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The Firm's Liquidity Risk Management Based on the Variable-investment Model

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Abstract: Based on the external financing analysis framework under asymmetric information, this study introduces a chance of reinvestment which could increase the probability of success. And then we analyze its influence on the firm's liquidity and risk management through variable-investment model. Both of the theory and numerical calculations show that this kind of profitable growth opportunity could significantly improve the firm's ability against liquidity risk.

Key words: Growth opportunity, liquidity, risk management, variable-investment model

INTRODUCTION

Liquidity risk is a widespread financial problem of enterprises. Over the years, enterprises have been deeply troubled by the "lack of liquidity" such as lack of funds, debt default, inventory backlog, etc. When the liquidity risk accident happens, it usually has a "domino" effect. In other words, it will not only affect the enterprise itself but also have an impact on a series of firms associated from different extent, especially for large enterprises. Therefore, it is very important to deeply study the reasons of liquidity risk and take reasonable measures to control liquidity risk.

Liquidity management is the further deepening and refinement of financial management. Corporate financial management can be roughly divided into three parts: funding, investment and profit distribution. Liquidity management is actually the coordination of the entire process of financial management. Therefore, the theoretical study for that is particularly significant.

The theory of liquidity risk management can be mainly divided into three kinds. Asset management theory has experienced commercial loan theory, asset shift ability theory and anticipated income theory. Commercial loan theory means that the financial business should focus on short-term self-liquidating loans (Smith, 1976); asset shift ability Theory indicates that asset liquidity is good or bad depends on the ability of the transfer of assets into cash (Moulton, 1918); Anticipated Income Theory shows that as long as expected future income is secure, long-term loans and consumer loans may maintain a certain liquidity and security (Prochnow, 1949). Liability Management Theory includes deposits theory, purchase theory. The former indicates that the stability of deposit will directly affect the use of funds for financial institutions. While the latter emerged in the 1970s holds that buying funds can enhance liquidity, in addition, purchase in liabilities is easier than assets. Asset-Liability Comprehensive Management Theory stresses the coordination management of assets risks and liabilities risk, through adjusting asset structure to maintain liquidity and minimize the risk (Baker, 1979).

Currently, the study for liquidity risk management could be divided into two major areas. On the one hand, from the view of holding liquid assets (Holmstrom and Tirole, 2000) find that firms in the process of implementing the investment projects will face liquidity risk because of external emergencies, however, if the enterprise hold a large amount of liquid assets, it is possible to reduce the potential liquidity risk; Guo and Jin (2007) also believe that holding of liquid assets could help companies avoid potential liquidity risk. On the other hand, from the nature of liquidity risk management, Liu (1998) put forward that the aim of liquidity risk management is the reasonable match of asset structure and the correct use of the fund-raising capacity. Shi and Meng (2008) point out that in order to deal with the liquidity risk, firms should begin with internal factors and then identify the potential liquidity risk.

ASSUMPTION

This study introduces growth prospects on the basis of the liquidity risk management model, in the contest of the variable-investment framework. The basic assumptions are as follows:

- **Participants:** An entrepreneur and investors
- **Three periods:** Date 0-2
- At date 0, the entrepreneur has a project requiring fixed investment I. He initially has "assets" A and needs to borrow I-A from investors

