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Investment Tax, Interest Rate Effect and The Entrepreneur' Welfare: Theory and Numerical Calculation

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Abstract: Based on the external financing analysis framework of asymmetric information, the study establishes a general equilibrium model which contains the investment tax. Firstly, this study investigates that the investment tax are how to influence the size of the equilibrium interest rate and investigates the internal mechanism of the investment tax impact of capital market equilibrium interest rate. Secondly, this study studies the investment tax impact on the investment activities and welfare of different classes of entrepreneurs, proves that the entrepreneurs with very weak financial strength are not suffered any injury from a tight fiscal policy; the entrepreneurs whose financial strength is in the borderline of policy are given the greatest effects from the investment tax; the entrepreneurs with strong financial strength are affected both positive and negative, under certain conditions, the total effect is positive. Finally, this study comparative analyzes that the investment tax have different effects on the capital market of information asymmetric and the capital market of symmetric information.

Key words: Fiscal policy, the equilibrium interest rate, asymmetric information, interest rate effect, welfare analysis

INTRODUCTION

As an important tool of fiscal policy, that the government tax has made impact on consumption, investment and the equilibrium interest rate, which is one of the important issues of Keynesian macroeconomics. Phelps (1967) firstly pointed out that change of fiscal policy possibly influences its implementation through expected mechanism. Lucas (1972) as the representative. The school of rational expectations shaken the status of Keynesian macroeconomics in that Keynesian macroeconomics strictly did not discussed the expected mechanism and solid micro-foundation. In particular, core propositions of the real business cycle theory, Kydland and Prescott (1982) as representative, revealed that investment, consumption, fluctuation interest rate and output are optimal reaction of economic behavior people for real impact as well as invalidity of macroeconomic policy. These core propositions and research methods which base on neoclassical paradigm are a complete denial for Keynesian macroeconomics.

Many scholars' research show that macroeconomic activity closely correlate with the principal-agent problem, liquidity, financial leverage and output, investment (King, 1994). Fisher (1936) proposed a famous "balance sheet effect", which pointed out that the non-indexation

debt contract as well as inflation make the transfer of wealth from borrowers to creditors and that the leverage effect which the company's cash flow reduces and the value of collateral declines suppress the investment and that the credit constraints play a important role in the economic recession in the late 1920s. The others study pointed out that bank run and bank panic generate a negative impacting on macroeconomic performance (Friedman, 1968), while the tax burden of the bank's statutory reserve actually fell on people who borrow from banks rather than savers (Fama, 1985). With overlapping generations model of asymmetric information, Suarez and Sussman (1997) proved that investment output and prices may occur cyclical fluctuations in the capital market. In the world of asymmetric information, Tirole (2006) pointed out that long-term gap between rich and poor may appear and that strong investor would reduce dependence on initial wealth and make income tend to be more equality within the long-term.

Most research in china focused on macroeconomic variables such as how fiscal policy impact output, investment and interest rates. Qualitative method mainly analyzes the effects of fiscal policy with the IS-LM model. While quantitative research mainly utilizes single equation or multi-equation econometric model to estimate fiscal policy impacting on macroeconomic variables such as

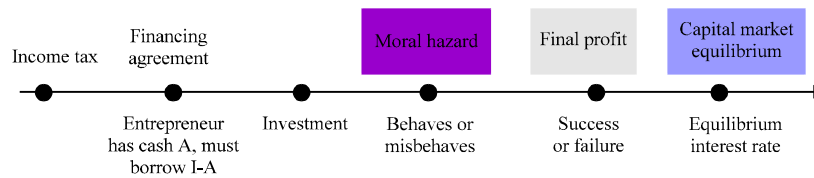


Fig. 1: Figure of the timing

economic growth, investment and interest rates. From the empirical perspective, Ma (2001) empirical study showed that the tax concessions for non-state-owned economy can promote its investment; With sample of survey data, Wu and Lu (2005) confirmed a positive correlation between the debt rate and the occurrence of insufficient investment, over-investment behavior. Xu and Chen (2012) verified that the private and foreign investments are more sensitive to the cost of capital by data of state-owned and large-scale industrial enterprises data.

