but it avoids discarding important structural information when performing a one-mode projection of the network. The algorithm provides a level of flexibility by incorporating independent clique thresholds for each of the non-overlapping node sets in the bipartite network.

Radicchi *et al*. (2004) introduced the link-pattern based community. A link-pattern based community is a group of nodes which have the similar link patterns, i.e., the nodes within a community link to other nodes in similar ways. Unlike the traditional link-dense, the link-pattern based community allows many-to-many correspondence between different types of communities and it is better suited to bipartite networks. As shown in Fig. 1b, such communities in an author-paper bipartite network are the way of link-pattern.

In general, finding an exact solution to partitioning communities is believed to be an NP-hard problem. Ant colony optimization proposed by Raghavan *et al*. (2007) who were inspired by the ants' foraging behavior is a swarm intelligence based optimization method which has been successfully applied to solve the NP-hard problems. In this paper, we take advantage the strong optimization ability of ACO to solving the problem of community detection in bipartite network.

BIPARTITE MODULARITY

In recent years, several bipartite modularity measures which can be applied to identify communities in bipartite networks are proposed. Guimera's bipartite modularity (Lehmann *et al*., 2008) focuses on the connectivity of only one type of nodes and Barber's bipartite modularity Liu and Murata (2009a) is based on the assumption that there is one-to-one correspondence between communities of different vertex types. Murata's bipartite modularity (Long *et al*., 2007), which gives consistent result as Newman's modularity when applied to unipartite networks, is one-to-many correspondence between communities of different vertex types.

Scott and Hughes (1980) advanced a modified version of Murata's bipartite modularity, which can reflect the multi-facet correspondence among communities. Suzuki's bipartite modularity is defined as an accumulation of pair-wise modularity between different types of communities. The pair-wise modularity between two different types of communities is weighted by connection density between them in order to reflect the strength of connection between them. In this study, we adopt Suzuki's bipartite modularity to measure the quality of bipartite network community detection.

Community detection in a bipartite graph $G = (V, E)$ is to partition its vertexes. $V = V^X \cup V^Y$, into a number of subsets $V_1^X, V_2^X, ..., V_{C^X}^X$ and $V_1^Y, V_2^Y, ..., V_{C^Y}^Y$, such that $V_i^X \subset V^X$, $V_i^Y \subset V^Y$:

$$U_{i=1}^{C^X} V_i^X = V^X, U_{i=1}^{C^Y} V_i^Y = V^Y$$

A community is a subset V_i^X ar V_i^Y, where $V_i^X \subset V^X$ and $V_i^Y \subset V^Y$. We call V_i^X and V_i^Y by X-vertex community and Y-vertex community respectively. Let M be the number of edges in a bipartite network. Suppose the bipartite network is partitioned into X-vertex communities and Y-vertex communities and the numbers of the communities are C^X and C^Y, respectively. Suppose $V_i^x \subset V^x$ and $V_m^y \subset V^Y$ are two communities, since the vertices in V_l and V_m are of different types, we can define elm and al as follows:

$$e_{lm} = \frac{1}{M} \sum_{i \in V_l^X} \sum_{j \in V_m^Y} A(i, j) \quad (1)$$

It can easily be seen that e_{lm} is the fraction of all edges in the network that connect vertices in community V_l^X to vertices in community V_m^Y. We further define a $C^X \times C^Y$ matrix E composed of e_{lm} as its (l, m) element and its row summations a_l and its column summations a_m:

$$a_l = \sum_{l \in C^X} e_{lm} = \frac{1}{M} \sum_{i \in V_l^X} \sum_{j \in V_m^Y} A(i, j) \quad (2)$$

$$a_m = \sum_{l \in C^X} e_{lm} = \frac{1}{M} \sum_{i \in V_m^Y} \sum_{j \in V^X} A(i, j) \quad (3)$$

Then Suzuki's bipartite modularity Q is defined as follows:

$$Q = \frac{1}{2} = \sum_{l \in C^X, m \in C^Y} (e_{lm} / a_l)(e_{lm} / a_l a_m) \quad (4)$$

Here, high Q value indicates better community partitioning in a bipartite network. Taking a closer look at the expression of Q in Eq. 5, you will find that the value of the bipartite modularity is not symmetric for the two sides of vertexes. In Eq. 5, we only define the bipartite modularity in the $V^X \rightarrow V^Y$ direction, which can be denoted as Q^{XY}. Similarly, we can define the bipartite modularity for the direction of $V^X \rightarrow V^Y$.

