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## Coordination and Optimization of Supply Chain under a New Buy-back Contract

<sup>1</sup>Songtao Zhang, <sup>2</sup>Min Zhang and <sup>1</sup>Juan Ni

<sup>1</sup>School of Management,

<sup>2</sup>Library, Harbin University of Commerce, Harbin, 150028, China

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**Abstract:** The problem of the coordination and optimization of supply chain is studied under a new buy-back contract. Firstly a supply chain system which consists of one manufacturer and two markets is constructed. Then a new buy-back contract is proposed to make the profit of the manufacturer maximize by the strategies of adding price and discriminating price. By compared to the normal price strategy, the maximum profit can be obtained for the manufacturer when the two price strategies are adopted for the two markets at the same time. Finally, a numerical example illustrates that the proposed new buy-back contract can coordinate and optimize the supply chain system.

**Key words:** Coordination, optimization, supply chain, buy-back contract

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### INTRODUCTION

The concept of supply chain has been widely accepted by people. Today, the competition among enterprises has turned from a simple competition between individual enterprises into the competition among supply chains. Supply chain is a corporate network which is composed of a number of independent subjects and in the process of cooperating, because of their different objectives, limited resources and their unique information resources, the enterprises of supply chain have a conflict of interest inevitably. As a result, the situation of the discoordination of supply chain arises.

In recent years, supply chain coordination is a hot issue. The contract mechanism of supply chain coordination is an effective way to achieve supply chain coordination. There are some contracts to solve the problem of supply chain coordination, such as quantity discounts contract and revenue sharing contracts. These contracts have their own adaptability and they are used widely in the enterprises (Liao *et al.*, 2009).

Qin (2012) studied the triple buy-back contracts and all members of supply chain can obtain the maximum profits. For the fuzzy demand of the customer, Sang (2013) proposed the revenue sharing contract and accordingly the optimal policy were designed. The conclusions were that the optimal order quantity of the entities fluctuated at the center of the fuzzy demand and increased with the raise of the retail price. Tang *et al.* (2012) studied the service supply chain consisting of one Service Vendor (SV) and one Service Integrator (SI) and developed information sharing coordination between the SV and SI

as well as performance according to different types of information transferring scenario. To illustrate the benefits of partnerships with information sharing, the three levels of information sharing were introduced. According to different situation of information sharing, the contribution of this study was the information sharing-based partnership can improve the overall performance of a service supply chain. Applying the options in the research of buyback contract and considering spot market in supply chain, Liang and Sheng (2010) analyzed the function of the expectation profit of the retailer. The study compared the optimal strategy with and without buyback options and the average buyback contract. And the retailer can get the most expectation profit. The ordering model of supply chain with delivery delay was constructed with buy-back contract (Liu *et al.*, 2012). The impact of delivery delay rate on the order quantity of the supplier and the retailer was analyzed as well as the profit. It was shown that the coordination buy-back price in delivery delay was less than that in non-delivery-delay. The supplier preferred high buy-back price and low delivery delay penalty and the retailer preferred low buy-back price and high delivery delay penalty. Both of them were willing to share more demand risk and less delivery delay risk. Liu and Chen (2010) established an incentive buy-back contract in order to coordinate supply chain. The contract allows suppliers to achieve the coordination of the supply chain at different ratios of reduced-price selling capacity by adjusting the purchase price. Gan and Ni (2010) established a buy-back contract model that incorporated sales rebate mechanism for two-echelon supply chains that consisted of a single

supplier and a single retailer, with transport cost borne by the supplier. This study analyzed the optimal decisions for the two parties both with and without the buy-back contract and arrived at the optimal ordering quantity for the retailer with the buy-back contract and the buy-back price for the supplier.

In this study, the supply-chain is composed of one manufacturer and two markets. The restrictive buy-back contract is proposed by the manufacturer in two markets. Restrictive buy-back contract is that the buy-back quantity is changed according to the demand quantity of market. Two making price strategies are adopted in the two markets. The adding price based on cost is expressed by the percent of cost of the series produces. And then, the adding price based on price is expressed by the percent of price of the series produces. Then, the manufacturer can decide the price according to the profit.

