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A Semi-Markov Process based Optimization Method for Availability of Hybrid Flow Shop

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Abstract: As Hybrid Flow Shops (HFS) are common manufacturing environments, availability of HFS is a basic indicator for measuring usage ability. Optimal maintenance strategy which achieves maximum availability with cost constraints, provides a better platform for its scheduling problems. We propose an availability model in this study by using Semi-Markov Process (SMP) under a general maintenance strategy which suit for general distribution of machines' life time distribution and maintenance time distribution. Based on the availability model, the maintenance site configuration optimization method is with total cost constrains. Furthermore, the method is applied to a simple hybrid flow shop and showed to be effective.

Key words: Optimization, semi-markov process, availability, hybrid flow shops

INTRODUCTION

In reliability theory, availability is the degree to which a system or equipment is in a specified operable and committable state at the start of a mission, when the mission is called for at random time. That means availability is the proportion of time when a system is in a functioning condition. Mathematically, it's expressed as one minus unavailability.

Hybrid Flow Shops (HFS) are common manufacturing environments in which a set of n jobs are to be processed in a series of m stages optimizing a given objective function (Ruiz and Vazquez-Rodriguez, 2010). There are a number of variants, all of which have most of the following characteristics in common:

- The number of processing stages m is at least 2
- Each stage p has $M_p \geq 1$ machines in parallel and in at least one of the stages $M_p>1$
- All jobs are processed following the same production flow: stage 1, stage 2,..., stage k

In the ''standard'' form of the Hybrid Flow Shops (HFS) problem, all jobs and machines are available at time zero, machines at a given stage are identical, any machine can process only one operation at a time and any job can be processed by only one machine at a time; setup times are negligible, preemption is not allowed. Normally, the data required is deterministic and known in advance, including machine life time and maintenance time. However, in many real situations, machines may be unavailable due to different reasons, such as the periods of unavailability when preventive maintenance-are also called gaps known in advance (deterministic unavailability) or breakdowns (stochastic unavailability) (Besbes *et al*., 2010). Machines are not always available during the scheduling horizon. So an optimal maintenance strategy provides a better platform for flow shop scheduling problem (Allaoui and Artiba, 2004).

HFS is a parallel-Series system with complicate system behaviors when consider maintenance. The availability of this kind of system has long research history. Coleman (1963) proposed ratio method for simple behavior system. Wood (1994) summarized and elaborated availability model with more complicate behavior which more practical maintenance strategy can be combined. But that method required strong Markov property which means it can only model for exponential residence time between states. To lift this restriction, a lot of authors began to use renewal process for modeling which can model more general distribution but less complicated system behaviors (Baxter *et al*., 1982; Mettas and Wenbiao, 2005; Zhang, 2002). To break the restriction of exponential distribution and lacking of system behavior, Semi-Markov process is used for modeling more practical system's availability (Cekyay and Ozekici, 2010; Gupta and Dharmaraja, 2011; Lopez Droguett *et al*., 2008; Ouhbi and Limnios, 2003; Tomasevicz and Asgarpoor, 2009).

So, we proposed a maintenance strategy optimization method based on Semi-Markov Process (SMP) in this study which can be divided into two steps. The first is to establish availability model of HFS system using SMP method. Another one is to study how to find maintenance strategy in order to achieve maximum HFS availability.

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DESCRIBTATION FOR HYBRID FLOW SHOP

An availability optimization problem of the hybrid flow shop in this study can be described as follows. It consists of k stages with each stage p ($1 \le p \le k$) comprising M_p parallel machines which have independent failure rate. Most importantly in our problem, there are p maintenance sites focusing on the machines on all k stages; each maintenance site can afford only one machine's maintenance task on each time; and when one machine is being repaired, the other broken machines have to wait until the under repair one's maintenance task is done. The efficiency and cost of the maintenance site is positively correlated which means the shorter time maintenance task consume, the higher cost of establishing the maintenance site and higher cost for single maintenance task. To ensure optimal maintenance site configuration, we begin with establishing availability model for each stage.