FRAMEWORK OF OUR ALGORITHM

Model formulation: Since finding an exact solution to partitioning communities is believed to be an NP-hard problem. To reduce the time complexity, we transform the problem into the one of combination optimization which can be solved by ant colony optimization. First, we construct a model graph, on which the ants search for the optimal solution. Meanwhile we define the pheromone and heuristic information according to the topological structure of the bipartite network. In the algorithm, each ant chooses its path according to the pheromone and heuristic information on the edges of the model graph to construct a solution.

Suppose a bipartite network $G = (V, E)$ is composed of X-vertices and Y-vertices and the numbers of X and Y-vertices are n and m, respectively. We label X-vertices with integers 1 to n and Y-vertices with n+1 to n+m, namely V^X and $V^Y = V^Y\{V_{n+1}, V_{n+1}\}$. We establish the model graph for ants foraging as a directed graph. The whole model graph consists of two parts, which correspond respectively to the X-vertices Fig. 2a and the of Y-vertices Fig. 2b in the network.

In the directed model graph, there are n+m+1 nodes: U_1 which represent vertices of bipartite network except the last one indicting the end of ants' foraging. Between each pair of neighboring X-vertices, there are n directed edges and each pair of neighboring Y-vertices is linked by m directed edges. That is to say, an ant can only select the same type of vertices into a community. For the part of X-vertices, let the set of directed edges between nodes V_i and V_{i+1} be $E_i = \{E_{i,1}, E_{i,2}.., E_{i,n}\}$. Each ant chooses its path according to the pheromone and heuristic information on each edge. If an ant arrives at node U_i and chooses the edge. $E_{i,k}$, it means the nodes V_i and V_k in the bipartite

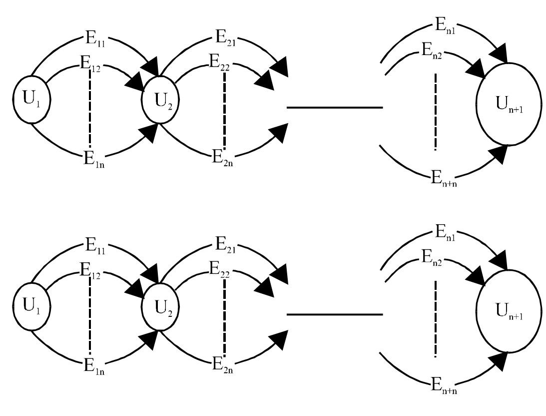

Fig. 2(a-b): Model graph for ants searching (a) n directed edges between neighboring X-vertices and (b) m directed edges neighboring Y-vertices

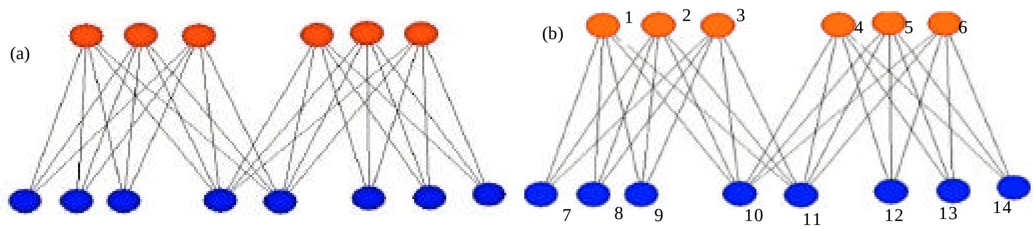

Fig. 3(a-b): An example of a division of a bipartite network, (a) Nodes of the same type are marked in the same color and (b) Labeled bipartite network

network are assigned into the same community. We use a vector S = (S_1, S_2,..., S_n, S_{n+1},..., S_{n+m}) to denote the solution the ant constructed. If the value of component S_i is k, node V_i and node V_k in the bipartite network are in the same temporary community. In the algorithm, each ant selects a component S_i for every node. V_i, so, as to construct the solution vector.

After constructing the vector S, we can merge the temporary communities to obtain the final result. If two temporary communities have common nodes, then we merge them into a larger temporary community. Repeating this process until there is no temporary communities can be merged.