**DESCRIPTION OF THE QUESTION**

In the study, two hypothesizes are proposed as follows:

- The first one is that two markets are independent
- The second one is that the buy-back quantity is the proportion of the demand quantity

Based on above two hypothesizes, the signs are proposed as follows:

- $c_0$ : The cost of producing unit product
- $p_1, p_2$ : The prices for two markets
- $p_1$ : The price for market one
- $p_2$ : The price for market two
- $p_{r1}, p_{r2}$ : The price of buy-back for two markets
- $p_{r1}$ : The buy-back price for market one
- $p_{r2}$ : The buy-back price for market two
- $D_i = \alpha_i - \beta_i p_i$ : The demand of two markets,  $i = 1, 2$  and the demand is random, so the  $\alpha$  and  $\beta$  are alterative
- $c_m$ : The residual value from reconstituted product

The restrictive buy-back contract is proposed, then  $G_i = 1/nD_i$ ,  $i = 1, 2$ , where  $G_i$  is the buy-back quantity from two markets.

**MATERIALS AND METHODS**

**Model establishment based on adding price:** Based on the description of model, this study sets up the model in which the parameters can be seen in Fig. 1.

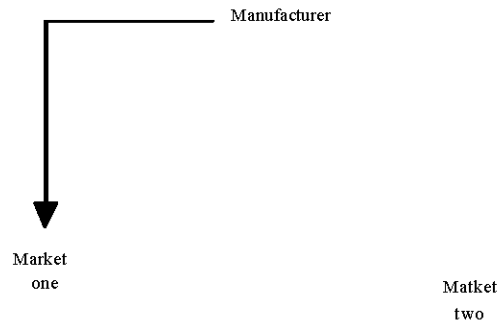


Fig. 1: Model of supply chain with the adding price

Let  $\Pi_{m1}$  and  $\Pi_{m2}$  represent respectively the profits of market one and market two and  $\Pi_m = \Pi_{m1} + \Pi_{m2}$ . The strategies of making two prices are denoted as  $p_1 = c_0 \xi$  and  $p_2 = p_{r2}(1 + \delta)$ . The demand quantity of market one is  $D_1$ , correspondingly, the demand quantity of market two is  $D_2$ , then  $D_1 = \alpha_1 - \beta_1 p_1$ ,  $D_2 = \alpha_2 - \beta_2 p_2$ . The buy-back quantities are expressed  $G_1 = 1/nD_1$  and  $G_2 = 1/nD_2$ , respectively.

Therefore, the profits of the manufacturer from market one and market two are:

$$\begin{cases} \Pi_{m1} = p_1 D_1 - p_{r1} G_1 - c_0 D_1 + c_m G_1 \\ \Pi_{m2} = p_2 D_2 - p_{r2} G_2 - c_0 D_2 + c_m G_2 \end{cases} \quad (1)$$

where,  $p_1$  denotes the price for market one,  $p_2$  denotes the price for market two,  $p_{r1}$  denotes the buy-back price for market one,  $p_{r2}$  denotes the buy-back price for market two,  $c_0$  denotes the cost of producing unit product,  $c_m$  denotes the residual value from reconstituted product.

So, the total profit can be denoted as follows:

$$\Pi_m = \Pi_{m1} + \Pi_{m2} = \sum_{i=1}^2 [(p_i - c_0) D_i - (p_{ri} + c_m) G_i]$$

**Model analysis based on adding price:** After  $p_1, G_1, D_1$  are calculated in Eq. 1 and 2 can be obtained as follows:

$$\begin{aligned} \Pi_{m1} &= p_1 D_1 - p_{r1} G_1 - c_0 D_1 + c_m G_1 \\ &= \alpha_1 c_0 \xi - \left( c_0 + \frac{1}{n} p_{r1} - \frac{1}{n} c_m \right) \alpha_1 \\ &\quad - \beta c^2 \xi^2 + \beta c_0 \xi \left( c_0 + \frac{1}{n} p_{r1} - \frac{1}{n} c_m \right) \end{aligned} \quad (2)$$

After the derivative for  $\xi$  from Eq. 3 is calculated Eq. 4 can be calculated as follows:

$$\frac{\partial \Pi_{m1}}{\partial \xi} = \alpha_1 c_0 - 2\beta_1 c_0^2 \xi + \beta_1 c_0 \left( c_0 + \frac{1}{n} p_{r1} - \frac{1}{n} c_m \right) \quad (3)$$

$$\frac{\partial^2 \Pi_{m1}}{\partial \xi^2} = -2\beta_1 c_0^2 \quad (4)$$

Let Eq. 4 be equal to zero, then:

$$\xi = \frac{\alpha_1 + \beta_1 \left( c_0 + \frac{1}{n} p_{r1} - \frac{1}{n} c_m \right)}{2c_0\beta_1}$$

Because:

$$\frac{\partial^2 \Pi_m}{\partial \xi^2} = -2\beta_1 c_0^2 < 0$$

$\xi$  can be obtained and the profit of the manufacturer can be maximized.