We present the notations used in this study in this section:

k	=	Number of stages
M_p	=	Number of machines on the stage p, p = 1, 2,..., k
$F^j_p(t)$	=	Life time distribution of machine j on the stage p
$G_p(t)$	=	Maintenance time distribution of maintenance site of stage P
$A_p(t)$	=	The stage p availability of flow shop
$X^p(t)$	=	State variable of state p at time t
λ_p	=	Failure rate of machines on stage p
μ_p	=	Mean Time To Failure (MTTR) of maintenance site on stage p
C_T	=	Total cost of maintenance site configuration
C_p	=	Maintenance configuration cost on stage p

AVAILABILITY MODEL OF FLOW SHOP

For each stage of flow shop, there are more than one possible states: Perfect state (all machines is working well), unperfected states (one or more than one but not all machines are broken), stop state (all machines in the stage is broken). We assume that the life time of machines in the same stage is independent and identically distributed (i.i.d) and maintenance time distribution of each stage are known. So how to find the availability model of hybrid flow shop can be answered by using a Semi-Markov Process (SMP).

Semi-markov processes: SMPs take a standard Markov process to another level by allowing the amount of time spent in each state to be any positive random variable and not just an exponential distribution. Sojourn time refers to the length of a visit in a particular state of a system. This, of course, is the significant difference between an SMP and a standard Markov process (Tomasevicz and Asgarpoor, 2009) Although, a standard Markov process is useful for the purpose of simplicity, semi-Markov models are often preferred, when possible, as the calculated results tend to be more comparable to actual data.

In this problem, we divide the whole hybrid flow shop availability into k availability of each stage. Each stage has M_p machines and several states shown as below:

$$X^p(t) = \begin{cases} 1 & \text{if all machines are working at time t} \\ 2 & \text{if only one machine is down at time t} \\ \vdots & \\ M_p+1 & \text{if all machines are down at time t} \end{cases} \quad (1)$$

The states transmission relationship is shown as Fig. 1.

Since, $Q(t) = (Q_{ij}(t), i, j \in E)$ is kernel of semi-Markov process of X(t) which can be defined as below:

$$Q_{ij}(t) = P(X(\tau_1) = j, \tau_1 \le t | X(0) = i)$$

where, $E \triangleq \{1, 2,..., M_i+1\}$.

And $\{\tau_n\}_{n\ge 0}$ is the nth state jumping moment.

In this case, we consider the machines are breaking down one by one which means the length of state jumping could not be longer than 1 and the maintenance task begins immediately when any machine is broken. So, the relationship of state transform matrix-SMP kernel is defined as Equation below:

$$Q_{ij}(t) = P(X(\tau_1) = j, \tau_1 \le t \mid X(0) = i)$$
$$= \begin{cases} \text{P(only one of the working machines break when maintenance task is unfinish)} & j-i=1 \\ \text{P(no working machine break when maintenance task is finish)} & j-i=-1 \\ 0 & \text{else} \end{cases}$$
$$= \begin{cases} \binom{M_i+1-i}{1} P(T_L \le t) P(T_L > t)^{M_i-i} P(T_M \ge t) & j-i=1 \\ P(T_L \ge t)^{M_i+1-i} P(T_M \le t) & j-i=-1 \\ 0 & \text{else} \end{cases}$$

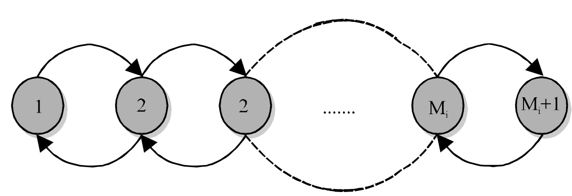

Fig. 1: State transmission relationship

As the life time distribution and maintenance time distribution is known as $F_{jp}(t)$ and $G_p(t)$, so the SMP kernel can be written as:

$$Q_{ij}(t)=\begin{cases}(M_p+1-i)F_p(t)(1-F_p(t))^{M_p-i}(1-G_p(t)) & j\text{-}i=1\\ (1-F_p(t))^{M_p+1-i}G_p(t) & j\text{-}i=\text{-}1\\ 0 & \text{else}\end{cases} \quad (2)$$

Availability model: The availability model based on Semi-Markov Process backward equations is obtained.