For example, Fig. 3a shows a bipartite network and then we label these nodes as shown in Fig. 3b. Suppose an ant constructs the solution S = (3, 3, 1, 5, 6, 4, 8, 9, 8, 11, 10, 14, 14, 12) from the bipartite network in Fig. 3. Then, the temporary communities in this solution are as shown in Fig. 4, where each column indicates a temporary community. For instance, the first column in Fig. 4 is (1, 3), which means nodes V1 and V3 form a temporary community.

From Fig. 4, we can see that the eight temporary communities are (1, 3), (2, 3), (3, 1), (4, 5), (5, 6), (6, 4), (7, 8), (8, 9), (9, 8), (10, 11), (11, 10), (12, 14), (13, 14), (14, 12). By merging (1, 3) and (2, 3), we get another temporary sub-community (1, 2, 3). Repeating the process of merging until we obtain the final solution consisting of two red-communities: (1, 2, 3), (4, 5, 6) and three blue-communities: (7, 8, 9), (10, 11) and (12, 13, 14).

IMPLEMENTATION OF THE ALGORITHM

Pheromone: To construct an effective solution, pheromone information τ_{ij} is assigned on each path E_{ij}. The pheromone information influences the choices the ants in their searching and the larger amount of pheromone deposited on an edge, the higher probability an ant will select this edge. Communications and cooperation between individual ants by pheromone

1	2	3	4	5	6	7	8	9	10	11	12	13	14
3	3	1	5	6	4	8	9	8	11	10	14	14	12

Fig. 4: Temporary communities of a solution

informaτtion enable the ant colony algorithm to have strong capability of finding the best solutions. In our algorithm, when all ants get their solutions after each iteration, we update the pheromone on each edge using the following pheromone updating equation:

$$\tau_{ij}(t+1) = (1-\rho)\,\tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \quad (4)$$

Here, ρ is the evaporation rate, m is the number of ants and $\Delta\tau_{ij}^{k}$ is the increment of pheromone laid on edge (i, j) by the k-th ant:

$$\Delta\tau_{ij}^{k} = \begin{cases} Q_B(S) & \text{if } V_j \in S \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

where, $Q_B(S)$ is the bipartite modularity of division communities constructed by the k-th ant.

HEURISTIC INFORMATION

In the algorithm, we also define the heuristic information η_{ij} to reflect the potential tendency for the ants to select the edge E_{ij} in the directed model graph. In the bipartite network, if two nodes v_i and v_k which of the same type have larger number directly connected links with nodes of the other type, they should be in the same community with a higher probability and the heuristic function η_{ij} on edge E_{ij} will be assigned higher value. The value of heuristic function is determined as follows:

$$\eta_{ij} = \frac{2C_{ij}}{d_i + d_j} \quad (7)$$

Here, d_i is the degree of node v_i and C_{ij} is the number of nodes connected with both v_i and v_j.

Transition probability: When constructing the solutions, the ants traverse on the model graph and select a directed edge at each node by a probabilistic decision. The transition probability for the k-th ant at the node ui choosing the path E_{ij} is given by:

$$p_{ij}^{k} = \begin{cases} \dfrac{\tau_{ij}^{\alpha},\ \eta\tau_{ij}^{\beta}}{\sum_{g=1}^{n+m} \tau_{ij}^{\alpha},\ \eta\tau_{ij}^{\beta}} & \text{if } i \neq j \\ 0 & \end{cases} \quad (8)$$

Here, α and β are the parameters which control the relative importance of the pheromone and the heuristic information. If α assigned a larger value than. β, the pheromone will have greater influence on the ants' searching, otherwise the heuristic information will have greater influence.

Framework of the Algorithm: Suppose the number of X-vertices in the bipartite network is n and the number of Y-vertices is m, we label these vertices from 1 to n+m. The framework of our algorithm MFCD (Multi-facet Community Detction) is described in Fig. 5.

EXPERIMENT RESULTS

Southern women network: To verify the accuracy of our algorithm, we first test on the southern women network (Suzuki and Wakita, 2009), which was collected by Davis *et al*. (1941) around Mississippi during the 1930s as part of an extensive study of class and race in the Deep South. Because the community structure of this network is known, this dataset has been widely used by social network researchers as a benchmark. The network describes the participation of 18 women in 14 social events. If a woman attended an event, there will be an edge linking their nodes.

