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Reasearch on the Real Linkage Effect Between Key Parameters in Inventory Management

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Abstract: With the expansion of competition and the growth of product variety, how to reduce safety stock in supply chain management has become a hot topic. Managers are seeking some areas that they can improve to make safety stock reduced meanwhile the cycle service level provided will not be influenced. Some pay more focus on the reduction of replenishment of lead time and the variability of lead time. Through analysing the normal approximation value of lead time demand distribution, the outcome shows that when cycle service level above 50%, these measures are effective. It also shows making the variability of lead time minimum has more effect on safety stock than reducing lead time. While the real situation is totally different, there is contradictory between the outcome from normal approximation value and formula derivation. By mathematic derivation, as the cycle service is above 50%, the reorder points will increase with the lead time variability decreasing. Therefore for enterprise with cycle service level above 50%, in order to reduce safety stock, the right choice should be to reduce the variability of lead time.

Key words: Linkage effect, inventory management, cycle service level, safety stock, mathematical derivation

INTRODUCTION

In supply chain management, lean concept has become widely accepted, more and more manager are faced with an increasingly pressure on how to keep the inventory level lower (Marklund, 2006), moreover, the object is to reduce inventory in the condition of keeping the service level for customers (Muckstadt, 2005). Safety stock as a key parameter in inventory management has relationship with the cycle service level, the uncertainty in demand. The lead time of replenishment is another key parameter. If the cycle service level is fixed, manager could control the safety stock through modifying two values, i.e., the feature of lead time and the characteristic of demand in every period. The feature of lead time includes the mean and the scope of variation. Here, the focus is the linkage effect between the variation of lead time and safety stock, some principles will be found out.

In reality, the service level between 50 and 70% is common in many enterprises. Because the rate of cost increasing is far higher that of service level, meanwhile that managers always pay attention to the product fill rate (Moinzadeh and Schmidt, 1991), regarding it as a method for measuring service not cycle service level. Product fill rate is a means to measure if the inventory can satisfy the demand of customers, while the cycle service level is to measure the rate between the cycles during which all the demand of customers will be satisfied and all the replenishment cycles. In the following Table 1, it demonstrates the corresponding relationship between cycle service level and product fill rate for a product under different reordering point (Liu, 2008). Weekly demand of the product is 2500, the variance of the demand is 500, lead time is 2 weeks, and order quantity is 10000.

Table 1: Corresponding relationship between cycle service level and product fill rate

Reordering point	Safety stock	Cycle service level	Fill rate
5000	0	0.5	0.9718
5040	40	0.523	0.9738
5080	80	0.545	0.9756
5120	120	0.567	0.9774
5160	160	0.59	0.9791
5200	200	0.611	0.9807

As Table 1 shows that the product fill rate between 97 and 98% corresponds to the cycle service level between 50 and 60% (Lim, 2001). Most enterprise wants to make the product fill rate between 97 and 98%, which means to make the cycle service level between 50 and 60%. So, more focus was put on the relationship between the characteristics of lead time and safety stock when the cycle service level (Li *et al*., 2007) is between 50 and 60%.

In usual scope of cycle service level where most enterprises are in, through the analysis of normal

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approximation, managers should decrease the variance of lead time, so that there is the possibility that the safety stock could be reduced, which is better than decreasing lead time (Chen and Yu, 2005). Then, the relationship between the variance of lead time and safety stock will be found out through mathematics derivation.

MATERIALS AND METHODS

Basic principles: For a specific cycle service level, the safety stock level which is required mostly depends on the features of demand distribution during the lead time. Suppose the demand in the specific day i is x_i, which is an independent demand and normally distributed (Xiong *et al*., 2006) with the mean μ_x, standard variation σ_x. The mean of lead time is L, the standard variation of lead time is S_L. The mean of the demand during lead time in the condition of normal distribution is $M = L\mu_x$, with the standard variation:

$$\sigma_L = \sqrt{L\sigma_x^2 + \mu_x^2 S_L^2}$$

The following Fig. 1 shows the relationship between the variance and reordering point under different service levels.