Similarly, after  $p_2, G_2, D_2$  are calculated in  $\Pi_{m2}$ , Eq. 5 can be obtain as follows:

$$\begin{aligned} \Pi_{m2} = & \alpha_2 \left( c_0 + \frac{1}{n} p_{r2} - \frac{1}{n} c_m \right) - \beta_2 p_{r2}^2 (1 + \delta)^2 \\ & + \beta_2 p_{r2} (1 + \delta) \left( c_0 + \frac{1}{n} p_{r2} - \frac{1}{n} c_m \right) \end{aligned} \quad (5)$$

After the derivative for  $\delta$  from Eq. 6 is calculated, Eq. 7 can be calculated as follows:

$$\begin{aligned} \frac{\partial \Pi_{m2}}{\partial \delta} = & \alpha_2 p_{r2} - 2\beta_2 p_{r2}^2 (1 + \delta) \\ & + \beta_2 p_{r2} \left( c_0 + \frac{1}{n} p_{r2} - \frac{1}{n} c_m \right) \end{aligned} \quad (6)$$

$$\frac{\partial^2 \Pi_{m2}}{\partial \delta^2} = -2\beta_2 p_{r2}^2 \quad (7)$$

Let Eq. 7 be equal to zero, then:

$$\delta = \frac{\beta_2 \left( c_0 + \frac{1}{n} p_{r2} - \frac{1}{n} c_m \right) + \alpha_2 - 2\beta_2 p_{r2}}{2\beta_2 p_{r2}}$$

Because:

$$\frac{\partial^2 \Pi_{m2}}{\partial \delta^2} = -2\beta_2 p_{r2}^2 < 0$$

$\delta$  can be obtained and the profit of the manufacturer can arrive at the maximum.

After  $\xi, \delta$  are calculated in  $p_1, p_2$ , Eq. 7 and Eq. 8 can be obtained as follows:

$$p_1 = \frac{\alpha_1 + \beta_1 \left( c_0 + \frac{1}{n} p_{r1} - \frac{1}{n} c_m \right)}{2\beta_1} \quad (8)$$

$$p_2 = \frac{\alpha_2 + \beta_2 \left( c_0 + \frac{1}{n} p_{r2} - \frac{1}{n} c_m \right)}{2\beta_2} \quad (9)$$

**Model establishment based on normal price:** This study adopts the normal price in buy-back contract and let the

price of market one  $p_1$  and market two  $p_2$  be the same. Let  $p_1 = p_2 = p$ .  $p_r$  is defined as the price of buy-back of supply chain.  $\Pi'_m$  is used to manifest the profit of the manufacturer.

$$\begin{aligned} \Pi'_m = & p(D_1 + D_2) - p_r(G_1 + G_2) - c_0(D_1 + D_2) \\ & + c_m(G_1 + G_2) = \sum_{i=1}^2 [ (p - c_0) D_i - (p_r - c_m) G_i ] \end{aligned} \quad (10)$$

**Model analysis based on normal price:** After  $D_1, D_2, G_1, G_2$  are calculated in Eq. 10, the following formula can be obtained:

$$\begin{aligned} \Pi'_m = & (p - c_0) [ (\alpha_1 + \alpha_2) - (\beta_1 + \beta_2) p ] \\ & - \frac{1}{n} (p_r - c_m) [ (\alpha_1 + \alpha_2) - (\beta_1 + \beta_2) p ] \end{aligned}$$

Let  $\alpha_1 + \alpha_2 = \alpha$   $\beta_1 + \beta_2 = \beta$ , then  $\Pi'_m$  can be shown as follow:

$$\Pi'_m = (\alpha - \beta p) \left[ p - c_0 - \frac{1}{n} (p_r - c_m) \right] \quad (11)$$

After the derivative for  $p$  from Eq. 12 is calculated, Eq. 13 and 14 can be calculated as follows:

$$\frac{\partial \Pi'_m}{\partial p} = -2\beta p + \beta \left[ c_0 + \frac{1}{n} (p_r - c_m) \right] + \alpha \quad (12)$$

$$\frac{\partial^2 \Pi'_m}{\partial p^2} = -2\beta \quad (13)$$

Because:

$$\frac{\partial^2 \Pi'_m}{\partial p^2} = -2\beta < 0$$

$p^*$  can be obtained and the profit of the manufacturer can be maximized.