Defining:

$$G_i(t)\triangleq\sum_{j\in E}Q_{ij}(t)<1, i\in E \quad (3)$$

$$P_{ij}(t) = p(X(t) = j, t\le\tau|X(0) = i), i, j\in E \quad (4)$$

So, we can describe the instantaneous availability as $A(t) = p_{11}(t)$. In order to obtain A(t), we have to transform P and Q as below:

$$\varphi_{iE}(s)=\int_0^\infty e^{-st}P_{iE}(t)ds, s\ge 0, i\in E$$

$$\hat{Q}_{ij}(s)=\int_{0^-}^\infty e^{-st}dQ_{ij}(t), s\ge 0, i\in E$$

$$h_i(s)=\int_{0^-}^\infty e^{-st}(1-G_i(t))ds, s\ge 0, i\in E$$

$$\hat{G}_i(s)=\int_{0^-}^\infty e^{-st}dG_i(t), s\ge 0, i\in E$$

So, we can write semi-Markov backward equation as Eq. 5:

$$\varphi(s)=\hat{Q}(s)\varphi(s)+H(s) \quad (5)$$

where, $H(s) = (\delta_{ij}h_i(s), i, j\in E)$, δ_{ij} is defined as:

$$\delta_{ij}=\begin{cases}1, i=j\\ 0, i\ne j\end{cases}$$

By solving the backward equation, we obtain that $\varphi(s)=(I-\hat{Q}(s))^{-1}H(s)$. So the Laplace form of availability can be obtained by $\varphi(s)$ (1, 1).

And the steady state availability of steady can be written as:

$$A_p=\lim_{s\to\infty}(\varphi(s)(1,1)\bullet s)$$

And the hybrid flow shop availability is:

$$A=\prod_{p=1}^{k}A_p$$

MAINTENANCE STRATEGY OPTIMIZATION

To make it easier to explain our method, we illustrate a two-stage hybrid flow shop with two maintenance site and 3 machines on each stage which is shown as Fig. 2 below.

Life times of machines on the same stage independent and identically distributed with CDF $F_p(t) = 1\text{-}\exp(-\lambda_p t)$, maintenance time distribution is $G_p(t) = u(t-\mu_p)$, where $\lambda_p>0$, $\mu_p>0$.

By Eq. 2, we can obtain the SMP kernel of each stage of this hybrid flow shop with definition of states according Eq. 1:

$$Q(t)=\begin{bmatrix}0 & 3(e^{-2\lambda_p t}-e^{-3\lambda_p t})u(\mu_p-t) & 0 & 0\\ e^{-2\lambda_p t}u(t-\mu_p) & 0 & 2(e^{-\lambda_p t}-e^{-2\lambda_p t})u(\mu_p-t) & 0\\ 0 & e^{-\lambda_p t}u(t-\mu_p) & 0 & (1-e^{-\lambda_p t})u(\mu_p-t)\\ 0 & 0 & u(t-\mu_p) & 0\end{bmatrix}$$

Using the method we proposed in section 3 we can obtain the availability of the whole flow shop and it has form as below:

$$A = A_1(\lambda_1, \mu_1)A_2(\lambda_2, \mu_2) = A(\lambda_1, \mu_1, \lambda_2, \mu_2)$$

We assume the total cost C_T includes the cost of each repair site configuration and the cost of