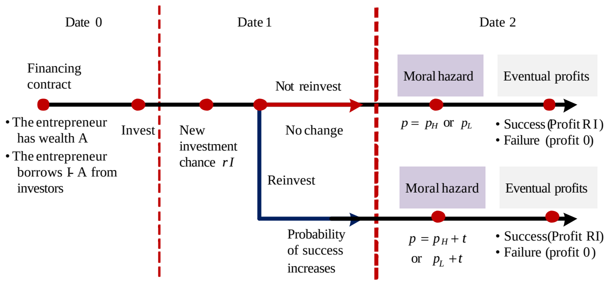

Fig. 1: Figure of the timing

- At date 1, the firm meets a new investment chance requiring an amount ρI, where ρ is ex ante unknown and has cumulative distribution function F(ρ) with density f(ρ) on ρ∈[0, ∞). The realization of ρ is learned at date 1
- The probability of success p is affected by the effort degree of the entrepreneur which is unobservable. Behaving yields probability $p = p_H$ of success and misbehaving results in probability $p = p_L<p_H$ of success and private benefit BI>0. Let $\Delta p = p_H-p_L>0$
- If the firm does not reinvest ρI, then it yields, at date 2, RI with probability p and 0 with probability 1-p
- If the firm reinvests ρI, then it yields, at date 2, RI with probability p+τ and 0 with probability 1-(p+τ), where τ>0
- The investment has positive NPV
- There exists in the economy a store of value. That is, 1 unit invested at date 0 delivers a return of 1 unit at date 1
- Both the entrepreneur and investors are risk neutral
- The entrepreneur is protected by limited liability
- The riskless rate is taken to be 0
- Investors behave competitively in the sense that the loan, if any, makes zero profit

We summarize the timing in Fig. 1.

OPTIMAL MODEL

Suppose that the financing contract takes the following state-contingent form:

$$\{I;\ \rho^c;\ (R_b,\ 0);\ (RI\text{-}R_b,\ 0)\}$$

- The contract specifies that ρ is a cutoff of reinvestment: only if $\rho\le\rho^c$, the firm reinvests
- The contract specifies investment level I
- If the project success, the entrepreneur and investors get R_b and RI-R_b, respectively; if the project fail, both of them get 0

For any cutoff of reinvestment ρ^c:

$$0<\frac{[\tau F(\rho^c)+p_H]R-[1+\int_0^{\rho^c}\rho f(\rho)]}{\tau F(\rho^c)+p_H}<\frac{B}{\Delta p}$$

the probability of reinvestment is:

$$\text{Prob}\{\rho\le\rho^c\} = F(\rho^c)$$

So, the optimization problem becomes:

$$\begin{cases}\max\limits_{R_b,\rho^c,I}\ F(\rho^c)(p_H+\tau)R_b+[1-F(\rho^c)]p_H R_b-A\\ \text{s.t.}\ \ (a1)\ \ F(\rho^c)(p_H+\tau)R_b\\ \qquad\qquad \ge F(\rho^c)[(p_L+\tau)R_b+BI]\\ \qquad (b1)\ \ [1-F(\rho^c)]p_H R_b\\ \qquad\qquad \ge[1-F(\rho^c)](p_L R_b+BI)\\ \qquad (c1)\ \ F(\rho^c)(p_H+\tau)(RI-R_b)+[1-F(\rho^c)]p_H\\ \qquad\qquad \times(RI-R_b)\ge I+\int_0^{\rho^c}\rho I f(\rho)d\rho-A\end{cases}\tag{1}$$

Where:

- The objective function is the entrepreneur's utility
- a1 is the entrepreneur's incentive-compatibility constraint if the firm can come up with enough cash to reinvest, b1 is the incentive-compatibility constraint if not and both of them could be simplified as:

$$R_b\ge\frac{BI}{\Delta p}$$

- c1 is the investors' individual-rationality constraint and it holds with equality. So the optimal model 1 will be simplified as:

$$\begin{cases}\max\limits_{R_b,\rho^c,I}\ \{F(\rho^c)(p_H+\tau)+[1-F(\rho^c)]p_H\}RI\\ \qquad -[I+\int_0^{\rho^c}\rho I f(\rho)d\rho]\\ \text{s.t.}\ \ (a2)\ \ R_b\ge BI/\Delta p\\ \qquad (b2)\ \ \{F(\rho^c)(p_H+\tau)+[1-F(\rho^c)]p_H\}\\ \qquad\qquad \times(RI-R_b)=I+\int_0^{\rho^c}\rho I f(\rho)d\rho-A\end{cases}\tag{2}$$

OPTIMAL CONTRACT

Next, we solve the model 2 in three steps:

Step 1: Solve R_b for a given ρ^c and I

The "feasible contract set" of 2 can be illustrated by Fig. 2 and it is constituted by the shaded area OEF. First of all, it is easy to get that the intercept of the line (b2) is $A/[\tau F(\rho^c)+pH]$ and the slope is:

$$k_1 = \frac{[\tau F(\rho^c) + p_H]R - [1 + \int_0^{\rho^c} \rho f(\rho)d\rho]}{\tau F(\rho^c) + p_H}$$

the intercept of the line (b1) is 0 and the slope is $B/\Delta p$. Since, $0<k_1<B/\Delta p$, the "feasible contract set" is not an empty set if $A\geq 0$.