Contribution and work of this study are that: The study tend to research the internal mechanism that investment tax impacts capital market equilibrium interest rate and study that investing activities and well-being of different categories entrepreneurs are impacted by heterogeneity under same fiscal policy and examine the invalidity which the fiscal policy effect on the information symmetry capital markets and on information asymmetric capital markets. This study bases on theoretical framework which Tirole (2006) established fixed investment analysis framework and external financing framework of asymmetric information in general equilibrium perspective.

MODELS AND ASSUMPTIONS

We adopt the fixed-investment model, The basic assumptions are as follows:

- **Participants:** Entrepreneurs, outside investors and the government
- **Consider a set of risk-neutral entrepreneurs:** Technically a continuum of mass 1 of them
- Each entrepreneur has a project requiring fixed investment I in period 0, everyone requires external financing.
- **We allow for one dimension of heterogeneity:** Entrepreneurs differ in their assets A . Namely, A , which recall is an index of a firm's strength of balance sheet, is distributed in the population of entrepreneurs according to the continuous cumulative distribution function $G(A)$ with support $[\underline{A}, \bar{A}]$ and density $g(A)$ we assume $\bar{A} \leq I$

- **Investment has risk:** If undertaken, the project either succeeds, that is, in period 1, yields verifiable income R , or fails and yields no income. The probability of success depends on the entrepreneur's behavior, it is equal to p_H if the entrepreneur works and p_L if she shirks. Shirking yields a private benefit $B > 0$ to the entrepreneur, the entrepreneur's behavior is unobservable. where $\Delta p = p_H - p_L > 0$
- Government imposes proportional tax $t \in [0, 1]$ on the final revenue of project
- We assume that the project has positive NPV if and only if the entrepreneur behaves. That is, in the relevant range for inter temporal rate r , there is:

$$p_H (1-t)R > (1+r)I > p_L (1-t)R + B$$

- The saving function $S(r)$ is monotonically increasing function
- Both the entrepreneur and the potential lenders (or "investors") are risk neutral
- The entrepreneur is protected by limited liability and so her income cannot take negative values
- Lenders behave competitively in the sense that the loan, if any, makes zero profit
- The entrepreneur has full bargaining power, she offers a take-it-or-leave-it financing contract to the investor
- Presum $p_H (1-t)R - (1+r)I < p_H B / (\Delta p)$

The game timing diagram as shown in Fig. 1.

OPTIMAL FINANCING CONTRACT OF REPRESENTATIVE ENTREPRENEURS

It is easy to see that one optimal contract will have the following structure:

- Suppose that investors are willing to finance the project of a representative entrepreneur, the entrepreneur contributes A and the investors $I - A$
- The optimal contract allocates the profit $(1-t)$ in case of success between representative entrepreneur R_s and investors R_i and gives 0 to both in case of failure

The optimal contract of the representative entrepreneurs and the investors is then the solution to the following maximization problem:

$$\begin{cases} \max_{R_b} & p_H R_b - (1+r)A \\ \text{s.t.} & (1) p_H R_b \geq p_L R_b + B \\ & (2) p_H [(1-t)R - R_b] \geq (1+r)(I - A) \end{cases} \quad (1)$$

The objective function is the representative entrepreneur's expected net revenue (the final value):

- **Constraint 1:** Is the representative entrepreneur's incentive compatibility constraint, can be reduced to $(\Delta_p)R_b \geq B$
- **Constraint 2:** Stands for the out investors' individual rationality constrain. Because this constraint will bind at the optimum, so it is known as investor's zero expected profit condition

In the case of success, the maximum expected income that can be pledged to investors without destroying incentives is equal to $(1-t)R - B/(\Delta p)$, so investor's expected pledge able income is equal to:

$$\begin{aligned} P &= \max_{R_b \in IC} p_H R_b = \max_{R_b \in IC} p_H [(1-t)R - R_b] \\ &= p_H [(1-t)R - \frac{B}{\Delta p}] \end{aligned}$$