Suppose the function F() is the cumulative distribution function of standard normal distribution with mean 0, standard variance 1. Suppose F (n) = α and ROP stands for the reordering point when cycle service level is α. In the condition of normal approximation, ROP = $M+n\sigma_L$, $n\sigma_L$ stands for safety stock. The uncertainty in demand (Robb and Silver, 2006) during lead time is determined by the features of demand in every cycle (represented by μ_x, σ_x) and the features of lead time (represented by L, S_L). Herein more focus was put on the effect of the features of lead time on safety stock. As Fig. 2 shows that when cycle service level is more than 50%, with the increase of L or S_L, safety stock increases as well (because at that time n>0). When cycle service level is below 50%, with the increase of L or S_L, safety stock decreases (because at that time n<0). When cycle service level equals to 50%, safety stock remains unchanged (because at that time n = 0).

Case analysis: Because the features of lead time is represented by two coefficients-L and S_L, a case is used in the following part to analyze the effect of decreasing lead time and the variance of lead time on safety stock. Suppose the demand obeys the normal distribution, the mean demand of a period is 20, the standard variance of demand is 10, the mean of lead time is 10 and the standard variance of lead time is 10.

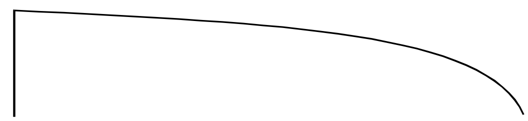

Fig. 1: Relationship between uncertainty of demand and safety stock during lead time when service level (>, <, =) 50%

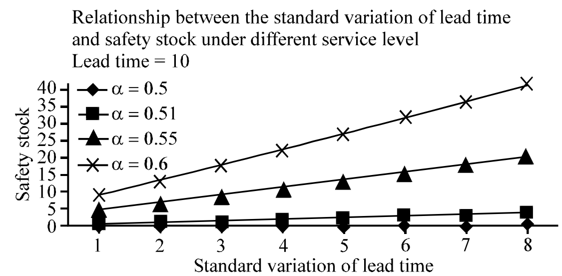

Fig. 2: Corresponding relationship between different standard variance of lead time and safety stock

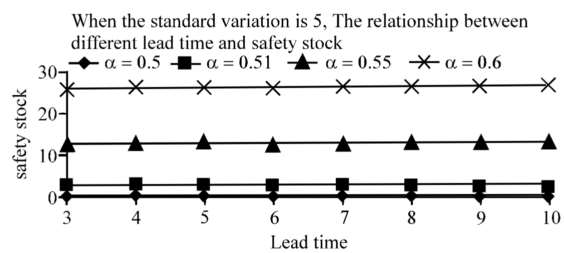

Fig. 3: Corresponding relation between different lead time and safety stock

To begin with, supposing the standard variance of lead time is between 1 and 8, under four different service level {0.5,0.51,0.55,0.6}, the corresponding relationship between different standard variance of lead time and safety stock is as Fig. 2 shows. As the cycle service level is at the interval (0.5, 0.6] to decrease the variance of lead time will make safety stock lower; the reordering point will be lower as well.

Secondly, supposing the standard variance of lead time is 5, as lead time varies is at the interval [3,10], under four different service level {0.5,0.51,0.55,0.6}, the corresponding relationship between lead time and safety stock is as Fig. 3 shows. So, when cycle service level is at the interval (0.5, 0.6] to decrease lead time will make safety stock lower.