Secondly, it is the bigger the better for R_b. So, the point F constitutes the optimal contract:

$$R^*_b = \frac{BI}{\Delta p}$$

Step 2: Find the optimal I for a given ρ^c

Actually, take R^*_b into the problem (2) and it can be further simplified as that:

$$\begin{cases} \max\limits_{\rho^c, I} \ m(\rho^c)I \\ \text{s.t.} \ \{F(\rho^c)(p_H + \tau) + [1 - F(\rho^c)]p_H\} \\ \qquad \times (RI - R_b) = I + \int_0^{\rho^c} \rho I f(\rho)d\rho - A \end{cases} \quad (3)$$

Where:

$$m(\rho^c) = [p_H + \tau F(\rho^c]R - [1 + \int_0^{\rho^c} \rho f(\rho)d\rho]$$

is the margin per unit of investment. In addition:

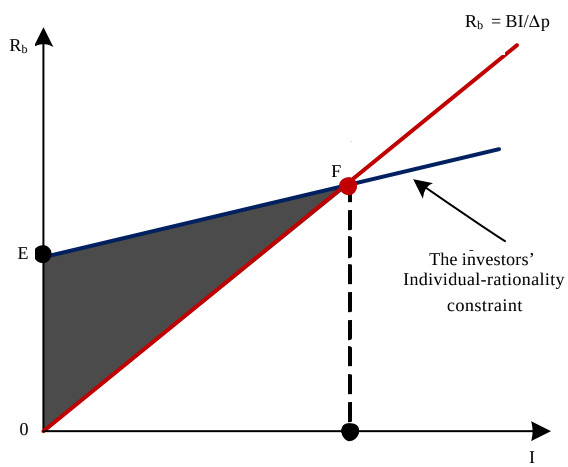

Fig. 2: Solution for the model

$$I^*(\rho^c) = k(\rho^c)A;\ U_b(\rho^c) = m(\rho^c)k(\rho^c)A$$

Where:

$$k(\rho^c) = \frac{1}{[1 + \int_0^{\rho^c} \rho f(\rho)d\rho] - [\tau F(\rho^c) + p_H](R - B/\Delta p)}$$

denote the financing capacity.

Step 3: Consider the optimal ρ^c

Proposition 1: Both of the investment level and financing amount reach their maximum if:

$$\rho^c = \breve{\rho}_0 = \tau\left(R - \frac{B}{\Delta p}\right)$$

Proof: The maximum $k(\rho^c)$ means that:

$$\min_{\rho^c} \int_0^{\rho^c} \rho f(\rho)d\rho - [\tau F(\rho^c) + p_H](R - B/\Delta p)$$

the first-order condition is:

$$\rho^c f(\rho) - \tau F(\rho^c)\left(R - \frac{B}{\Delta p}\right) = 0 \Leftrightarrow \rho^c = \breve{\rho}_0$$

Proposition 2: The entrepreneur's margin per unit of investment reaches its maximum if $\rho^c = \breve{\rho}_1 = \tau R$.