Firstly, we label the nodes of 18 women as 1-18 and the nodes of 14 events as 19-32 as shown in Fig. 6a. Then we use our MFCD algorithm to detect the communities on southern women network and the experimental result is shown in Fig. 6b. Nodes assigned in the same community are marked with a unique color. From Fig. 6b we can see that six woman-communities and five event-communities are obtained: {woman 1-7}, {woman 8}, {woman 9}, {woman 10-15}, {woman 16}, {woman 17-18}, {event 19-24}, {event 25}, {event 26}, {event 27, 29}, {event 28, 30-32}. The bipartite modularity of the result is 0.2367.

We also test the BRIM algorithm proposed by Baber and the result obtained is shown in Fig. 6c. However there is only one-to-one correspondence between woman-communities and event-communities. To indicate the correspondence relation between different

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Algorithm MFCD (G,A,Q)
Input: G: the bipartite network;
   A: the adjacency matrix of G;
Output: S_best: solution community division;
   Q_best: the modularity of the solution;
Begin
   Initialize the various parameters;
   Initialize values of pheromone and heuristic information;
 While not the terminate condition do
  For k = 1 to k do   /*k ants*/
  For i = 1 to n do /* n X-nodes*/
   Ant k selects S_i according to probability (8);
  Endfor i
  For i = n+1 to n+m do /* m Y-nodes*/
   Ant k selects S_i according to probability (8);
  Endfor i;
   Calculate the modularity Q_B of solution S;
   If Q_B>Q_best then
            Q_best = Q_B;   S_best = S;
          /*Q_best is the highest modularity obtained
 so far, S_best is the best solution obtained so far */
          endif
  Endfor k;
          Update the pheromone on the edges;
 End While;
End
```

Fig. 5: Framework of algorithm MFCD

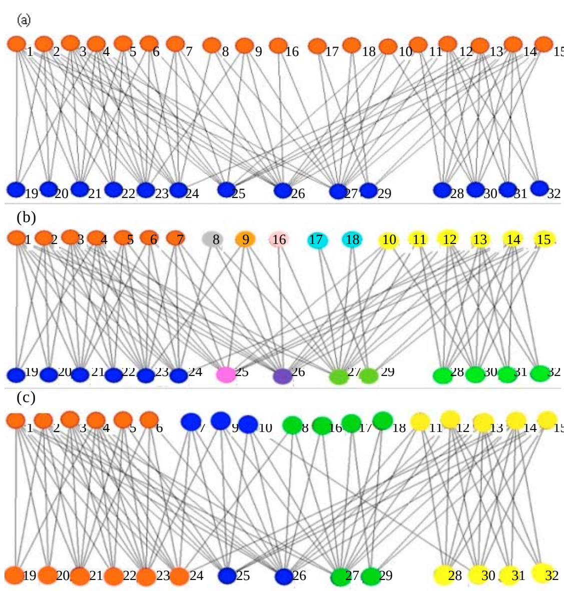

Fig. 6: Communities of the Southern women bipartite network, (a) Numbering bipartite network, 18 yellow circles are 18 women and 16 green circles are 16 events, (b) Final division obtained by our MFCD algorithm for optimizing Suzuki-Modularity and (c) Final division obtained by BRIM, an algorithm for optimizing Baber-Modularity

types of communities, we use the same color on the nodes assigned on the same community. Partitioning the women and events in a symmetric manner as presented by Baber seems to cause unreasonable grouping. For example, women 8 and 16 in the group {8, 16, 17, 18} participate in both of events 26 and 27 but not in 29 in their corresponding event set {27, 29}. It is obvious that women 8 and 16 have more tight relation with event set {26, 27} than {27, 29}.

Present study is identical to that of Scott and Hughes (1980), bipartite clustering algorithm which is a simplified variant of Blondel's algorithm. The algorithm performs hierarchical clustering and obtains high quality many to many communities partitioning. Moreover, our algorithm does not require a predefined number of communities and can obtain the higher quality of community partitioning without previously known parameters.

Divorce in US: As the second example, we test on a bipartite network of divorce status in the fifty states in America Watts and Strogatz (1998). The dataset consists of 50 states in United States and 9 statuses of divorce. The most notable characteristic of this bipartite network is that the numbers of nodes in different types are seriously imbalanced. Firstly, we label the nodes of 50 states as 1-50 and the nodes of 9 statuses of divorce as 51-59. Figure 7 shows the community structure depicted by PAJEK, which is software for analyzing and visualizing large networks. From Fig. 7, we can see that there are obviously three status-communities: {51}, {52, 53, 54, 55, 56, 57}, {58, 59} but the number of state-community is not unambiguous. However, we can confirm that one state-community existing is {3, 6, 15, 17, 22, 23, 25, 26, 27, 37}; because these nodes just connect with the node 51.