Through the above two cases, the corresponding relationship between cycle service level and safety stock under the following three conditions, as the following Fig. 4:

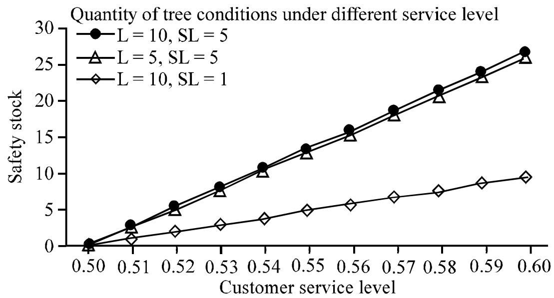

Fig. 4: Corresponding relationship between service level and safety stock (Under three conditions)

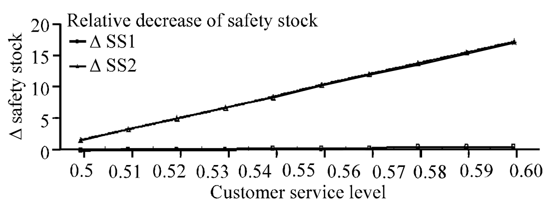

Fig. 5: Relative reduction of safety stock

- Q_1: Lead time is 10, the standard variance of lead time is 5
- Q_2: Lead time is 5, the standard variance of lead time is 5
- Q_3: Lead time is 10, the standard variance of lead time is 1

Suppose ΔSS_1 = SS (Q_1)-SS (Q_2), ΔSS_2 = SS (Q_1)-SS (Q_3), then we may get the following Fig. 5.

Through the normal approximation method, the following two features are found: The first is when cycle service level above 50%, to decrease the variance of lead time will make safety stock and reordering point lower. The second is when cycle service level above 50%, in order to make safety stock lower, decreasing the variance of lead time is more effective than decreasing lead time. Because the former will make safety stock decreased more obviously (Dai *et al*., 2006). It means that when the enterprise is under the normal operation condition (i.e., when service level is at the interval (50, 60%), if the enterprise wants to decrease safety stock and keep the service level unchanged, then to decrease the variance of lead time is the best method. While, what on earth is the real situation? Then, by establishing mathematical model and derivation, what on earth is the effect of decreasing the variance of lead time on safety stock is the research focus.

FEATURES OF LEAD TIME SAFETY STOCK AND RELATIONSHIP

Basic mathematical model: Suppose periodical demand obeys normal distribution and lead time obeys discrete uniform distribution, with the mean of Y and the range of Y±y. If lead time is uniformly distributed in the range of Y±y, the reordering point is R, P_y (R) stands for the probability that the demand is below or equivalent to R during lead time, μ_x stands for the expected demand during the period, σ_x is the standard variance of the demand in the period, then the following equation is got:

$$\varphi_Y(R) = (R - Y\mu_x) / (\sigma_x \sqrt{Y}) \quad (1)$$

From the definition of ϕ_Y (R), obviously it indicates that under the premise of the given lead time Y,ϕ_Y (R) stands for the standard variance which R is far from the expected value of demand. Suppose F (ϕ_Y (R)) represents the probability that standard normal is less than or equivalent to ϕ_Y (R), then the following equation is inferred:

$$P_y(R) = \left(\frac{1}{2y+1}\right)\sum_{z=Y-y}^{Y+y} F(\varphi_z(R)) \quad (2)$$

From Eq. 1 and 2, the following Eq. 3 can be derived, that is:

$$P_y(R_1) > P_y(R_2) \text{ if and only if } R_1 > R_2 \quad (3)$$

If y = 0, the lead time is determined. Here the situation of P_y (R) is the target when the lead time is changeable because of the change of y. To begin with, the effect of increasing lead time by y in a period should be examined. From Eq. 2, the following outcomes can be derived that:

$$P_{y+1}(R) - P_y(R) = \left(\frac{1}{2y+3}\right)\left[F(\varphi_{Y+y+1}(R)) + F(\varphi_{Y-y-1}(R)) - 2P_y(R)\right] \quad (4)$$