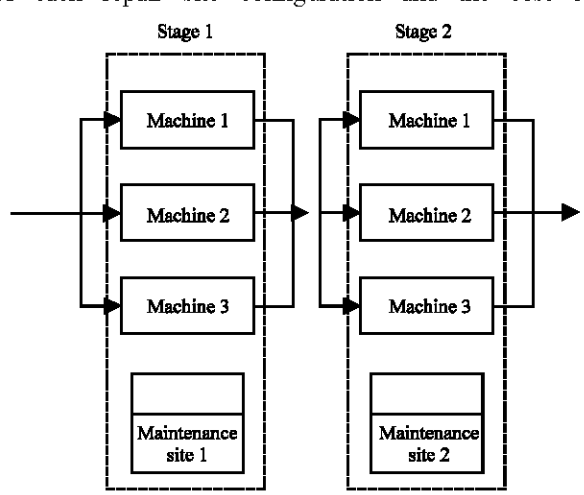

Fig. 2: Two-stage HFS

configuration are inversely related to Mean Time To Repair (MTTR) of each maintenance site. So total cost can be expressed as equation:

$$C_T = \sum_{i=1}^{k} C_{Mi}(\mu_i)$$

and in this case $C_T = C_{M1}(\mu_1)+C_{M2}(\mu_2)$.

Bound of the total cost by adjusting the parameters of each repair site in order to achieve the maximum availability of the entire hybrid flow shop.

We can achieve the maximum availability of the entire hybrid flow shop by adjusting maintenance sites configuration cost with total cost constraining. So our problem is converted to an optimization problem shown as below:

$$\begin{cases} \text{Maximize} \\ A(\lambda_1, \lambda_2, \mu_1(C_{M1}), \mu_2(C_{M2})) \\ \text{subject to} \\ C_T = C_{M1} + C_{M2} \end{cases}$$

It is a constrained optimization problem of a single target. When the two stages machines' failure rate λ_1 = 0.02, λ_2 = 0.03 are given and relationship between maintenance site cost and MTTR is:

$$C_{Mi} = \frac{3000}{\mu_i}$$

which means the minimize MTTR can reach 0.6.

We can obtain the relationship between availability of flow shop system and stage one maintenance site configuration cost C1 by plotting the availability expression of flow shop system with independent variable C1, shown as Fig. 3.

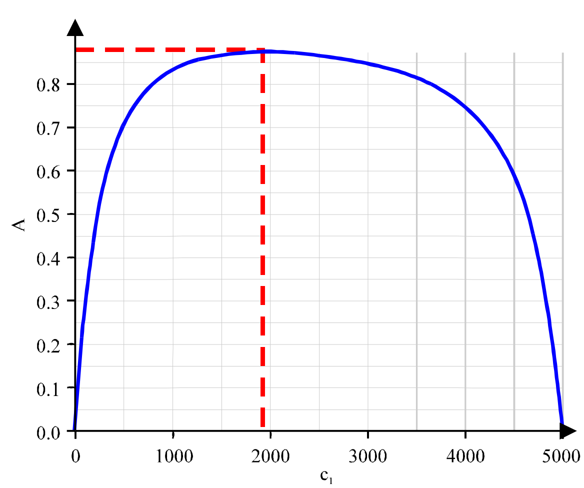

Fig. 3: Cost-availability relationship

Thus we find that C_1 = 1961.5 and C_2 = 3038.5 can maximize the hybrid flow shop availability to 0.873. That means the small difference of single machine in different state can result in bigger difference in maintenance site configuration cost. And these conclusions can guide follow-up scheduling works.

CONCLUSION

Maintenance sites can afford one machine's maintenance at the same time and may follow "first-come, first-served" principal. In this study, we investigated the hybrid flow shop availability optimizing problem with cost constraints due to maintenance site's MTTR on each stage. So a two-stage HFS system is illustrated to show how to establish availability model and how to optimize it. Then availability model is established by using Semi-Markov Process (SMP) for more general distribution of life time of machines and maintenance time. By finding the global optimal solution, the optimal maintenance site cost configuration is obtained by a simple cost model given.

However, in the case, we assume the life time distributions of machines are exponentially distributed for simplicity. Numeral method for Laplace transform (Cohen, 2006) is needed, if life time distribution comply with general distribution such as the Weibull distribution. Moreover, we can also establish availability model considering preventive maintenance tasks or imperfect maintenance task (Botta-Genoulaz, 2000; Carpov *et al.*, 2012; Hmida *et al.*, 2011; Lin and Liao, 2003; Linn and Zhang, 1999; Liu *et al.*, 2008; Uetake *et al.*, 1995; Voss and Witt, 2007) by more detailed definition of Semi-Markov Process states.

