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Study on Equilibrium and Analytical Solution of Mobile Phone Virus spreading

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Abstract: In this study, under investigation is a propagation system of mobile phone virus via., epidemic model (SEIR model). Via., symbolically computation, an analytic solution for the system is obtained which has higher accuracy and can describe the propagation more completely than those in the previous results. Moreover, several propagation situations on the system are illustrated and figures are plotted for understanding the propagation mechanism based on the obtained solutions. Results of this study might be helpful for the future development of some diffusion in the modern social network, such as online review and user generated contents.

Key words: Mobile phone virus, SEIR model, analytic solution, symbolic computation

INTRODUCTION

With the rapid development of mobile communication technology and the smart phones, mobile phone virus emerged. How to deeply understand the spreading trend of mobile phone virus and then design efficient prevention and containment strategies becomes an urgent task. It is also of high relevance for the department of networking management to understand the diffusion or spreading mechanism of computer virus, worm and Trojan on the internet. In fact, it is very important for us to release some related warning information of malcodes and take corresponding measures which are beneficial to keep large scale networking system running stably and safely (Hypponen, 2006). Furthermore, monitoring the trend of rumor and opinion disseminating is also an efficient means to collect public voices. Therefore, modeling and analyzing the evolution of mobile phone virus and public opinions on real systems has become an actively interdisciplinary subject which attracts the increasing attention and interest of various researchers from computer science, system engineering, statistical physics, mathematics, public hygienics and so on (Kleinberg, 2007; Eubank *et al*., 2004).

The self-replication and propagation behavior of mobile phone virus is similar to that of infectious diseases, so we can make use of existing mathematical methods in epidemic areas to study the propagation of mobile phone virus (Wang and Chen, 2003; Cui *et al*., 2010; Xu *et al*., 2011). However, in classical model Susceptible-Infected-Recovered (SIR) of infectious disease spread, the propagation of virus obeys the following rule: $S \xrightarrow{\beta} I \xrightarrow{\gamma} R$, where β is the infection rate and γ is the immunization rate (Kephart and White, 1991; Qu *et al*., 2013). The problem of SIR model is that it assumes the changes with time of three states in a closed group are only related with its own and two other constants β and γ which cannot consider other factors' effects. As a matter of fact, the individuals cannot be infected until some time delay. Xu *et al*. (2006) discuss the impact of infection delay on the spreading behaviors in complex networks through numerical simulations but the authors do not obtain the specific relationship between transmission delay and infection threshold. Considering the practical propagation process of mobile phone virus in the network, it is necessary to introduce an exposed state (E),i.e., once susceptible mobile phone receives the message with virus, it enters the exposed state immediately. Furthermore, if the mobile phone installs the virus attachment automatically, the user changes the exposed state into the infected one (Mukhopadhyay and Bhattacharyya, 2007). Based on the above mentioned discussions, we will pay attention to the extended SIR model (SEIR) which combines the infection delay to study the virus spreading behaviors within complex network.

Taking into account prevention awareness and pre-immune behaviors, we need to introduce state conversion $S \rightarrow R$, $E \rightarrow R$ and $R \rightarrow I$. The susceptible state and exposed state gain pre-immune ability via., updating virus database, patching and so on, directly entering the

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recovered state. We assume that all individuals can be divided into four classes: Susceptible (S), Exposed (E), Infected (I) and Recovered (R). The SEIR model can be depicted in Fig. 1a. Figure 1a shows the different states and state conversions of mobile phones. We divide the phone nodes into (S, E, I, R) states and 6 kinds of state conversions. Where, α is the rate at which the susceptible phone node becomes infected one; β is the rate at which the exposed phone node is infected; γ is the probability at which the infected phone node removes the infected virus by anti-virus software, patches and so on and immunes to the virus; ω is the probability at which the susceptible phone node gains pre-immunity by defense technologies such as updating virus database and patches; δ is the probability at which the exposed phone node gains pre-immunity by defense technologies; ε is the probability at which the recovered phone node comes back to being susceptible once again because of losing defense ability to virus variants. Then the SEIR model can be described with the following differential equation system:

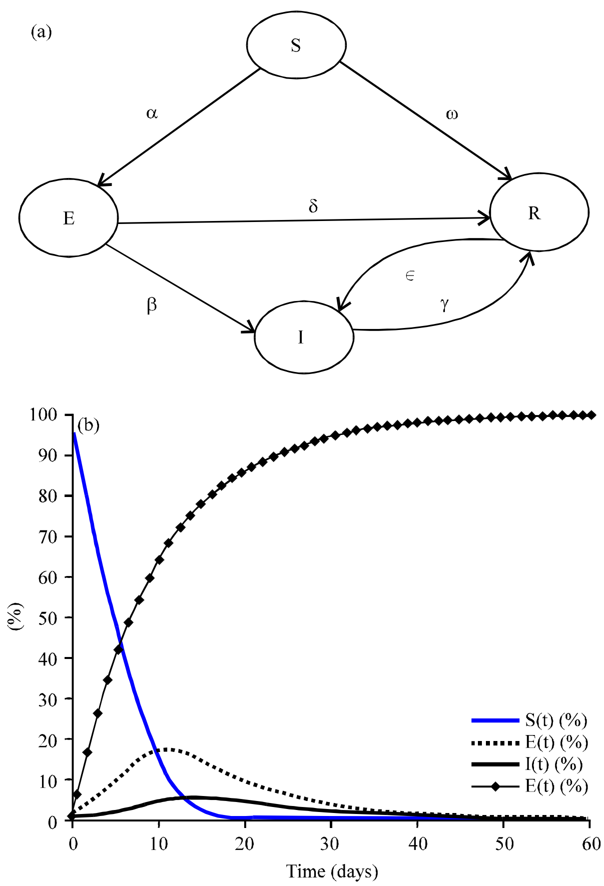

Fig. 1(a-b): (a) State conversions of mobile phone virus and (b) Time evolution of the partial populations

$$\begin{cases} \frac{dS(t)}{dt} = \alpha S(t) - \omega S(t) \\ \frac{dE(t)}{dt} = \alpha S(t) - \delta E(t) - \beta E(t) \\ \frac{dI(t)}{dt} = \beta S(t) - \mu I(t) - \varepsilon R(t)I(t) \\ \frac{dR(t)}{dt} = \omega S(t) - \delta E(t) - \gamma R(t)I(t) - \lambda R(t) \end{cases} \quad (1)$$

where, S(t) is the number of susceptible nodes at time t; E(t) is the number of exposed nodes at time t; I(t) is the number of infected nodes at time t; R(t) is the number of recovered nodes at time t; N is the total number of nodes in the whole short message network and the initial conditions are S(0) = N, E(0) = 0, I(0) = 0, R(0) = 0.

The remainder of this study is structured as follows. In section 2, via., symbolic computation, analytic solution for system 1 under the initial condition will be given. In section 3, analysis and discussions will be performed and some figures will be shown to illustrate the evolution of system 1 for understanding the virus propagation mechanism. Meanwhile, some intuitive countermeasures are presented to stop the virus spreading. Section 4 will be allotted for some concluding remarks.

ANALYSIS OF THE PROPOSED MODEL

Here, we will investigate the stability of the equilibrium of constant system 1 as follows:

For determination of the equilibrium of system 1, let:

$$\begin{cases} \frac{dS(t)}{dt} = \alpha S(t) - \omega S(t) \\ \frac{dE(t)}{dt} = \alpha S(t) - \delta E(t) - \beta E(t) \\ \frac{dI(t)}{dt} = \beta E(t) - \gamma I(t) - \varepsilon R(t)I(t) \\ \frac{dR(t)}{dt} = \omega S(t) - \delta E(t) - \gamma I(t) - \varepsilon R(t)I(t) \end{cases} \quad (2)$$

Considering the specific feature of the first two equations of model 2, the state equations in model 2 can be rewritten as below:

$$\frac{dI(t)}{dt} = rS(t) - \gamma I(t) - \varepsilon R(t)I(t) = 0$$
$$\frac{dR(t)}{dt} = \varepsilon R(t)I(t) - \lambda R(t) = 0 \quad (3)$$

Solving Eq. 2, we can obtain:

- When S = E = 0, solving Eq. 2, we can get that $\gamma I(t) = \varepsilon R(t)$. So we capture the non-trivial equilibrium:

$$I^+ = \left(0,\ 0,\ \frac{\lambda}{\varepsilon},\ \frac{r}{\lambda},\ \frac{\gamma}{\varepsilon}\right)$$

- Additionally, we can easily obtain that a trivial equilibrium:

$$I^0 = \left(0,\ 0,\ \frac{r}{\lambda},\ 0\right)$$

We have solved the two equilibriums for model 1. And then, we will check the stability of the equilibrium of model 1.