And:

$$P_{y+1}(R) = \left(\frac{1}{2y+3}\right)\left[F(\varphi_{Y+y+1}(R)) + F(\varphi_{Y-y-1}(R))\right] + \left(\frac{2y+1}{2y+3}\right)P_{y+1}(R) \quad (5)$$

Because Y+yy+1>Y-y-≥0, then $\phi_{Y+y+1}(R) \le \phi_{Y-y-1}(R)$ so that:

$$0 < F(\phi_{Y+y+1}(R)) < F(\phi_{Y-y-1}(R)) < 1 \quad (6)$$

Supposed lemma and proving

Lemma 1: Suppose R_y and R_{y+1} satisfy the following conditions, i.e., $P_y(R_y) = P_{y+1}(R_{y+1}) = \alpha$, then R_{y+1}>(<) R_y, if and only if:

$$F(\phi_{Y+y+1}(R_y))+F(\phi_{Y-y-1}(R_y))>(<)\ 2P_y(R_y)$$

If, $F(\phi_{Y+y+1}(R_y))+F(\phi_{Y-y-1}(R_y))>(<)\ 2P_y(R_y)$ then from Eq. 3 and 4, we can infer $P_{y+1}(R_y)<(>)\ P_y(R_y) = \alpha$. Because $P_{y+1}(R_{y+1}) = \alpha$, using Eq. 3, we get $R_{y+1} > (<)\ R_y$ On the other side, if $R_{y+1}>(<)\ R_y$, using Eq. 3, we have $\alpha = P_{y+1}(R_{y+1})<(>)\ P_{y+1}(R_y)$.

Because $\alpha = P_y(R_y)$, then $P_y(R_y)>(<)\ P_{y+1}(R_y)$. From Eq. 4, we have:

$$F(\phi_{Y+y+1}(R_y))+F(\phi_{Y-y-1}(R_y))>(<)\ 2P_y(R_y)$$

Mathematical derivation: Through mathematical derivation we investigate whether safety stock wil come down through making the variance of lead time decreasewhen $50\%<\alpha<1$. Firstly, Firstly we investigate if there is $50\%<\alpha<1$, $y>0$, then $R_y(\alpha)<R_0(\alpha)$.

Suppose the lead time of independent demand is constant Y, when the reordering point is $R\mu_x$ ($R\in(0, Y)$), the cycle service level can be expressed as:

$$F(\frac{R-Y}{c\sqrt{Y}})$$

Then, the mean of lead time is Y, which is uniformly distributed among the three values {Y-y, Y, Y+y}, y is a small positive number. Then, what about the condition of reordering point when reordering point is fixed at $R\mu_x(R\in(0, Y))$, for some y (y>0). The key point depends on if the following formula is set up:

$$F(\frac{R-(Y+y)}{c\sqrt{Y+y}})+F(\frac{R-(Y-y)}{c\sqrt{Y-y}})>2F(\frac{R-Y}{c\sqrt{Y}}) \quad (7)$$

As for Eq. 7, if y = 0, two sides of the formula is the same. Through the partial derivative of the left side with respect to y, we can get the following expression of the partial derivative:

$$Z_y' = f(\frac{R-Y+y}{c\sqrt{Y-y}})\frac{c\sqrt{Y-y}-(R-Y+y)\frac{-c}{2\sqrt{Y-y}}}{c^2(Y-y)}+$$
$$f(\frac{R-Y-y}{c\sqrt{Y+y}})\frac{-c\sqrt{Y+y}-(R-Y-y)\frac{c}{2\sqrt{Y+y}}}{c^2(Y+y)}$$
$$\Rightarrow Z_y' = f(\frac{R-Y+y}{c\sqrt{Y-y}})\frac{Y-y+R}{2c(Y-y)^{3/2}}-f(\frac{R-Y-y}{c\sqrt{Y+y}})\frac{R+Y+y}{2c(Y+y)^{3/2}},$$

herein $f(.) = F^{-1}(.)$.