Proof: Because the margin per unit of investment:

$$m(\rho^c) = [p_H + \tau F(\rho^c]R - [1 + \int_0^{\rho^c} \rho f(\rho)d\rho]$$

the first-order condition is:

$$\tau F(\rho^c)R - \rho^c f(\rho) = 0 \Leftrightarrow \rho^c = \breve{\rho}_1$$

Proposition 3: If the threshold liquidity shock is equal to the expected unit cost of effective investment, the entrepreneur's welfare reaches its maximum and it lies between the increase in the expected pledgeable income and the increase in expected income that is:

$$c(\rho^{c*}) = \rho^{c*};\ \breve{\rho}_0 < \rho^{c*} < \breve{\rho}_1$$

Where:

$$c(\rho^c) = \frac{1 + \int_0^{\rho^c} \rho f(\rho)d\rho}{p_H/\tau + F(\rho^c)}$$

Proof: In fact:

$$U_b(\rho^c)=\frac{\{[p_H+\tau F(\rho^c)]R-[1+\int_0^{\rho^c}\rho f(\rho)d\rho]\}A}{[1+\int_0^{\rho^c}\rho f(\rho)d\rho]-[\tau F(\rho^c)+p_H](R-B/\Delta p)}$$
$$=\frac{\tau R-c(\rho^c)}{c(\rho^c)-\tau(R-B/\Delta p)}A$$

so the maximum $U_b(\rho^c)$ means that:

$$\min_{\rho^c} c(\rho^c)=\frac{1+\int_0^{\rho^c}\rho f(\rho)d\rho}{p_H/\tau+F(\rho^c)}$$

The first-order condition is:

$$\rho^c f(\rho^c)\frac{p_H}{\tau+F(\rho^c)}-f(\rho^c)-f(\rho^c)\int_0^{\rho^c}\rho f(\rho)d\rho=0$$

that is:

$$\rho^{c*}\frac{p_H}{\tau}+\int_0^{\rho^{c*}}F(\rho)d\rho=1 \qquad (4)$$

Assume that:

$$Z=1-\rho^{c*}(p_H/\tau)-\int_0^{\rho^{c*}}F(\rho)d\rho=0$$

then from Eq. 4 we can get that:

$$c(\rho^{c*})=\frac{Z+\rho^{c*}[p_H/\tau+F(\rho^{c*})]}{p_H/\tau+F(\rho^{c*})}$$
$$=\rho^{c*}$$

that means:

$$\rho^{c*}=c(\rho^{c*})=\frac{1+\int_0^{\rho^{c*}}\rho f(\rho)d\rho}{p_H/\tau+F(\rho^{c*})}$$

Because of:

$$U_b^*=\frac{\breve{\rho}_1-\rho^{c*}}{\rho^{c*}-\breve{\rho}_0}A$$

we may get that:

$$\breve{\rho}_0<\rho^{c*}<\breve{\rho}_1$$

Figure 3 shows the conclusion of proposition 1-3. The margin $m(\rho^c)$ and the multiplier $k(\rho)$ are both decreasing above $\breve{\rho}_1$ and increasing below $\breve{\rho}_0$. That means

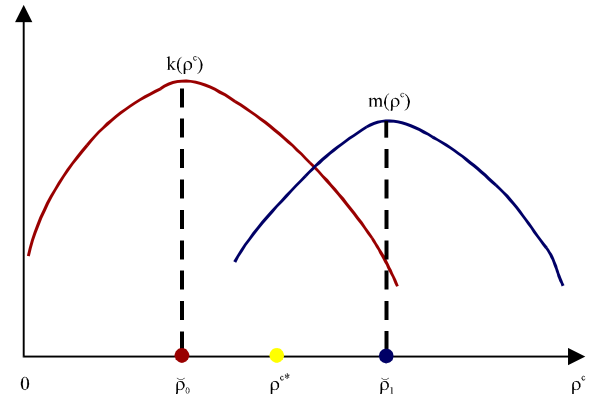

Fig. 3: Comparative static analysis

if $\rho^c>\breve{\rho}_1$, the project could not be financed profitably and if $p^c<\breve{\rho}_0$, the financing capacity and the entrepreneur's utility would be infinite.

Proposition 4: The "first-best cutoff" of reinvestment is increasing with the increase of τ/pH.

From Eq. 5, we may know that ρ^{c*} is increasing with the increase of τ and decreasing with the increase of pH which means it is increasing with the increase of τ/pH.