REFERENCES

Allaoui, H. and A. Artiba, 2004. Integrating simulation and optimization to schedule a hybrid flow shop with maintenance constraints. Comput. Ind. Eng., 47: 431-450.

Baxter, L.A., E.M. Scheuer, D.J. McConalogue and W.R. Blischke, 1982. On the tabulation of the renewal function. Technometrics, 24: 151-156.

Besbes, W., J. Teghem and T. Loukil, 2010. Scheduling hybrid flow shop problem with non-fixed availability constraints. Eur. J. Ind. Eng., 4: 413-433.

Botta-Genoulaz, V., 2000. Hybrid flow shop scheduling with precedence constraints and time lags to minimize maximum lateness. Int. J. Prod. Econ., 64: 101-111.

Carpov, S., J. Carlier, D. Nace and R. Sirdey, 2012. Two-stage hybrid flow shop with precedence constraints and parallel machines at second stage. Comput. Oper. Res., 39: 736-745.

Cekyay, B. and S. Ozekici, 2010. Mean time to failure and availability of semi-Markov missions with maximal repair. Eur. J. Oper. Res., 207: 1442-1454.

Cohen, A.M., 2006. Numerical Methods for Laplace Transform Inversion. Springer, New York.

Coleman, J.J., 1963. Evaluation of automatic checkout for system availability. Society of Automotive Engineers, Warrendale, PA., USA.

Gupta, V. and S. Dharmaraja, 2011. Semi-Markov modeling of dependability of VoIP network in the presence of resource degradation and security attacks. Reliab. Eng. Syst. Safety, 96: 1627-1636.

Hmida, A.B., M. Haouari, M.J. Huguet and P. Lopez, 2011. Solving two-stage hybrid flow shop using climbing depth-bounded discrepancy search. Comput. Ind. Eng., 60: 320-327.

Lin, H.T. and C.J. Liao, 2003. A case study in a two-stage hybrid flow shop with setup time and dedicated machines. Int. J. Prod. Econ., 86: 133-143.

Linn, R. and W. Zhang, 1999. Hybrid flow shop scheduling: A survey. Comput. Ind. Eng., 37: 57-61.

Liu, B., l. Wang and Y.H. Jin, 2008. An effective hybrid PSO-based algorithm for flow shop scheduling with limited buffers. Comput. Operat. Res., 35: 2791-2806.

Lopez Droguett, E., M. das Chagas Moura, C. Magno Jacinto and M. Feliciano Silva, 2008. A semi-Markov model with Bayesian belief network based human error probability for availability assessment of downhole optical monitoring systems. Simul. Modell. Pract. Theory, 16: 1713-1727.

Mettas, A. and Z. Wenbiao, 2005. Modeling and analysis of repairable system with general repair. Proceedings of the Annual Reliability and Maintainabilty Symposium, January 24-27, 2005, Alexandaria, Virginia, pp: 176-182.

Ouhbi, B. and N. Limnios, 2003. Nonparametric reliability estimation of semi-Markov processes. J. Stat. Planning Inference, 109: 155-165.

Ruiz, R. and J.A. Vazquez-Rodriguez, 2010. The hybrid flow shop scheduling problem. Eur. J. Oper. Res., 205: 1-18.

Tomasevicz, C.L. and S. Asgarpoor, 2009. Optimum maintenance policy using semi-Markov decision processes. Electr. Power Syst. Res., 79: 1286-1291.

Uetake, T., H. Tsubone and M. Ohba, 1995. A production scheduling system in a hybrid flow shop. Int. J. Prod. Econ., 41: 395-398.

Voss, S. and A. Witt, 2007. Hybrid flow shop scheduling as a multi-mode multi-project scheduling problem with batching requirements: A real-word application. Int. J. Prod. Econ., 105: 445-458.

Wood, A., 1994. Availability modeling. IEEE Circ. Devices Mag., 10: 22-27.

Zhang, Y.L., 2002. A geometric-process repair-model with good-as-new preventive repair. IEEE Trans. Reliab., 51: 223-228.