If we accept the definition in unipartite networks that communities consist of densely linked nodes, that is to say, there is only one-to-one correspondence between state-communities and status-communities. No matter what algorithm you use, you can get only one community from this bipartite network. Such result does not make any sense and cannot help to analyze the community structure of the network. For example, we adopt BRIM algorithm to partition this bipartite network. We obtain only one community (one status-community and one state-community). Because the numbers of communities of both node types have to be equal and this weaknesses is fatal for dividing real-world networks since the number of communities of both node types are often imbalanced.

We use our algorithm to detect the communities on this network and three state-communities and three status-communities are obtained. Careful observation of the final partitioning communities, one state-community is {3, 6, 15, 17, 22, 23, 25, 26, 27, 37} and three status-communities are {51}, {52, 53, 54, 55, 56, 57}, {58, 59}. It is shown that our algorithm can obtain high quality of community partitioning and is suitable for the networks where the numbers of nodes in different types are greatly imbalanced. Bipartite modularity of the result is 0.0726 which is very small and maybe unreasonable. The reason is that Sukuzi's bipartite modularity is not suitable for bipartite network where the number of nodes in different types are greatly imbalanced. This problem is left for our further study on evaluating community structure in bipartite networks.

Scotland corporate interlock: As the third example, we test on a network of corporate interlocks in Scotland in the early twentieth century. The data set characterizes 108 Scottish firms during 1904-1905, detailing the corporate sector, capital and board of directors for each firm. The dataset includes only those board members who held multiple directorships, totaling 136 individuals as shown in Fig. 9.

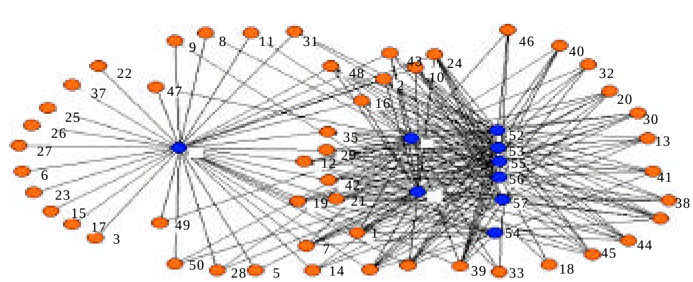

Fig. 7: Community structure of the network of divorce in us. The red nodes are represented as 50 states in us and the blue nodes are represented as 9 statuses of divorce

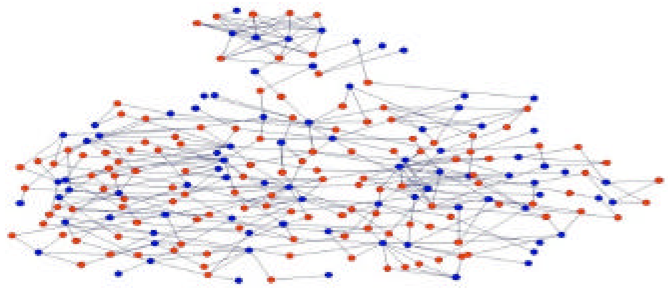

Fig. 8: Largest component of the network of scotland. The blue nodes represent 86 firms and the red nodes represent 131 directors

Here, we focus on the bipartite network of firms and directors, with edges existing between each firm and its board members. Unlike the Southern Women network, the Scotland corporate interlock network is not connected. We conduct the experiments on the largest component of the network containing 131 directors and 86 firms as shown in Fig. 8.

In the experiment, present algorithm MFCD divides this largest component of bipartite network into 16 firm-communities and 22 director-communities and gets a bipartite modularity 0.4043. The number of firm-community is not equal to the number of director-community, so the final partitioning communities, to some extent, reflects the many-to-many correspondence between firm-communities and director-communities.

The experiment of optimizing Baber's bipartite modularity is also performed. Baber's method is based on an assumption that there is one-to-one correspondence between communities of both vertex types. Baber declares that if restricting the number of communities being less than twenty, BRIM algorithm can get the maximum value of modularity. The number of communities obtained by our algorithm MFCD approximately agrees with Baber's conclusion, the community detecting results and their modularity are very close. But in contrast to BRIM algorithm, our algorithm can straightly obtain the number of communities and the specific division without any prior knowledge of the network.

CONCLUSION AND FURTHER WORK