A necessary and sufficient condition for an entrepreneur with assets A to receive financing is:

$$\begin{aligned} p_H [(1-t)R - \frac{B}{\Delta p}] &\geq (1+r)(I - A) \\ A \geq A^c(r, t) &= I - \frac{p_H [(1-t)R - B/\Delta p]}{1+r} \end{aligned} \quad (2)$$

Let $A^c(r, t)$ be the smallest level of cash on hand that enables funding. Obviously there are:

$$\frac{\partial A^c(r, t)}{\partial r} > 0, \quad \frac{\partial A^c(r, t)}{\partial t} > 0$$

When the conditions (2) are established, representative entrepreneur in case of success paid for:

$$R_b^*(A) = (1-t)R - \frac{(1+r)(I - A)}{p_H}$$

And the representative entrepreneur's balanced net utility or net payoff is:

$$U_b^*(A) = \begin{cases} 0 & \text{if } A < A^c(r, t) \\ p_H(1-t)R - (1+r)I & \text{if } A \geq A^c(r, t) \end{cases}$$

CAPITAL MARKET EQUILIBRIUM AND THE EQUILIBRIUM INTEREST RATE

The financial market clears when corporate net investment $I(r, t)$ is equal to investors' savings; corporate net investment is the summation of all entrepreneurs' net investment, where $I(r, t)$:

$$\begin{aligned} I(r, t) &\equiv \int_{A^c(r, t)}^{\bar{A}} (I - A)g(A)dA - \int_{\bar{A}}^{A^c(r, t)} Ag(A)dA \\ &= [1 - G(A^c(r, t))]I - A^c \end{aligned} \quad (3)$$

Where:

$$A^c \equiv \int_{\bar{A}}^{\bar{A}} Ag(A)dA$$

is the average entrepreneur wealth. About the relationship between corporate net investment and interest rate and tax rate, there is a proposition:

- **Proposition 1:** In the fixed-investment model, an increase in interest rate or tax rate leads to a smaller corporate net investment. Contrarily, a decrease in interest rate or tax rate leads to a higher corporate net investment
- **Proof:** In fact, according to Eq. 3, we have:

$$\frac{\partial I(r, t)}{\partial r} = -g(A^c(r, t)) \frac{\partial A^c(r, t)}{\partial r} I < 0 \quad (4)$$

$$\frac{\partial I(r, t)}{\partial t} = -g(A^c(r, t)) \frac{\partial A^c(r, t)}{\partial t} I < 0 \quad (5)$$

Proposition 1 shows that impact of interest rate and tax rate on corporate net investment merely express as more credit rationing effect. In fact, a increase in interest rate or tax rate leads to a smaller the outside investor's pledge able income, the smallest level of cash on hand that entrepreneur enables funding is increase, only those firms with strong balance sheets are able to continue to receive financing.

According to market clearing condition, equilibrium interest rate satisfies the following equation:

$$I(r, t) = S(r) \quad (6)$$

In fact, equilibrium interest rate is determined by the intersection of the net investment function and the saving function.

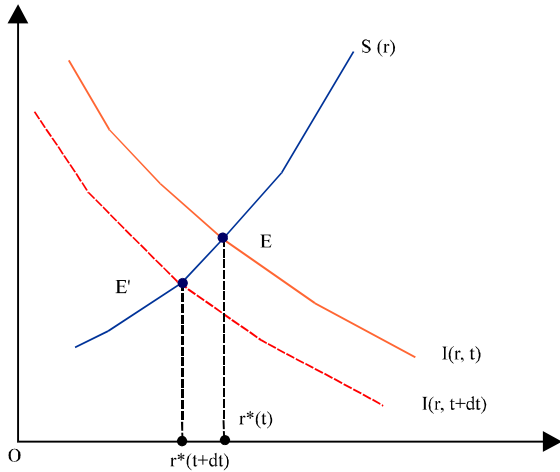


Fig. 2: Tax policy and the interest rate effects

COMPARATIVE STATICS

According to equilibrium Eq. 6, equilibrium interest rate depends on the entrepreneur's own funds and investment tax etc. Change of investment tax will affect the capital market equilibrium interest rate, it is depicted as the following proposition:

- **Proposition 2:** In the fixed-investment model, capital market equilibrium interest rate is scaled down when the government adopts a contractionary fiscal policy (t increases); capital market equilibrium interest rate is mounted up if the government adopts an expansionary fiscal policy (t decreases)
- **Proof:** According to Eq. 6, we have:

$$S'(r^*) \frac{dr^*}{dt} = -g(A^*(r^*(t), t)) \times \left[\frac{p_H [(1-t)R - B/(\Delta p)]}{(1+r^*)^2} \cdot \frac{dr^*(t)}{dt} + \frac{p_H R}{1+r^*(t)} \right]$$

Hence, $dr^*/dt < 0$

- **Proposition 3:** In the fixed-investment model, when the capital market equilibrium, the smallest level of cash on hand that entrepreneur enables funding is increase if the government adopts a contractionary fiscal policy, but decrease if the government adopts an expansionary fiscal policy
- **Proof:** Suppose the equilibrium interest rate is $r^*(t)$, the smallest level of cash on hand that entrepreneur enables funding is:

$$A^*(r^*(t), t) = 1 - \frac{p_H [(1-t)R - B/(\Delta p)]}{1+r^*(t)}$$

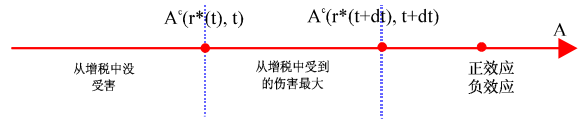


Fig. 3: Tax policy and welfare

So we have:

$$\begin{aligned} & \frac{dA^*(r^*(t), t)}{dt} \\ &= \frac{p_H [(1-t)R - B/(\Delta p)]}{(1+r^*(t))^2} \cdot \frac{dr^*(t)}{dt} + \frac{p_H R}{1+r^*(t)} \\ &= -[g(A^*(r^*(t), t))]^{-1} S'(r^*(t)) \frac{dr^*(t)}{dt} > 0 \end{aligned}$$

Colligates Eq. 6 and proposition 3, it is easy to see that the fiscal policy will produce different effects to different entrepreneurs. The case is illustrated in Fig. 3.

Firms with very weak balance sheets can't to invest and are therefore net savers, so they are not suffered any injury from a contractionary fiscal policy. Because net savers have no bargaining power, they only get zero expected utility. In fact, although capital market equilibrium interest rate is scaled down when the government adopts a contractionary fiscal policy, but as net savers, the final claim is compensation for their loss, they still get zero utility.

Firms with weak balance sheets are hurt by a contractionary fiscal policy, a decline in interest rate squeezes out the marginal firms (with A just above $A^*(r^*(t), t)$). In a contractionary fiscal policy, the smallest level of cash on hand that entrepreneur enables funding is increase, which turn investors into net savers.

A contractionary fiscal policy has an ambiguous impact on firms with strong balance sheets, on the one hand, they may gain in a contractionary fiscal policy to the extent that the reduced financing costs (positive effects), on the other hand, their expected revenue from the project fall (negative effects), the sign of the total effect depends on the size of the positive and negative effects which one dominates:

- **Proposition 4:** In the fixed-investment model, if and only if $dr^*(t)/dt < -p_H R/I$, firms with strong balance sheets benefit from the contractionary fiscal policy; if and only if $dr^*(t)/dt > -p_H R/I$, firms with strong balance sheets are hurt by the contractionary fiscal policy
- **Proof:** In fact, for entrepreneurs whose financial strength is over $A^*(r^*(t+dt), t+dt)$, their welfare is:

$$U_b^*(A) = p_H (1-t)R - (1+r^*(t))I$$

So:

$$\frac{dU_b^*(A)}{dt} = -p_H R - I \frac{dr^*(t)}{dt}$$

Because:

$$-p_H R - I \frac{dr^*(t)}{dt} > 0 \Leftrightarrow \frac{dr^*(t)}{dt} < -\frac{p_H R}{I}$$

Therefore, proposition 4 is proved.