Therefore:

$$\begin{aligned} p^* = & \frac{\alpha + \beta \left[ c_0 + \frac{1}{n} (p_r - c_m) \right]}{2\beta} \\ = & \frac{(\alpha_1 + \alpha_2) + (\beta_1 + \beta_2) \left[ c_0 + \frac{1}{n} (p_r - c_m) \right]}{2(\beta_1 + \beta_2)} \end{aligned} \quad (14)$$

## RESULTS AND DISCUSSION

The values for the numerical analysis can be seen in Table 1.

According to Eq. 9, 10 and 14,  $p_1, p_2, D_1, D_2$  and  $\Pi'_m$  can be calculated and the results can be seen in Table 2.

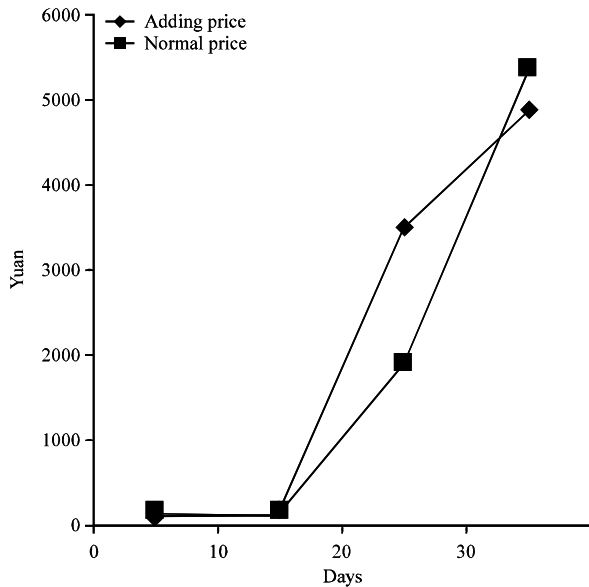


Fig. 2: Comparison results of the total profit for the manufacturer between adding price and normal price

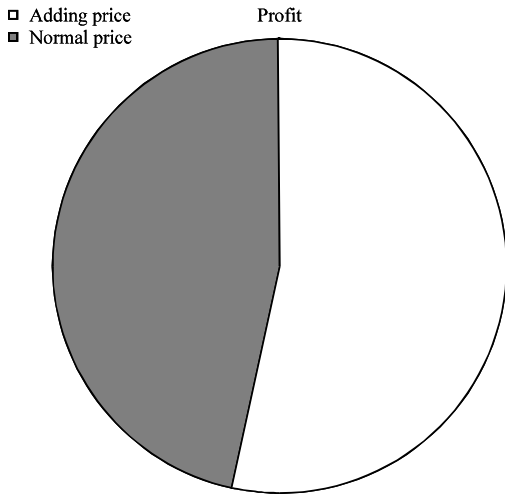


Fig. 3: Proportion of the profits between adding price and normal price

The strategies of price and demand quantity can be figured by graphs.

From Fig. 2, the conclusion can be drawn that the total profit of the manufacturer is higher by adopting adding price than normal price. Consequently, the strategy of the distinction fixing price can coordinate supply chain and the profit can be maximized.

The profits from two strategies of making price can be represented in Fig. 3.

Table 1: Values of all variables in the numerical analysis

Variables	Numerical
$c_0$	10
$p_{11}$	6
$\beta_1$	20
$\alpha_2$	10000
$p_{12}$	4
$c_m$	8
$\alpha_1$	5000
$n$	10
$\beta_2$	30
$p_r$	5

Table 2: Comparison results between adding price and normal price

Price parameter	Adding price	Normal price
$p_1$	129	155
$D_1$	3510	1900
$p_2$	171	155
$D_2$	4870	5350
$\Pi_m$	1204410	1053425

### CONCLUSION

In this study supply chain system is established with one manufacturer and two markets. A strategy is adopted to make the profit maximize by the manufacturer which uses respectively adding price based on cost and adding price based on price for market one and market two. Compared to the normal price, a conclusion can be drawn that the profit of supply chain is more by using adding price instead of normal price. Finally, a numerical example in which adding price is calculated reveals the validity for coordinating supply chain.

More research works such as considering the cooperation of different kinds of contracts and different kinds of fixing price in complex supply chain will be carried on in the future.

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