For the purpose of treating the stability of the equilibrium of system 3, we get the threshold:

$$T_0 = \frac{r\varepsilon}{\gamma\lambda}$$

which is viewed as reproductive number of virus. T_0 is a criterion to predict whether the influence of virus will be increasing or decreasing. Meanwhile, we build a set:

$$\Omega = \left\{(I,\ R)\,|\,I,\ R > 0,\ I + R \le \frac{r}{\gamma}\right\}$$

From the expression of T_0, we can infer that $T_0 \le 1$ or $T_0 > 1$. So we will analyze the stability of the equilibrium into the following two conditions:

- When $T_0 \le 1$, we only need to investigate the stability of:

$$I^0 = \left(0,\ 0,\ \frac{r}{\lambda},\ 0\right)$$

in system 2. Taking the equilibrium:

$$I^0 = \left(0,\ 0,\ \frac{r}{\lambda},\ 0\right)$$

to the origin by virtues of a coordinate transformation:

$$I' = I - \frac{r}{\gamma}$$

system 3 can be rewritten as follows:

$$\begin{aligned} \frac{dI'}{dt} &= -\gamma I - \varepsilon R\left(I' + \frac{r}{\gamma}\right) = 0 \\ \frac{dR}{dt} &= \varepsilon R\left(I' + \frac{r}{\gamma}\right) - \lambda R = 0 \end{aligned} \tag{4}$$

While $(I', R) \in \Omega' = \{(I', R)|I'R > 0, I' + R \le 0\}$. We take:

$$L_1 = \frac{I'^2}{2} + \frac{r}{\gamma}$$

as the Lyapunov function. Deriving L_1 with time t by means of system 4 results in:

$$\begin{aligned} \frac{dL_1}{dt} &= -\gamma I'^2 - \varepsilon\left(I' + \frac{r}{\gamma}\right)I'R + \frac{r}{\gamma}\left[\varepsilon R\left(I' + \frac{r}{\gamma}\right) - \lambda R\right] \\ &= -\gamma I'^2 - \varepsilon I'^2 R + \frac{r}{\gamma}R\frac{\varepsilon r + \lambda}{\gamma} \end{aligned}$$

Because $T_0 \le 1$ and $\varepsilon r - \lambda \le 0$. So the derived function of L_1 is not positive in Ω'. From the first equation of system 4, we can easily make a conclusion that R must be zero if I' = 0 is the solution of system 4. Therefore, there is no non-trivial solution in the set Ω in which each element satisfies the condition $dL_1/dt = 0$. And then the equilibrium I_0 is globally asymptotically stable in the neighborhood Ω

- When $T_0 > 1$, there is a positive equilibrium I^+ in addition to I_0 for system 3. Now we testify the stability of the equilibrium. For treating of the stability of I^+ in system 3, we transform the equilibrium to the origin by means of the following coordinate transformation for system 3:

$$\begin{cases} I' = I - I^+ \\ R' = R - R^+ \end{cases}$$

So that system 3 can be rewritten to the following form:

$$\begin{aligned} \frac{dI'}{dt} &= -\gamma I - \varepsilon I'\left(\frac{r}{\gamma} - \frac{\gamma}{\varepsilon}\right) - \lambda R' \\ \frac{dR'}{dt} &= \varepsilon I'\left(R' + \frac{r}{\lambda} - \frac{\gamma}{\varepsilon}\right) \end{aligned} \tag{5}$$

After that, taking:

$$L_2 = \frac{I'^2}{2} + \frac{\lambda}{\varepsilon}\left(R' - \frac{\ln R}{R'}\right)$$

as the Lyapunov function, we differentiate L_2 which get:

$$\frac{dL_2}{dt} = -\gamma I'^2 - \varepsilon I'^2 R^+$$

It is suggested that I' = 0 is necessary and sufficient for:

$$\frac{dL_2}{dt} = 0$$

Additionally, if I' = 0 is the solution of system 5, R' = 0 (i.e., $R = R^+$). So the set of solutions of system 5 meeting:

$$\frac{dL_2}{dt} = 0$$

does not include the non-trivial solution. Furthermore, we can get the conclusion that the equilibrium $/^+$ keeps global asymptotic stability in neighborhood Ω while $/_0$ is unstable