For positive y, whether $Z_y'>0$ is set up? Suppose $R = (1+\lambda)Y$ and $y = \beta Y$ ($0<\beta<\lambda$) then:

$$Z_y' = f(\frac{(\lambda+\beta)Y}{c\sqrt{Y(1-\beta)}})\frac{(2+\lambda-\beta)Y}{2cY^{3/2}(1-\beta)^{3/2}}-f(\frac{(\lambda-\beta)Y}{c\sqrt{Y(1+\beta)}})\frac{(2+\lambda+\beta)Y}{2cY^{3/2}(1+\beta)^{3/2}}$$
$$=\frac{1}{2c\sqrt{2\pi Y}}\left[\exp(-k\frac{(\lambda+\beta)^2}{1-\beta})\frac{2+\lambda-\beta}{(1-\beta)^{3/2}}-\exp(-k\frac{(\lambda-\beta)^2}{1+\beta})\frac{2+\lambda+\beta}{(1+\beta)^{3/2}}\right],$$
$$(k=\frac{Y}{2c^2})Z_y' = \frac{\exp(-k\frac{(\lambda-\beta)^2}{1+\beta})}{2c\sqrt{2\pi Y}}$$
$$\left[\exp(k\left(\frac{(\lambda-\beta)^2}{1+\beta}-\frac{(\lambda+\beta)^2}{1-\beta}\right))\frac{2+\lambda-\beta}{(1-\beta)^{3/2}}-\frac{2+\lambda+\beta}{(1+\beta)^{3/2}}\right]$$

In order to prove $Z_y'\geq 0$ when k is proper, we need to prove:

$$\exp(k\left(\frac{(\lambda-\beta)^2}{1+\beta}-\frac{(\lambda+\beta)^2}{1-\beta}\right))\frac{2+\lambda-\beta}{(1-\beta)^{3/2}}-\frac{2+\lambda+\beta}{(1+\beta)^{3/2}}\geq 0$$

i.e.:

$$\exp(-k\left(\frac{4\lambda\beta+2\lambda^2\beta+2\beta^3}{1-\beta^2}\right))\frac{2+\lambda-\beta}{(1-\beta)^{3/2}}-\frac{2+\lambda+\beta}{(1+\beta)^{3/2}}\geq 0 \quad (8)$$

When:

$$k\leq\frac{1}{4\lambda+4\lambda^2},$$

it can be derived that:

$$-k\left(\frac{4\lambda\beta+2\lambda^2\beta+2\beta^3}{1-\beta^2}\right)\geq\frac{-\beta}{1-\beta^2}.$$

Then:

$$\exp(-k\left(\frac{4\lambda\beta+2\lambda^2\beta+2\beta^3}{1-\beta^2}\right))\geq\exp(\frac{-\beta}{1-\beta^2})$$

Suppose:

$$Z(\lambda,\beta)=\exp(\frac{-\beta}{1-\beta^2})\frac{2+\lambda-\beta}{(1-\beta)^{3/2}}-\frac{2+\lambda+\beta}{(1+\beta)^{3/2}} \quad (9)$$

Because Z (λ, 0) and:

$$\frac{2+\lambda+\beta}{(1+\beta)^{3/2}}$$

goes down with the increase of β ($\beta>0$), then if Eq. 9>0 is set up when ($\beta>0$), the change of:

$$\exp(\frac{-\beta}{1-\beta^2})\frac{2+\lambda-\beta}{(1-\beta)^{3/2}}$$

with the change of β ($\beta>0$) should be examined.