EQUILIBRIUM INTEREST RATE AND THE POLICY EFFECT UNDER SYMMETRIC INFORMATION

If the entrepreneurs' behavior can be observed, there is no asymmetric information between outside investors and entrepreneurs.

First, under symmetric information, the entrepreneurs and the investors' optimal compensation contract is then the solution to the following maximization problem:

$$\begin{cases} \max_{R_b} & p_H R_b - (1+r)A \\ \text{s.t.} & (1)p_H [(1-t)R - R_b] \geq (1+r)(I-A) \end{cases} \quad (7)$$

Where:

- The objective function is the entrepreneurs' expected net reward
- Constraint 1 stands for the outside investors' individual rationality constraint

It's easy to prove, the first-best solution is:

$$R_b^* = (1-t)R - \frac{(1+r)(I-A)}{p_H}$$

Compared with balanced outcome which can access to financing under asymmetric information, the entrepreneurs' reward is the same, but there is no credit rationing under symmetric information. Specifically, all entrepreneurs can borrow money to invest in an efficient risk project.

Second, under symmetric information, corporate net investment is:

$$I^* = \int_{\bar{A}}^{\bar{A}} (I-A)g(A)dA = I - A^* \quad (8)$$

- **Proposition 5:** Under the analytical framework of symmetric information and the

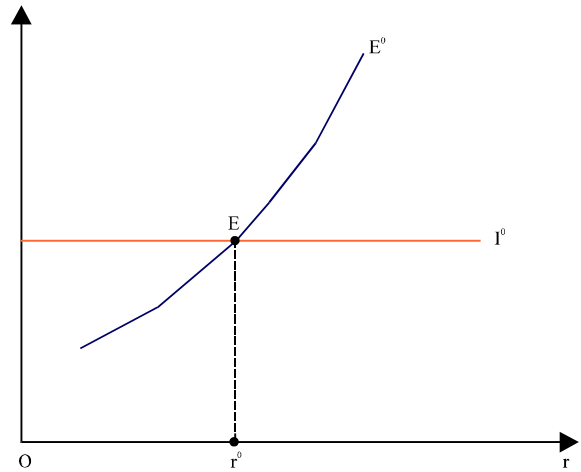


Fig. 4: Capital market equilibrium under symmetric information

fixed-investment, corporate net investment is independent of interest rate and tax rate

- **Proof:** in fact, according to Eq. 8, we have:

$$\frac{\partial I^0}{\partial r} = 0 \quad (9)$$

$$\frac{\partial I^0}{\partial t} = 0 \quad (10)$$

according to market clearing condition, equilibrium interest rate satisfies the following equation:

$$I^0 = S(r) \quad (11)$$

In fact, the capital market equilibrium interest rate is determined by the intersection (E) of net investment function (I^0) and saving function ($S(r)$). Change of investment tax will affect the capital market equilibrium interest rate, it is depicted as the following proposition:

- **Proposition 6:** Under the analytical framework of symmetric information and the fixed-investment, the capital market equilibrium interest rate is invariable regardless of the government adopts a contractionary fiscal policy or an expansionary fiscal policy
- **Proof:** According to equilibrium Eq. 11, we have:

$$S'(r^0) \frac{dr^0}{dt} = 0$$

So:

Table 1: Effects of investment tax on entrepreneurial welfare under asymmetric information

t	$S(r) = 625 - \frac{625}{(1+r)^2}$			$S(r) = 225 - \frac{225}{(1+r)^2}$		
	r*	A _e	U [*] _b	r*	A _e	U [*] _b
0.030	0.066	320	190	0.124	342	146.3
0.035	0.062	323	188	0.117	344	146.7
0.040	0.058	327	186	0.109	346	147.1
0.045	0.054	330	184	0.102	348	147.5
0.050	0.050	333	181	0.095	350	147.9
0.055	0.047	337	179	0.088	352	148.2
0.060	0.043	340	177	0.080	354	148.6
0.650	0.039	343	175	0.073	356	148.9
0.070	0.035	347	172	0.066	358	149.2
0.075	0.031	350	170	0.059	361	149.5
0.080	0.027	354	168	0.052	363	149.8