An algorithm for detecting communities from bipartite networks based on ant colony optimization is presented. The algorithm firstly transforms the problem of community detection into the one of combination optimization and establishes a model graph for the ants' searching. Meanwhile we define heuristic information according the topological structure of the network. Each ant chooses its path according to the pheromone and heuristic information on each edge to construct a solution. The quality of solution obtained by each ant is measured by its bipartite modularity. Experiment results show that our algorithm can not only extract multi-facet community structure from bipartite networks but also accurately identify the number of communities and true community structure from bipartite network. Since Sukuzi's bipartite modularity is not suitable for bipartite network where the number of nodes in different types is great imbalanced, how to define a proper measurement for evaluating the multi-facet community part

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REFERENCES

Barabasi, A.L. and Z.N. Oltvai, 2004. Network biology: Understanding the cell's functional organization. Nat. Rev. Genet., 5: 101-113.

Barber, M.J., 2007. Modularity and community detection in bipartite networks. Phys. Rev. E, 76: 1-9.

Broder, A., R. Kumar, F. Maghoul, R. Stata and A. Tomkins *et al*., 2000. Graph structure in the web. Comput. Networks, 33: 309-320.

Davis, A., B.B. Gardner and M.R. Gardner, 1941. Deep South. University of Chicago Press, USA.

Dorigo, M., V. Maniezzo and A. Colorni, 1996. Ant system: Optimization by a colony of cooperating agents. IEEE Trans. Syst. Man Cybern. Part B: Cybern., 26: 29-41.

Girvan, M. and M.E.J. Newman, 2002. Community structure in social and biological networks. Proc. Natl. Acad. Sci. USA., 99: 7821-7826.

Guimera, R., M. Sales-Pardo and L.A. Amaral, 2007. Module identification in bipartite and directed network. Phys. Rev. E, Vol. 76. 10.1103/PhysRevE.76.036102

Lehmann, S., M. Schwartz and L.K. Hansen, 2008. Biclique communities. Phys. Rev. E, Vol. 78. 10.1103/PhysRevE.78.016108

Liu, X. and T. Murata, 2009a. Community detection in large-scale bipartite networks. Proceedings of the IEEE/WIC/ACM International Conference on Web Intelligence and Intelligent Agent Technology, September 15-18, 2009, Milan, Italy, pp: 50-57.

Liu, X. and T. Murata, 2009b. How does label propagation algorithm work in bipartite networks? Proceedings of the IEEE/WIC/ACM International Joint Conference on Web Intelligence and Intelligent Agent Technology, Volume 3, September 15-18, 2009, Washington, DC., USA., pp: 5-8.

Long, B., X. Wu, Z. Zhang and P.S. Yu, 2007. Community learning by graph approximation. Proceedings of the 7th IEEE International Conference on Data Mining, October 28-31, 2007, Omaha, NE., pp: 232-241.

Newman, M.E.J., 2006. Modularity and community structure in networks. Proc. Nat. Acad. Sci. USA., 103: 8577-8582.

Porter, M.A., J.P. Onnela and P.J. Mucha, 2009. Communities in networks. Notices Am. Math. Soc., 56: 1082-1097.

Radicchi, F., C. Castellano, F. Cecconi, V. Loreto and D. Parisi, 2004. Defining and identifying communities in networks. Proc. Natl. Acad. Sci. USA., 101: 2658-2663.

Raghavan, U.N., R. Albert and S. Kumara, 2007. Near linear time algorithm to detect community structures in large-scale networks. Phys. Rev. E, Vol. 76. 10.1103/PhysRevE.76.036106

Scott, J. and M. Hughes, 1980. The Anatomy of Scottish Capital: Scottish Companies and Scottish Capital, 1900-1979. Croom Helm, London, ISBN: 9780773505285, Pages: 291.

Suzuki, K. and K. Wakita, 2009. Extracting multi-facet community structure from bipartite networks. Proceedings of the International Conference on Computational Science and Engineering, Volume 4, August 29-31, 2009, Vancouver, BC., pp: 312-319.

Watts, D.J. and S.H. Strogatz, 1998. Collective dynamics of small-world networks. Lett. Nat., 393: 440-442.