Suppose:

$$Z_1(\lambda,\beta) = \exp(\frac{-\beta}{1-\beta^2})\frac{2+\lambda-\beta}{(1-\beta)^{3/2}}$$

then:

$$\frac{\partial Z_1(\lambda,\beta)}{\partial\beta} = \exp(\frac{-\beta}{1-\beta^2})\frac{-(1-\beta)^{3/2}+\frac{3}{2}(2+\lambda-\beta)\sqrt{1-\beta}}{(1-\beta)^3}$$
$$+\exp(\frac{-\beta}{1-\beta^2})(\frac{2+\lambda-\beta}{(1-\beta)^{3/2}})(\frac{-(1-\beta)^2-2\beta^2}{(1-\beta^2)^2})\frac{\partial Z_1(\lambda,\beta)}{\partial\beta}\Big|_{\beta=0}$$
$$= -1+\frac{3}{2}(2+\lambda)-(2-\lambda)>0$$

The outcome:

$$\frac{\partial Z_1(\lambda,\beta)}{\partial\beta}>0,(\beta>0)$$

is inferred, then Z_1 (λ, β) is an increasing function, that is Z (λ, β) is an increasing function also. So, if:

$$k = \frac{Y}{2c^2} \le \frac{1}{4\lambda+4\lambda^2}$$

Eq. 7>0, then:

$$\Rightarrow P_y(R_0(\alpha))\text{-}P_0(R_0(\alpha))>0 \Rightarrow P_y(R_0(\alpha))\text{-}P_y(R_y(\alpha))>0$$
$$\Rightarrow R_y(\alpha)<R_0(\alpha) \Rightarrow P_y(\alpha)\text{-}Y\mu_x < P_0(\alpha)-Y\mu_x \Rightarrow SS_y < SS_0$$

From the above mathematical derivation, it shows that as the uncertainty of lead time increases, the relative reordering point will go down. Because the mean of lead time remain fixed, safety stock will become lower as the variance of lead time increases.

CONCLUSION

Through normal approximation method, the key point of the linkage relationship between the uncertainty of lead time and safety stock is mainly based on the features of demand during lead time. When cycle service level is above 50%, normal approximation value shows that through cutting down the uncertainty of lead time, safety stock will decrease. While through precise mathematical derivation, the outcome is different that the normal approximation value is wrong when the cycle service level is above 50%. At that time, the safety stock will increase as the uncertainty of lead time decreases. When the cycle service level in the scope, in order to reduce safety stock, decreasing lead time should be put more attention than decreasing the variance of lead time. Obviously, compared with the outcome from normal approximation method, there is contradiction. There needs further specific research on the numerical analysis in the further.

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REFERENCES

Chen, F. and B. Yu, 2005. Quantifying the value of leadtime information in a single-location inventory system. Manuf. Serv. Operat. Manage., 7: 144-151.

Dai, G.X., Y.Z. Hou and Q.D. Yu, 2006. An optimized method for two-echelon inventory system based on stochastic lead time. Math. Pract. Theory, 3: 51-56.

Li, X., X. Wang and S. Sun, 2007. Research on the relative stability of cycle service level under different ordering strategy. Sci. Technol. Progress Policy, 24: 38-40.

Lim, W.S., 2001. Product-supplier contracts with incomplete information. Manage. Sci., 47: 709-715.

Liu, H.L., 2008. Two kinds of new inventory order models. Ind. Eng. J., 3: 101-103.

Marklund, J., 2006. Controlling inventories in divergent supply chains with advance-order information. Operat. Res., 54: 998-1010.

Moinzadeh, K. and C.P. Schmidt, 1991. An (S-1, S) inventory system with emergency orders. Operat. Res., 39: 308-321.

Muckstadt, J., 2005. Analysis and Algorithms for Service Parts Supply Chain. Springer, New York, USA.

Robb, D.J. and E.A. Silver, 2006. Inventory management under date-terms supplier trade credit with stochastic demand and leadtime. J. Oper. Res. Soc., 57: 692-702.

Xiong, Y.H., H.C. Shen and G.J. Xing, 2006. The effect of normality assumption of demand features on the inventory system. Ind. Technol. Econ., 6: 76-80.