Table 2: Effects of investment tax on entrepreneurial welfare under symmetric information

t	$S(r) = 2500 - \frac{2500}{(1+r)^2}$			$S(r) = 3600 - \frac{3600}{(1+r)^2}$		
	r*	A _e	U [*] _b	r*	A _e	U [*] _b
0.035	0.078	0	176	0.053	0	195
0.040	0.078	0	171	0.053	0	190
0.045	0.078	0	165	0.053	0	185
0.050	0.078	0	160	0.053	0	180
0.055	0.078	0	155	0.053	0	175
0.060	0.078	0	150	0.053	0	169
0.065	0.078	0	145	0.053	0	164
0.070	0.078	0	140	0.053	0	159
0.075	0.078	0	135	0.053	0	154
0.080	0.078	0	130	0.053	0	149
0.085	0.078	0	125	0.053	0	144

$$\frac{dr^0}{dt} = 0$$

- **Proposition 7:** Under the analytical framework of symmetric information and the fixed-investment, the welfare of all entrepreneurs is scaled down when the government adopts a contractionary fiscal policy, but increase if the government adopts an expansionary fiscal policy
- **Proof:** In fact, under symmetric information, the welfare of the entrepreneur who owns A is:

$$U_b^0(A) = p_H(1-t)R - (1+r^0)I$$

And:

$$\frac{dU_b^0(A)}{dt} = -p_H R < 0$$

so proposition 7 is proved.

NUMERICAL CALCULATION

The numerical simulation of the theoretical model as follows, the benchmark parameters are:

$$p_H = 0.85; p_L = 0.45; R = 1200; B = 250; I = 750; A \sim U(100, 700)$$

Table 1 shows the numerical results under asymmetric information, saving function is divided into two cases. An increase in investment tax lowers the interest rate and raises the smallest level of cash on hand; In particular, for the first type of savings function, the welfare of firms with strong balance sheets is scaled down when the investment tax is increase. For the second type of savings function, an increase in investment tax raises the welfare of firms with strong balance sheets.

Table 2 shows the numerical results under symmetric information, saving function is divided into two cases. Under the analytical framework of symmetric information and the fixed-investment, the equilibrium interest rate is independent of the investment tax, all entrepreneurs have access to finance, the welfare of all entrepreneurs is scaled down. The slopes of the saving function only affect the size of the equilibrium interest rate, don't change the balanced nature.

CONCLUSION

About the important theoretical issue of Keynesian's macroeconomics economy "what impact of economic policies on the economy", this study conducts in-depth theoretical investigation and numerical simulation under the analytical framework of asymmetric information and the fixed-investment, we have the following conclusions:

- If there is asymmetric information between entrepreneurs and outside investors, it's effective for fiscal policy to adjust the interest rate, because asymmetric information leads to credit rationing
- In the capital market with asymmetric information, an increase in the investment tax has different effects on the welfare of entrepreneurs with different financial strength. None but the entrepreneurs with strong financial strength benefit from the contractionary fiscal policy; strong balance sheets are hurt by the contractionary fiscal policy. The entrepreneurs with weak financial strength are in the margin of access to finance, they are hurt by a contractionary fiscal policy
- Fiscal policy will produce different effects in the capital market of information asymmetric and the capital market of symmetric information, because there is no credit rationing in the capital market of symmetric information

This study provides a solid microeconomics foundation for the effects of fiscal policy in Keynesian macroeconomics and shows that the implementation of fiscal policy should take into account the different impact on different type of entrepreneurs. The conclusion of this study is derived from some strict assumptions, for example, we didn't consider the open economy and dynamic, didn't introduce currency etc. For the factors which are not involved in this study, we also can think deeply about in the framework of this study. In fact, macroeconomic operation of a country is extremely complex, this is the reason why macroeconomic policy has never been so easy to be implemented, the significance of academic research is to make use of improved knowledge to understand complex macroeconomic phenomenon and put forward better economic policy.

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