ANALYTICAL SOLUTION FOR SYSTEM 1

Solving Eq. 1a, we have:

$$S(t) = c_1 e^{-(\alpha+\omega)t} \tag{6}$$

where, c_1 is a constant that needs to be determined according to the initial condition. Here, S(0) = N, so we can get c_1 = N and $S(t) = N\ e^{-(\alpha+\omega)t}$. Submitting 6 into 1b and using the initial condition E(0) = 0 yields:

$$E(t) = \frac{N\alpha}{\alpha-\beta-\delta+\omega}[e^{-(\beta+\delta)t} - e^{-(\alpha+\omega)t}] \tag{7}$$

Taking (1d) into (1c), we get:

$$\frac{d[I(t)+R(t)]}{dt} = \omega S(t) + (\beta+\delta)E(t) \tag{8}$$

Submitting S(t) and E(t) into Eq. 8 yields:

$$G(t) = I(t) + R(t) = \frac{N}{\alpha-\beta-\delta+\omega}[(\beta+\delta-\omega)e^{-(\alpha+\omega)t} - \alpha e^{-(\beta+\delta)t} + N]$$

Assuming

$$R(t) = a_1 e^{-(\alpha+\omega)t} + b_1 e^{-(\beta+\delta)t} + c_1 e^{-(\gamma+\xi)t} + d_1$$

and taking it into (1c), we can obtain:

$$\begin{aligned} a_1 &= \frac{[(\delta-\omega)(\alpha-\gamma+\omega)-\beta(\gamma-\omega)]N}{(\alpha-\beta-\delta+\omega)(\alpha-\gamma-\xi+\omega)} \\ b_1 &= \frac{\alpha(\gamma-\delta)N}{(\alpha-\beta-\delta+\omega)(\alpha-\gamma-\xi+\omega)} \\ c_1 &= \frac{[(\delta-\omega)(\alpha-\gamma+\omega)-\beta(\gamma-\omega)]N}{(\gamma+\xi)(\beta-\gamma+\delta-\xi)(\alpha-\gamma-\xi+\omega)} \\ d_1 &= \frac{\gamma N}{\gamma+\xi} \end{aligned} \tag{9}$$

So, we can obtain that:

$$R(t) = G(t)\text{-}I(t)$$

where a1 and b1 are constants determined by Eq. 9.

So, far, we have derived the analytical solution for System (1) under the initial conditions:

$$S(0) = N,\ E(0) = 0,\ I(0) = 0,\ R(0) = 0$$

ANALYSIS AND DISCUSSION

The propagation of mobile phone virus is influenced by a lot of factors. In the following, we will make use of the analytic solution of system 1 to plot some figures to illustrate the evolution of system 1 and perform some analysis for understanding the propagation mechanism. The time evolution of the respective populations (percentage respect the total population) is displayed in Fig. 1b. It can be seen that the model tends to suitable equilibrium point as time goes to infinity since all the population becomes asymptotically recovered nodes, i.e., the infection would be cured in a relative short time period, approximately 50 days. As seen in Fig. 1b, S(t) decays exponentially from the initial value 1. As t approaches 15, S(t) drops to 1.8363 which almost vanishes. Since, the rate of reduction of S(t) corresponds to the transformation of susceptible nodes, the transformation proceeds rapidly.

The procedure of E(t) can be divided into two parts as shown in Fig. 1b. In the first stage, with t rushing from 0 to 12, E(t) surges from the initial value 0 to the maximum value. In the second stage, E(t) decreases and then slowly approaches the final value 0. In fact, it indicate that with occurrence of virus, most susceptible individuals have been attacked by virus and transfer into the exposed state. As time goes, the exposed individuals become infected. Time evolution of I(t) is similar to E(t). For recovered individuals, the growth rate is large at first and then smaller and smaller with the recovered individual into infected ones. At last, it approaches the final value 1 which represents that the spreading of mobile phone virus has finished completely.

CONCLUSION

Because of the mobility, mobile phone has some relevant characteristics: moving velocity, moving scope etc., which make the epidemic model of mobile phone virus very different from the model of computer virus and worm. We can make use of stochastic mobile model (such as Random Waypoint model, Random Direction model) to build spreading model of mobile phone virus. But these stochastic models have some limitations and can't accord with the fact preferably. For simplification of this problem, we make use of the extend SIR model to investigate the propagation of the mobile phone virus. Via., symbolic computation, we have derived an analytic solution for system 1. Figure is plotted to illustrate the evolution of mobile phone virus and analysis is performed for understanding the spreading mechanism. Through the analysis of this model, some measures can be taken for control of mobile phone virus by adjusting the propagation parameters. In summary, the results derived in this study are of interest in the complex network and need further experimental verification. We anticipate that those results might be of applications in the future development of mobile phone.

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