Proposition 5: The relationship between $k(\rho^{c*})$ and $k = 1/[pH(R-B/\Delta p)]$ is that:

- If:

$$N>\int_0^{\rho^{c*}}\rho f(\rho)d\rho$$

  then $k(\rho^{c*})>k$
- If:

$$N<\int_0^{\rho^{c*}}\rho f(\rho)d\rho$$

  then $k(\rho^{c*})<k$
- If:

$$N=\int_0^{\rho^{c*}}\rho f(\rho)d\rho$$

  then $k(\rho^{c*})=k$

Where:

$$N=\tau F(\rho^{c*})\left(R-\frac{B}{\Delta p}\right)$$

Table 1: First-best solution

pH	τ	M	ρ^{c*}	k*	k
0.6	0.02	0.8	0.03	1.92	1.92
0.6	0.10	0.8	0.16	1.92	1.92
0.6	0.18	0.8	0.29	1.92	1.92
0.6	0.26	0.8	0.40	1.94	1.92
0.6	0.34	0.8	0.50	1.97	1.92
0.9	0.10	0.8	0.11	3.61	3.57
0.8	0.10	0.8	0.12	2.80	2.78
0.7	0.10	0.8	0.14	2.28	2.27
0.6	0.10	0.8	0.16	1.92	1.92
0.5	0.10	0.8	0.20	1.66	1.67
0.8	0.10	0.3	0.12	1.31	1.32
0.8	0.10	0.5	0.12	1.66	1.67
0.8	0.10	0.7	0.12	2.28	2.27
0.8	0.10	0.9	0.12	3.62	3.57
0.8	0.10	1.1	0.12	8.77	8.33

Proposition 6: The "wait and see" policy, under which the entrepreneur tries to raise funds from the investors on the capital market at date 1 in order to cover the liquidity shock, is suboptimal.

In fact, even under perfect coordination, the new investors will provide new credit only if $\rho<\breve{\rho}_0$. And since $\rho^{c*} > \breve{\rho}_0$, it is optimal for the entrepreneur to get more assurance against the firm's shortage of funds other than "wait-and-see".

Proposition 7: The entrepreneur should hoard more liquidity when the liquidity risk reduces.

Proof: An increase in the liquidity risk shows that G(ρ) is the second-order stochastic dominance compared with F(ρ), since:

$$\int_0^{\rho_F^{c*}} F(\rho)d\rho = 1$$

we can get that $\rho_G^{c*} < \rho_F^{c*}$, that is to say the "first-best cutoff" of reinvestment increases if the liquidity risk reduces.

NUMERICAL SIMULATION

Now, we make some numerical calculations and assume that M = R-B/Δp, ρ~U[0, 1]. The results are shown by Table 1:

- ρ^{c*} is increasing with the increase of τ and decreasing with the increase of pH. That means the "first-best cutoff" of reinvestment is increasing with τ/pH
- Comparing the multiplier with reinvestment and without reinvestment that is $k^* = k(\rho^{c*})$ and k = 1/pH (R-B/Δp), the relationship between them is uncertain

CONCLUSION AND SUGGESTIONS

In order to implement the optimal reinvestment policy, the entrepreneur has to hoard and plan his liquidity. There are three ways for the enterprise to plan his liquidity:

- The bank grants a nonrevokable and full amount of line of credit. The optimum can be implemented by a nonrevokable line of credit granted by a bank at level $\rho^{c*}I$. The entrepreneur, who is always better off continuing, will always take advantage of part of this line of credit as long as $\rho \le \rho^{c*}$
- The bank grants a smaller line of credit and allows the entrepreneur to issue new claims. The bank grant a smaller line of credit $(\rho^{c*} - \breve{\rho}_0)I$ and give the entrepreneur the right to dilute claimholders by issuing new claims at date 1. Overall, the entrepreneur can gather $(\rho^{c*} - \breve{\rho}_0)I + \breve{\rho}_0 I = \rho^{c*}I$ and it is enough to withstand the liquidity shock
- The investors could invest more money at the beginning. As an alternative to providing a credit line for the future, the lenders can invest $(1+\rho^{c*})I - A$ in the firm at the start. And then the entrepreneur can invest I and keep $\rho^{c*}I$ in safe

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